New Jersey Center for Teaching and Learning
Progressive Mathematics Initiative®

Algebra II

Quadratic Functions

2014-10-14

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Key Terms

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Key Terms

Quadratic Equation: An equation that can be written in the standard form $ax^2 + bx + c = 0$. Where a, b and c are real numbers and a does not equal 0.

$$ex: \quad 3x^2 + 5x - 12 = 0$$

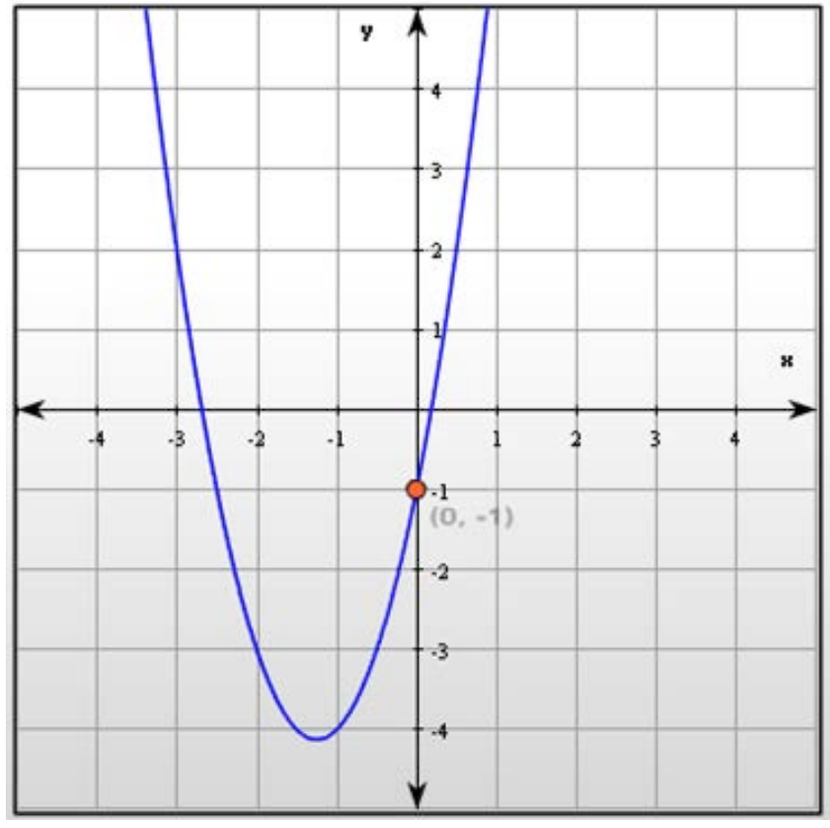
Quadratic Function: Any function that can be written in the form $y = ax^2 + bx + c$. Where a, b and c are real numbers and a does not equal 0.

$$ex: \quad y = -2x^2 + 10x + 7$$

Key Terms

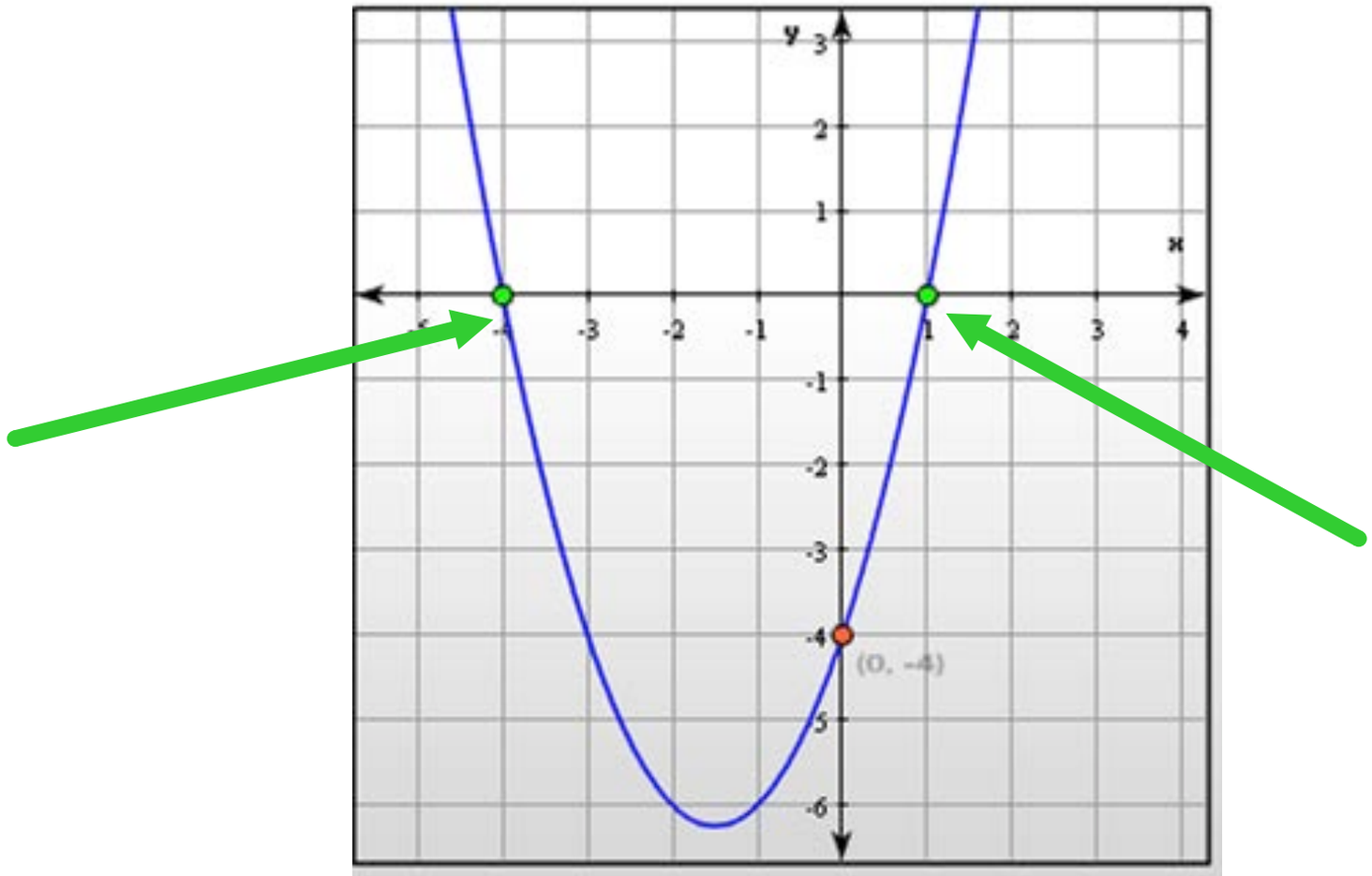
Parabola: The curve result of graphing a quadratic equation

$$y = 2x^2 + 5x - 1$$



Key Terms

Zero(s) of a Function: An x value that makes the function equal zero. Also called a "root," "solution" or "x-intercept"

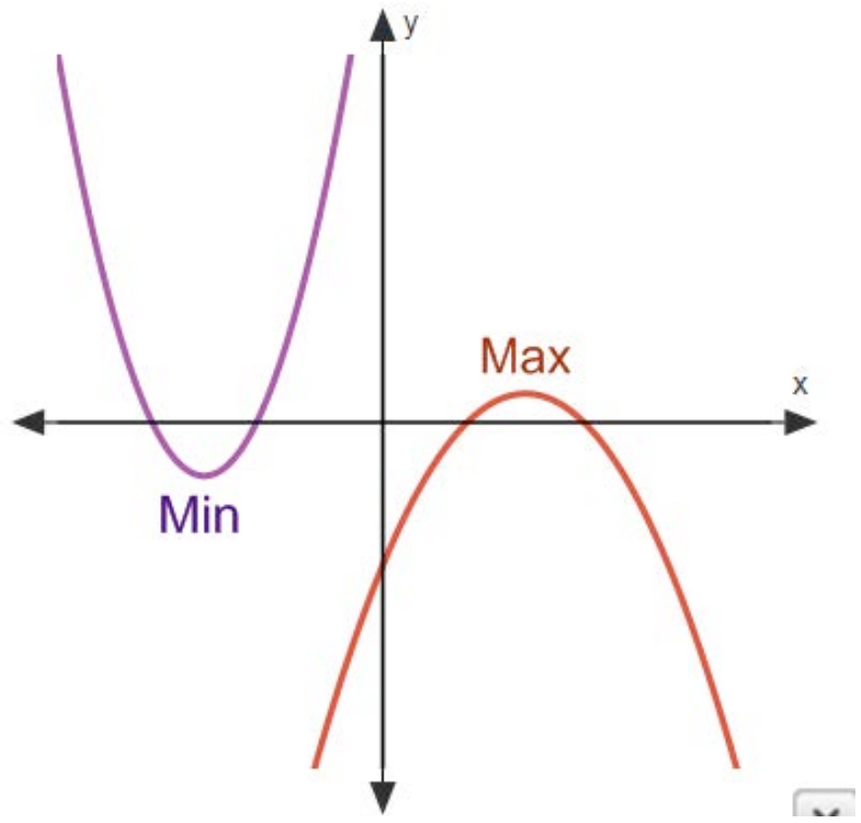


Key Terms

Vertex: The highest or lowest point on a parabola.

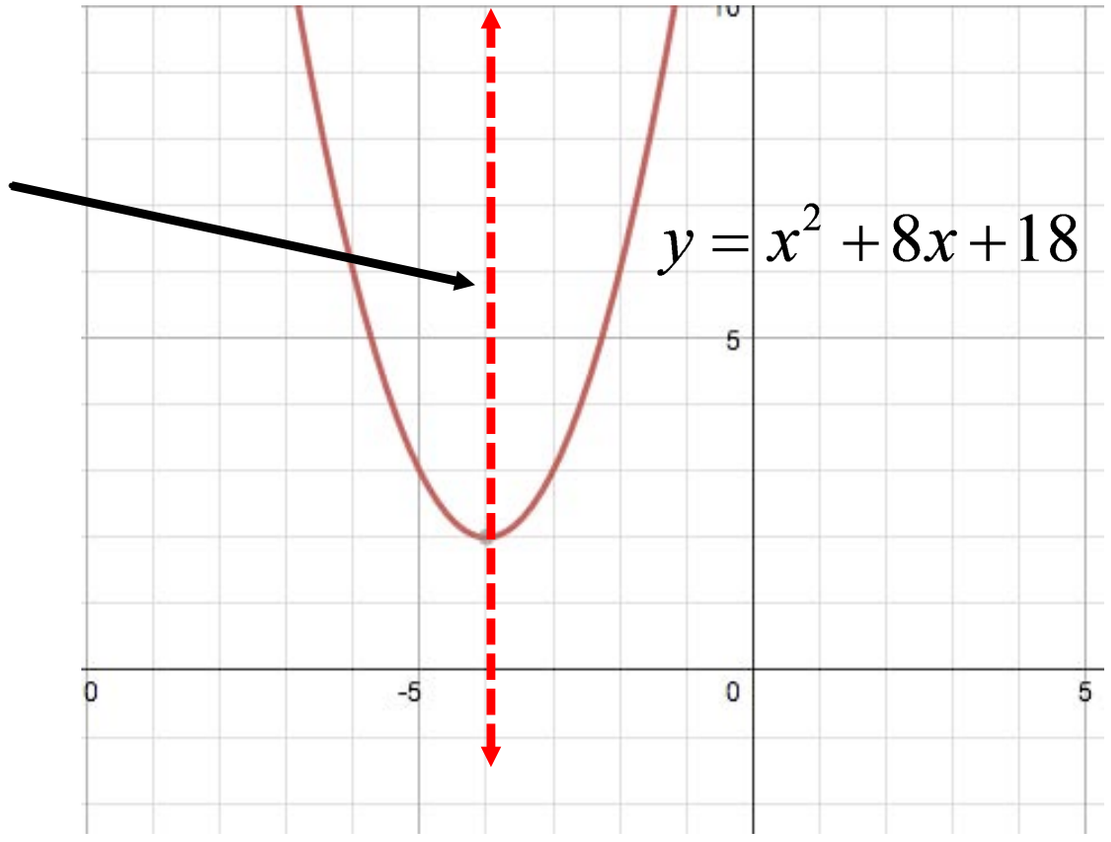
Minimum Value: The y-value of the vertex if a > 0 and the parabola opens upward

Maximum Value: The y-value of the vertex if a < 0 and the parabola opens downward



Key Terms

Axis of symmetry: The vertical line that divides a parabola into two symmetrical halves



Explain Characteristics of Quadratic Functions

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Characteristics of Quadratics

Remember: A quadratic equation is any equation that can be written in the form $ax^2 + bx + c = 0$
Where a, b, and c are real numbers and $a \neq 0$

Answer

Question 1: Is $2x^2 = x + 4$ a quadratic equation?

Question 2: Is $3x - 4 = x + 1$ a quadratic equation?

Characteristics of Quadratics

The form $ax^2 + bx + c = 0$ is called the standard form of a quadratic equation.

The standard form is not unique.

For example,

$x^2 - x + 1 = 0$ can also be written $-x^2 + x - 1 = 0$.

Also,

$4x^2 - 2x + 2 = 0$ can be written $2x^2 - x + 1 = 0$.

Standard Form

Practice writing quadratic equations in standard form:
(Simplify if possible.)

Write $2x^2 = x + 4$ in standard form:

Answer

Standard Form

Write $3x = -x^2 + 7$ in standard form, if possible:

Answer

Standard Form

Write $6x^2 - 6x = 12$ in standard form and simplify, if possible:

Answer

Standard Form

Write 3x - 2 = 5x in standard form:

Answer

Standard Form

Similar to Quadratic Equations, the standard form of a Quadratic Function is $y = ax^2 + bx + c$, where $a \neq 0$.
Notice, a can be positive or negative.

$$y = -3x^2 + 4x - 10$$

$$y = 5x^2 - 9$$

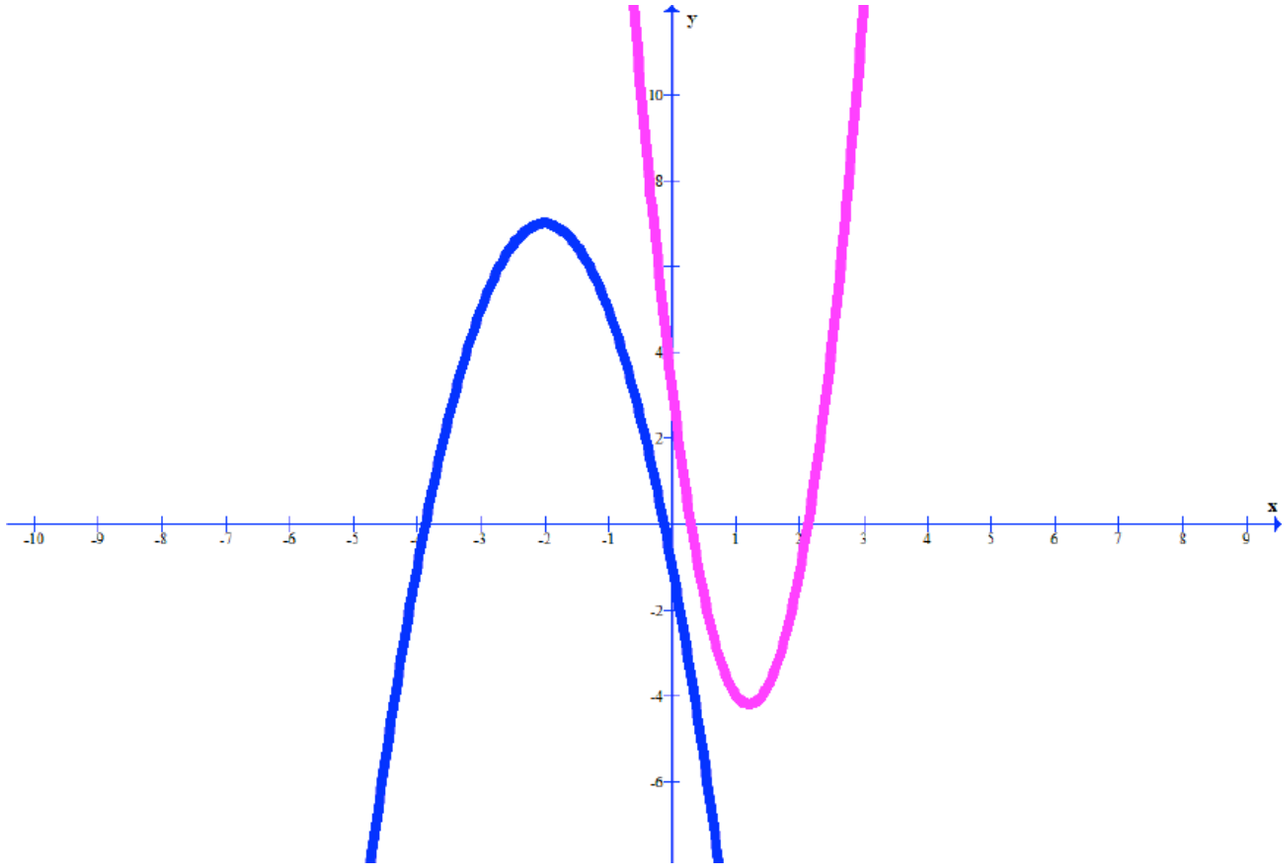
$$y = x^2$$

$$y = \frac{1}{4}x^2 + 5x - 20$$

Graph

When graphed, a quadratic function will make the shape of a parabola.

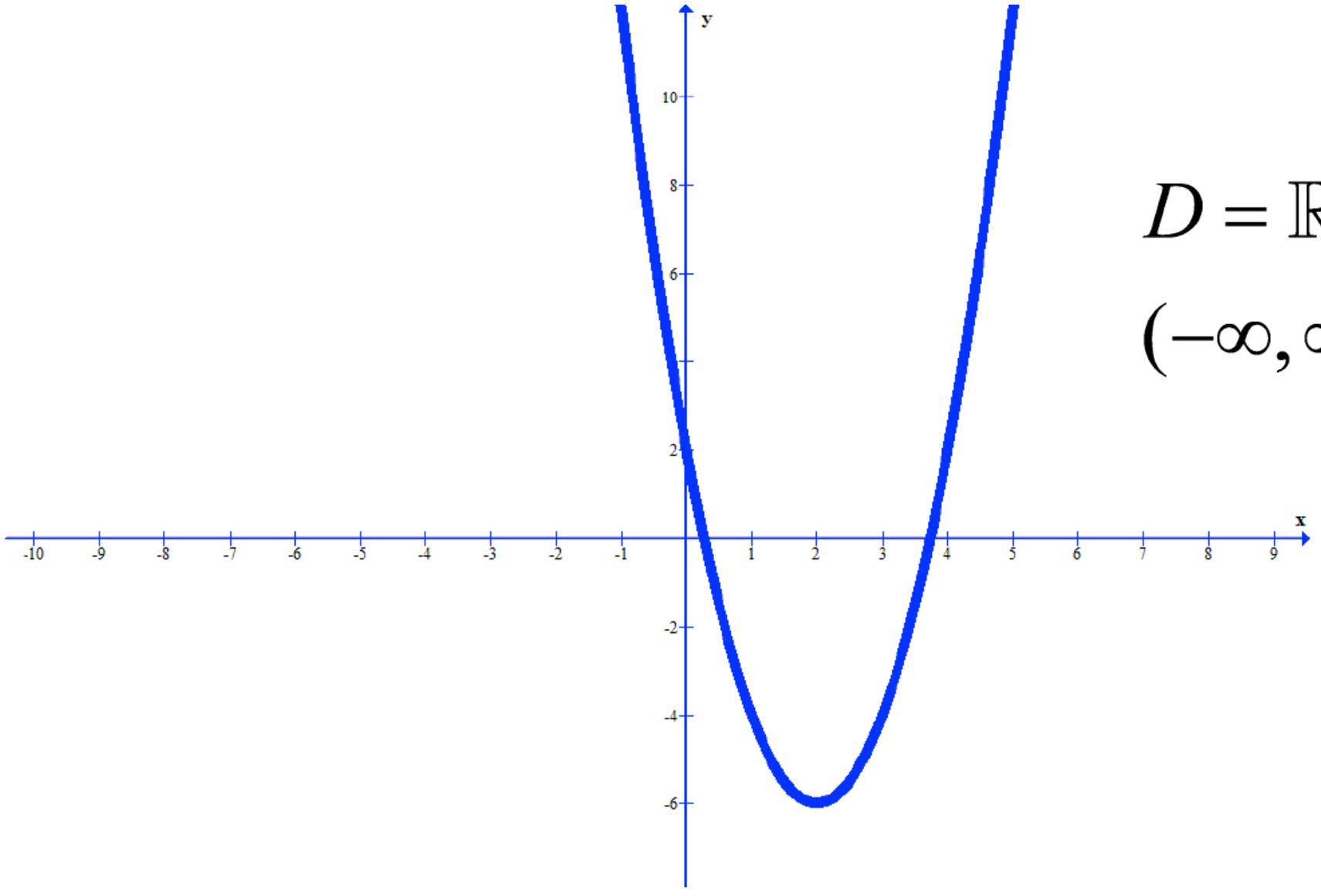
The parabola will open upward if a > 0 or downward if a < 0.



Teacher Notes

Domain

The domain of a quadratic function is all real numbers.



$$D = \mathbb{R}eals$$

$$(-\infty, \infty)$$

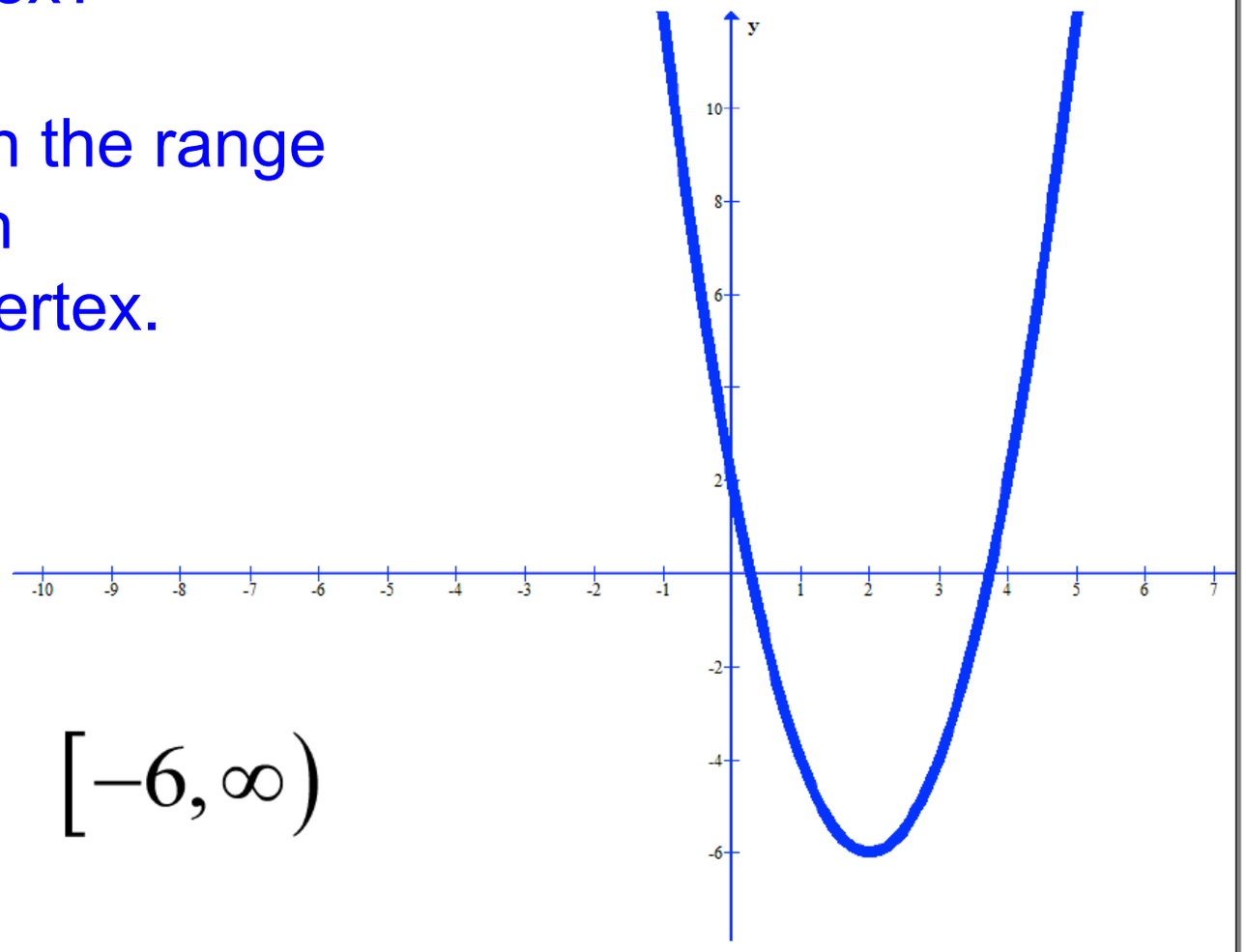
Range

To determine the range of a quadratic function, ask yourself two questions:

Is the vertex a minimum or maximum?

What is the y-value of the vertex?

If the vertex is a minimum, then the range is all real numbers greater than or equal to the y-value of the vertex.

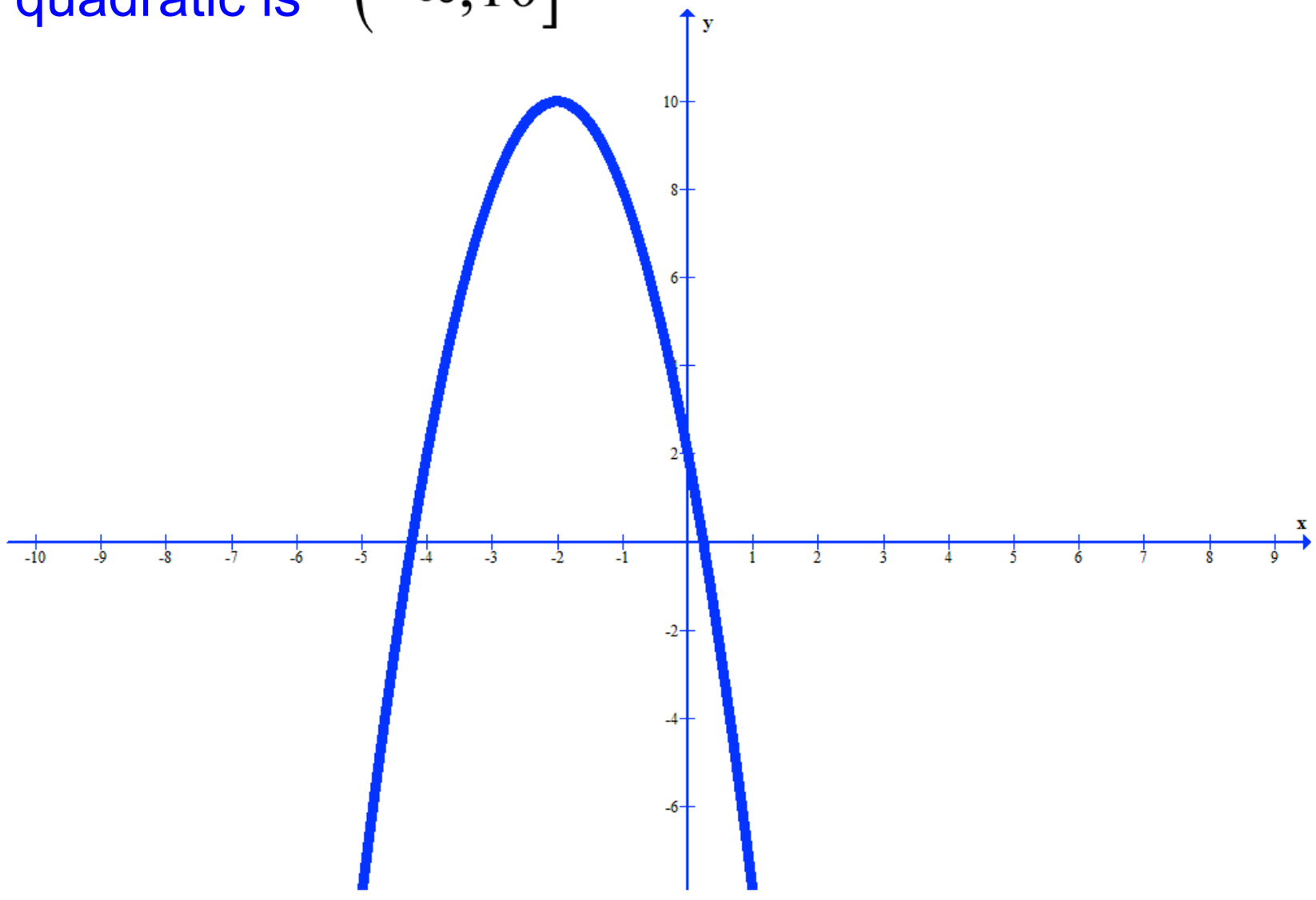


The range of this quadratic is $[-6, \infty)$

Range

If the vertex is a maximum, then the range is all real numbers less than or equal to the y-value of the vertex.

The range of this quadratic is $(-\infty, 10]$



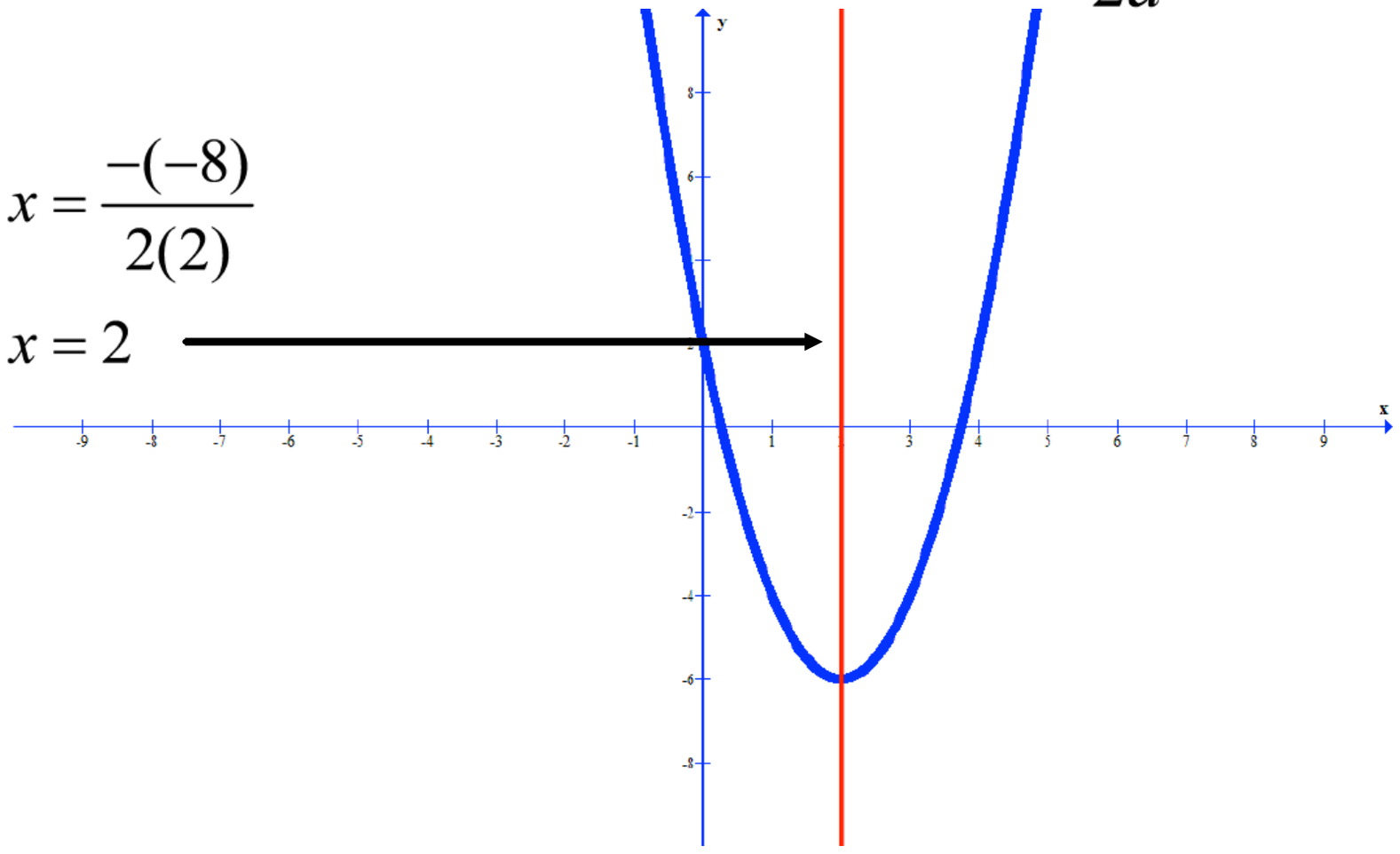
Axis of Symmetry

An axis of symmetry (also known as a line of symmetry) will divide the parabola into mirror images.

The line of symmetry is always a vertical line of the form $x = \frac{-b}{2a}$

$$y = 2x^2 - 8x + 2 \longrightarrow x = \frac{-(-8)}{2(2)}$$

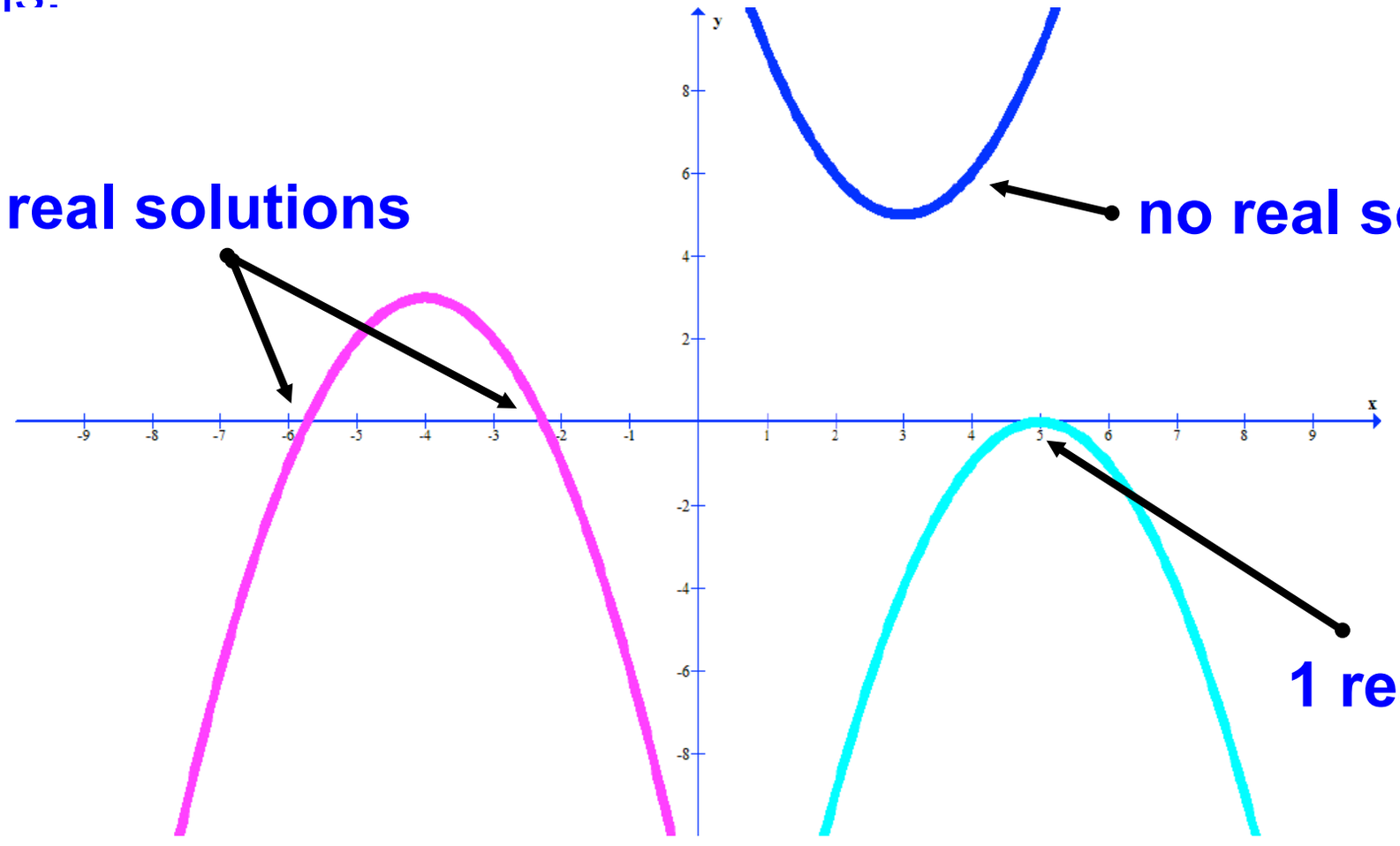
$$x = 2$$



X-Intercepts

The x-intercepts are the points at which a parabola intersects the x-axis. These points are also known as zeros, roots or solutions and solution sets. Each quadratic function will have 0, 1, or 2 or real solutions.

2 real solutions



no real solutions

1 real solution

1 If a parabola opens downward, the vertex is the highest value on the parabola.

True

False

Answer

2 If a parabola opens upward then...

A $a > 0$

B $a < 0$

C $a = 0$

Answer

3 The vertical line that divides a parabola into two symmetrical halves is called...

A discriminant

B quadratic equation

C axis of symmetry

D vertex

E maximum

Answer

4 Which of the following shows a quadratic equation correctly written in standard form?

A $3x - 5x^2 + 8 = 0$

B $3x - 5x + 8 = 0$

C $-5x^2 + 8 = -3x$

D $-5x^2 + 3x + 8 = 0$

E $3x = 5x^2 - 8$

Answer

5 What is the equation for the axis of symmetry for the quadratic function $y = 2x^2 + 12x - 7$?

A $x = 12$

B $x = -6$

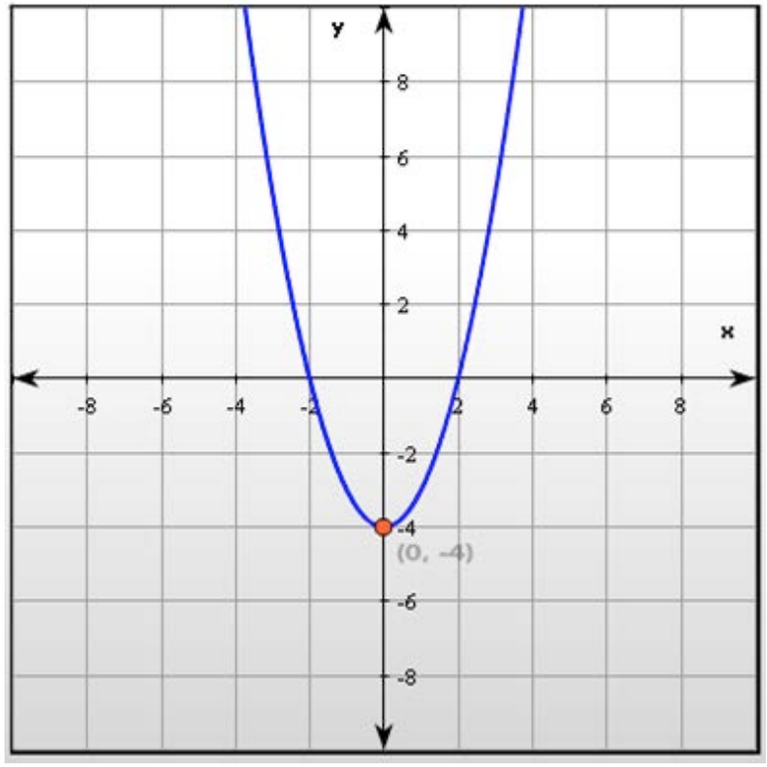
C $x = 2$

D $x = -3$

E $x = -7$

Answer

6 What is the domain of the quadratic function below?



A $[-4, \infty)$

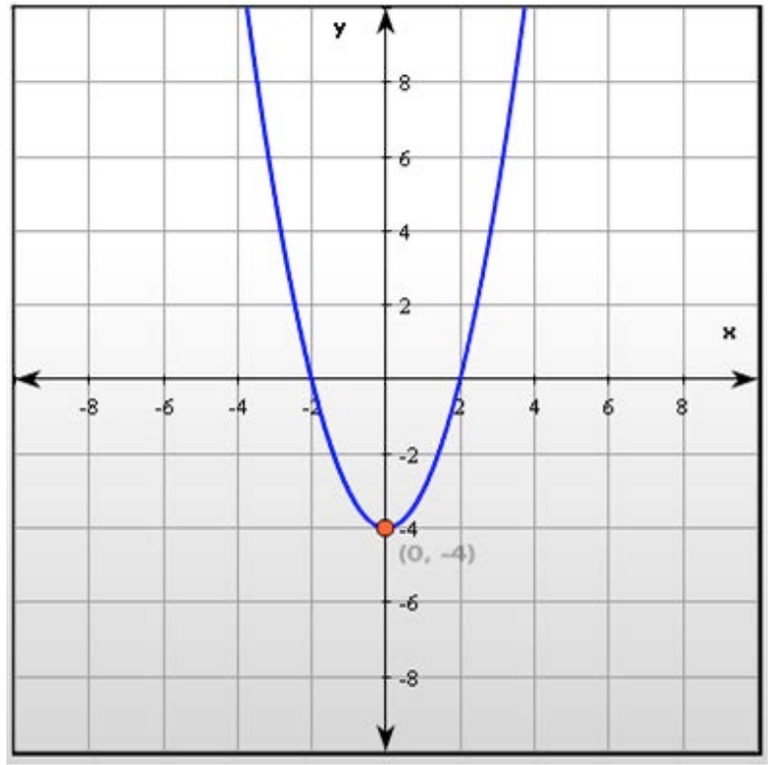
B $[-2, 2]$

C $(-\infty, 4]$

D $\mathbb{R}$

Answer

What is the range of the quadratic function below?



Answer

Graph Quadratic Functions

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Graph by Following Five Steps:

Step 1 - Find Axis of Symmetry

Step 2 - Find Vertex

Step 3 - Find y-intercept

Step 4 - Locate another point

Step 5 - Reflect and Connect

Graphing

Task: Graph $y = 3x^2 - 6x + 1$

Step 1: Find the Axis of Symmetry

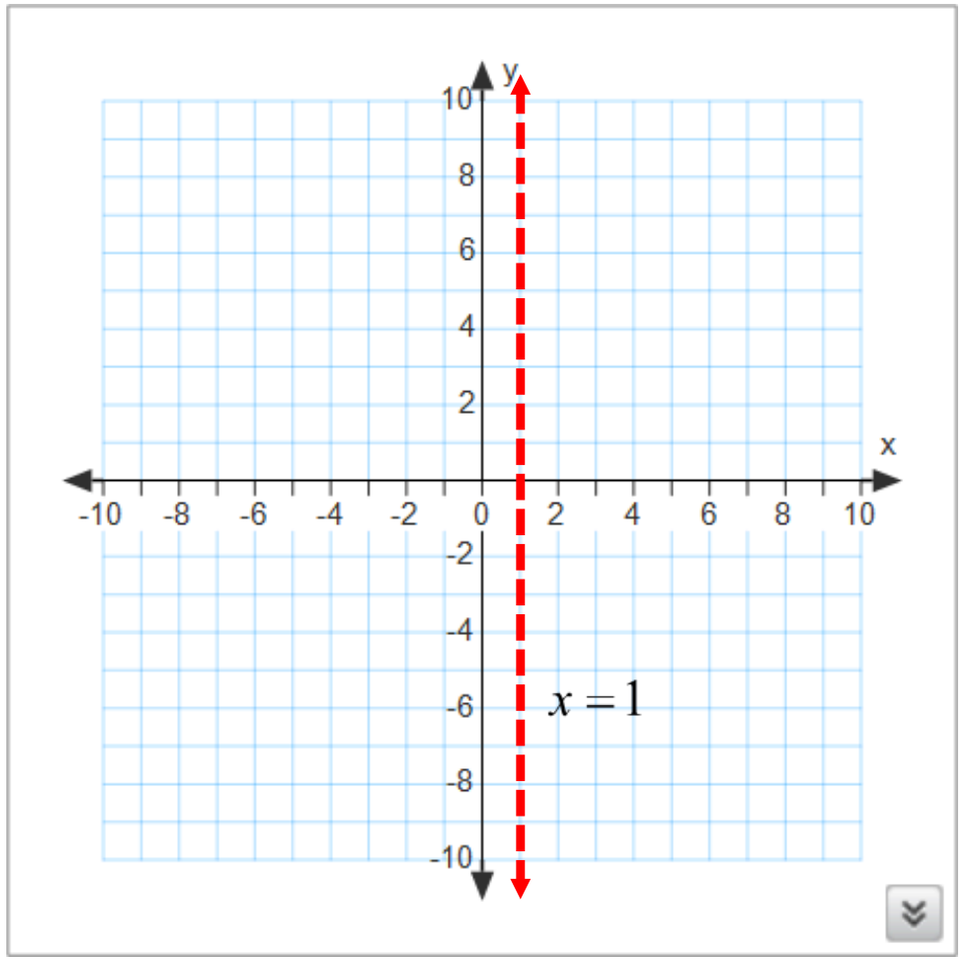
Recall the Formula: $x = \frac{-b}{2a}$

a = 3

b = -6

$x = \frac{-(-6)}{2(3)} = \frac{6}{6} = 1$

Therefore, the axis of symmetry is x = 1.



Graphing

Task: Graph $y = 3x^2 - 6x + 1$

Step 2: To find the vertex, substitute $\frac{-b}{2a}$ for x in the equation and find y.

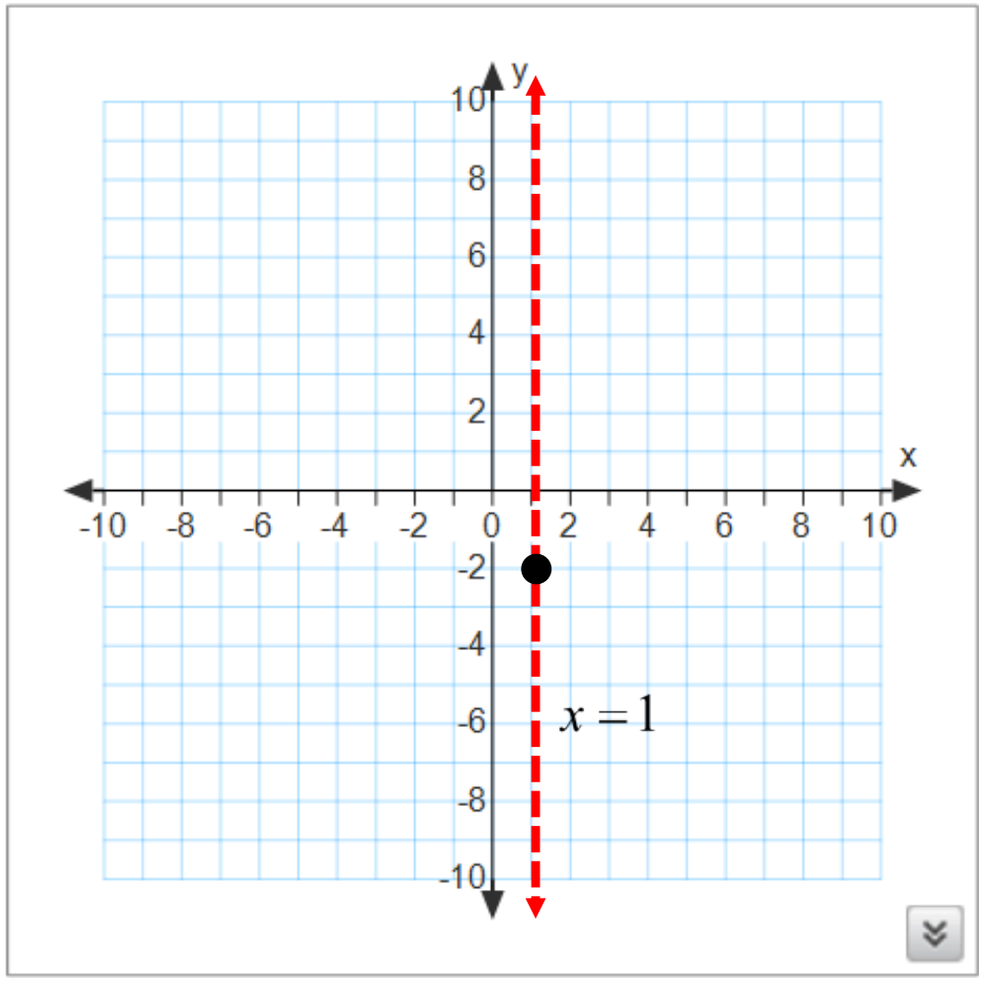
$y = 3x^2 - 6x + 1$

$y = 3(1)^2 + -6(1) + 1$

$y = 3 - 6 + 1$

$y = -2$

Vertex = (1 , -2)



Graphing

Task: Graph $y = 3x^2 - 6x + 1$

Step 3: Find the y-intercept.

The y-intercept occurs when x = 0, so substitute zero for x in the equation.

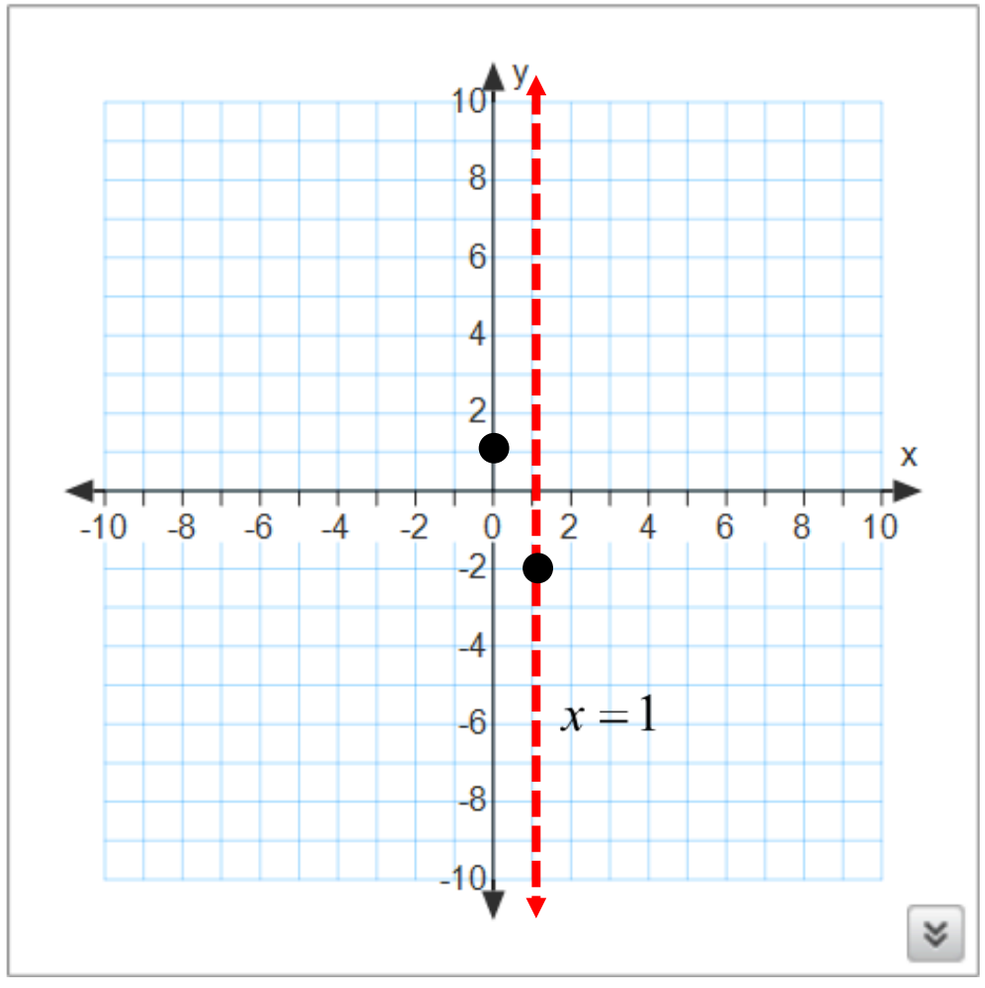
$y = 3x^2 - 6x + 1$

$y = 3(0)^2 + -6(0) + 1$

$y = 0 - 0 + 1$

$y = 1$

y intercept = (0, 1)



Graphing

Task: Graph $y = 3x^2 - 6x + 1$

Step 4: Plot an additional point.

Choose an x-value to substitute into the function.

Using x = -1

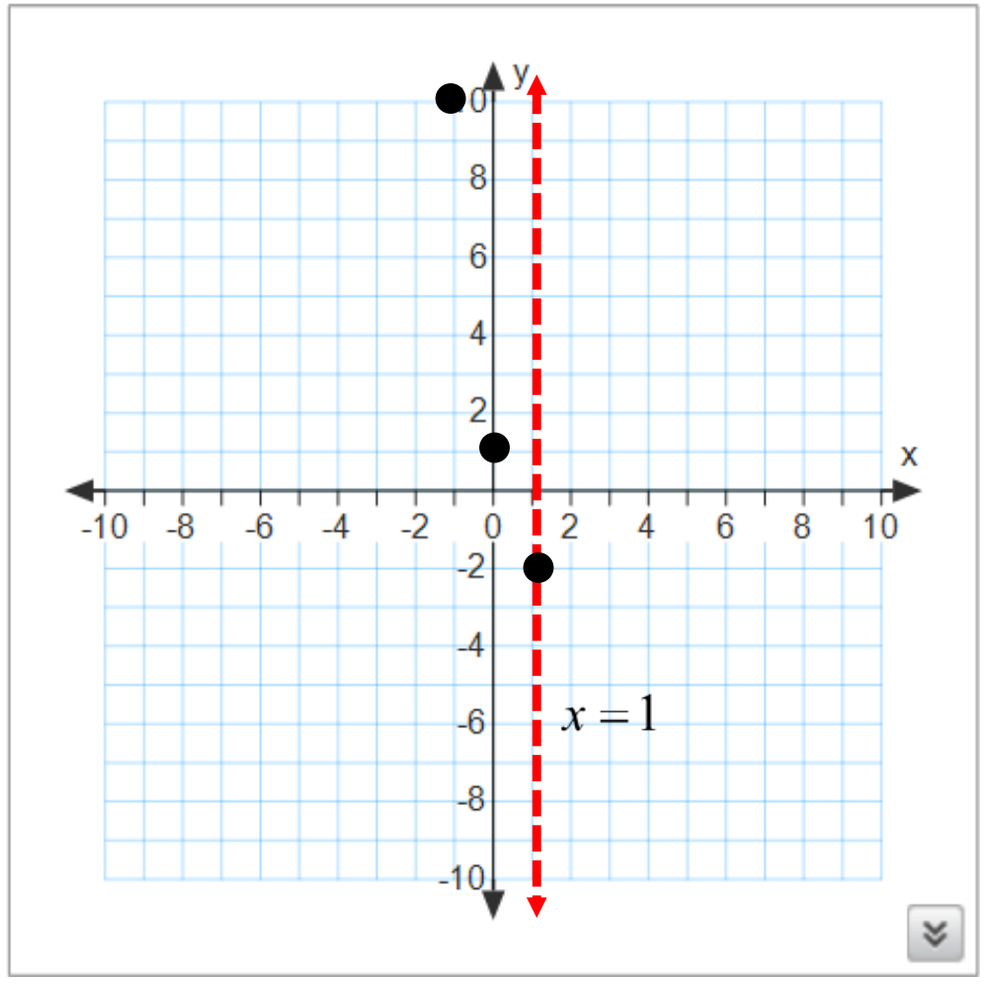
$y = 3x^2 - 6x + 1$

$y = 3(-1)^2 + -6(-1) + 1$

$y = 3 + 6 + 1$

$y = 10$

point = (-1, 10)



Graphing

Task: Graph $y = 3x^2 - 6x + 1$

Step 5: Using the axis of symmetry, reflect the points to get the other half of the parabola. Connect with a smooth curve.