Graph real numbers on a number line

Graph the real numbers
$$-\frac{5}{4}$$
 and $\sqrt{3}$ on a number line. SOLUTION

Note that $-\frac{5}{4} = -1.25$. Use a calculator to approximate $\sqrt{3}$ to the nearest tenth:

 $\sqrt{3} \approx 1.7$. (The symbol \approx means *is approximately equal to*.)

So, graph $-\frac{5}{4}$ between -2 and -1, and graph $\sqrt{3}$ between 1 and 2, as shown on the number line below.



http://www.classzone.com/cz/books/algebra_2_2011_na/book_home.htm

Standardized Test Practice

The table shows the lowest elevations of six continents. Which list shows the elevations from lowest to highest?

Continent	Africa	Asia	Australia	Europe	North America	South America
Lowest elevation	–156 m	-408 m	–16 m	–28 m	-86 m	-40 m
(A) −408, −156, −86, −28, −40, −16 (B) −408, −156, −28, −86, −40, −16						
C −16, −28, −40, −86, −156, −408 D				-408, -156, -86, -40, -28, -16		

SOLUTION

EXAMPLE 2

From lowest to highest, the elevations are -408, -156, -86, -40, -28, and -16.

ANSWER The correct answer is D. (A) (B) (C) (D)

1. Graph the numbers -0.2, $\frac{7}{10}$, -1, $\sqrt{2}$, and -4 on a number line.

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Graph Exponential Growth and Decay Functions for Examples 1 and 2

2. Which list shows the numbers in increasing order?

(A)
$$-0.5, 1.5, -2, -0.75, \sqrt{7}$$

(B) $-0.5, -2, -0.75, 1.5, \sqrt{7}$
(C) $-2, -0.75, -0.5, 1.5, \sqrt{7}$
(D) $\sqrt{7}, 1.5, -0.5, -0.75, -2$

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ANSWER The correct answer is C.

EXAMPLE 3 Identify properties of real numbers

Identify the property that the statement illustrates.

a.
$$7 + 4 = 4 + 7$$

SOLUTION Commutative property of addition

b.
$$13 \cdot \frac{1}{13} = 1$$

SOLUTION Inverse property of multiplication

Use properties and definitions of operations to show that a + (2 - a) = 2. Justify each step.

Use properties and definitions of operations

SOLUTION

EXAMPLE 4

$$a + (2 - a) = a + [2 + (-a)]$$
$$= a + [(-a) + 2]$$

Definition of subtraction

Commutative property of addition

Graph Exponential Growth and Decay Functions

- = [a + (-a)] + 2 Associative property of addition
- = 0 + 2 Inverse property of addition

= 2 Identity property of addition

Identify the property that the statement illustrates.

3.
$$(2 \cdot 3) \cdot 9 = 2 \cdot (3 \cdot 9)$$

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SOLUTION Associative property of multiplication.

4.
$$15 + 0 = 15$$

SOLUTION Identity property of addition.

Identify the property that the statement illustrates.

5.
$$4(5+25) = 4(5) + 4(25)$$

SOLUTION Distributive property.

6.
$$1 \cdot 500 = 500$$

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SOLUTION Identity property of multiplication.

Use properties and definitions of operations to show that the statement is true. *Justify* each step.

7.
$$b \quad (4 \div b) = 4$$
 when $b \neq 0$

SOLUTION

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$$b \cdot (4 \div b) = b \cdot (4 \cdot \frac{1}{b})$$
$$= b \cdot (\frac{1}{b}) \cdot 4$$
$$= (b \cdot \frac{1}{b}) \cdot 4$$
$$= 1 \cdot 4$$
$$= 4$$

Def. of division

Comm. prop. of multiplication

Assoc. prop. of multiplication Inverse prop. of multiplication Identity prop. of multiplication

Use properties and definitions of operations to show that the statement is true. *Justify* each step.

8.
$$3x + (6 + 4x) = 7x + 6$$

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SOLUTION

3x + (6 + 4x) = 3x + (4x + 6) Comm. prop. of addition = (3x + 4x) + 6 Assoc. prop. of addition = 7x + 6 Combine like terms.

Use unit analysis with operations

a. You work 4 hours and earn \$36. What is your earning rate?

SOLUTION

EXAMPLE 5

 $\frac{36 \text{ dollars}}{4 \text{ hours}} = 9 \text{ dollars per hour}$

b. You travel for 2.5 hours at 50 miles per hour. How far do you go?

SOLUTION

$$(2.5 \text{ hours}) \left(\frac{50 \text{ miles}}{1 \text{ hour}} \right) = 125 \text{ miles}$$



Use unit analysis with operations

c. You drive 45 miles per hour. What is your speed in feet per second?

SOLUTION

EXAMPLE 5



= 66 feet per second

Graph Exponential Growth and Decay Functions

Use unit analysis with conversions

Driving Distance

EXAMPLE 6

The distance from Montpelier, Vermont, to Montreal, Canada, is about 132 miles. The distance from Montreal to Quebec City is about 253 kilometers.

- a. Convert the distance from Montpelier to Montreal to kilometers.
- b. Convert the distance from Montreal to Quebec City to miles.



Graph Exponential Growth and Decay Functions

Use unit analysis with conversions

SOLUTION

EXAMPLE 6

- a. 132 miles $\cdot \frac{1.61 \text{ kilometers}}{1 \text{ mile}} \approx 213 \text{ kilometers}$
- **b.** 253 kilometers $\frac{1 \text{ mile}}{1.61 \text{ kilometers}} \approx 157 \text{ miles}$

Solve the problem. Use unit analysis to check your work.

9. You work 6 hours and earn \$69. What is your earning rate?

GUIDED PRACTICE

SOLUTION

- **SOLUTION** $\frac{69 \text{ dollars}}{6 \text{ hours}} = 11.5 \text{ dollars per hour}$
- **10.** How long does it take to travel 180 miles at 40 miles per hour?

$$\left(\frac{180 \text{ miles}}{40 \text{ miles}}\right) \left(\frac{1 \text{ hour}}{40 \text{ miles}}\right) = 4.5 \text{ hour}$$

Solve the problem. Use unit analysis to check your work.

11. You drive 60 kilometers per hour. What is your speed in miles per hour?

SOLUTION

GUIDED PRACTICE

$$\left(\frac{60 \text{ km}}{1 \text{ hour}}\right) \left(\frac{1 \text{ mile}}{1.61 \text{ km}}\right) = \text{about } 37 \text{ mph}$$

Perform the indicated conversion.

12. 150 yards to feet

GUIDED PRACTICE



= 450 **ft**

Perform the indicated conversion.

13. 4 gallons to pints

GUIDED PRACTICE

SOLUTION 4 gallon · 8 pints 1 gallon

= 32 **pints**

Graph Exponential Growth and Decay Functions for Examples 5 and 6

Perform the indicated conversion.

14. 16 years to seconds

GUIDED PRACTICE

SOLUTION

16 years $.\frac{365}{1}$ days $.\frac{24}{1}$ hours $.\frac{60}{1}$ minute $.\frac{60}{1}$ seconds1 year1 day1 hour1 minute

= 504,576,000 **sec**



a.
$$(-5)^4 = (-5) \cdot (-5) \cdot (-5) \cdot (-5) = 625$$

b.
$$-5^4 = -(5 \cdot 5 \cdot 5 \cdot 5) = -625$$

Graph Exponential Growth and Decay Functions

Evaluate an algebraic expression

Evaluate $-4x^2 - 6x + 11$ when x = -3.

EXAMPLE 2

 $-4x^{2}-6x + 11 = -4(-3)^{2}-6(-3) + 11$ Substitute -3 for x. = -4(9) -6(-3) + 11 Evaluate power. = -36 + 18 + 11 Multiply. = -7 Add.

Use a verbal model to solve a problem

Craft Fair

EXAMPLE 3

You are selling homemade candles at a craft fair for \$3 each. You spend \$120 to rent the booth and buy materials for the candles.

- Write an expression that shows your profit from selling *c* candles.
- Find your profit if you sell 75 candles.

Graph Exponential Growth and Decay Functions

Use a verbal model to solve a problem

SOLUTION

EXAMPLE 3

STEP 1 Write: a verbal model. Then write an algebraic expression. Use the fact that profit is the difference between income and expenses.



An expression that shows your profit is 3c - 120.

Graph Exponential Growth and Decay Functions

Use a verbal model to solve a problem

STEP 2 Evaluate: the expression in Step 1 when c = 75.

- 3c 120 = 3(75) 120 Substitute 75 for *c*.
 - = 225 120 Multiply.
 - = 105 **Subtract.**

ANSWER Your profit is \$105.

EXAMPLE 3



Graph Exponential Growth and Decay Functions for Examples 1, 2, and 3

Evaluate the expression.

1. 6³



$$6^3 = 6 \cdot 6 \cdot 6 = 216$$

2.
$$-2^6$$



$$-2^6 = -(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) = -64$$



3. (-2)⁶

SOLUTION

$$(-2)^6 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) = 64$$

4. 5x(x-2) when x = 6

SOLUTION

$$5x(x-2) = 5(6) (6-2)$$
$$= 30 (4)$$
$$= 120$$

Substitute 6 for *x*.

Graph Exponential Growth and Decay Functions

Multiply.

for Examples 1, 2, and 3

GUIDED PRACTICE for

for Examples 1, 2, and 3

5.
$$3y^2 - 4y$$
 when $y = -2$

SOLUTION

$$3y^2 - 4y = 3(-2)^2 - 4(-2)$$
$$= 3(4) + 8$$
$$= 20$$

Substitute –2 for *y*.

Multiply.

GUIDED PRACTICE

6. $(z + 3)^3$ when z = 1

SOLUTION

$$(z + 3)^3 = (1 + 3)^3$$

= $(4)^3$
= 64

Substitute 1 for *z*.

for Examples 1, 2, and 3

Evaluate Power.

7. What If? In Example 3, find your profit if you sell 135 candles.

SOLUTION

GUIDED PRACTICE

STEP 1 Write a verbal model. Then write an algebraic expression. Use the fact that profit is the difference between income and expenses.





An expression that shows your profit is 3c - 120.

STEP 2 Evaluate: the expression in Step 1 when c = 135.

- 3c 120 = 3(135) 120 Substitute 135 for *c*.
 - = 405 120 Multiply.
 - = 185 **Subtract.**
- **ANSWER** Your profit is \$185.

GUIDED PRACTICE

Simplify by combining like terms

a. 8x + 3x = (8 + 3)x **Distributive property**

EXAMPLE 4

- = 11x Add coefficients.
- **b.** $5p^2 + p 2p^2 = (5p^2 2p^2) + p$ Group like terms. = $3p^2 + p$ Combine like terms.
- c. 3(y+2) 4(y-7) = 3y + 6 4y + 28 Distributive property
 - = (3y 4y) + (6 + 28) Group like terms.

= -y + 34 Combine like terms.

Simplify by combining like terms

d. 2x - 3y - 9x + y = (2x - 9x) + (-3y + y) Group like terms.

EXAMPLE 4)

=-7x-2y Combine like terms.

GUIDED PRACTICE

8. Identify the terms, coefficients, like terms, and constant terms in the expression $2 + 5x - 6x^2 + 7x - 3$. Then simplify the expression.

for Example 5

SOLUTION

Terms:2, 1Coefficients:5 fLike terms:5xConstants:2 aSimplify: $2 + 5x - 6x^2 + 7x - 3$ $= -6x^2 + 5x + 7x - 3 + 2$

 $=-6x^2+12x-1$

2, 5*x*, $-6x^2$, 7*x*, -3

- 5 from 5x, -6 from $-6x^2$, 7 from 7x
- 5*x* and 7*x*, 2 and –3
- 2 and -3

State the problem. Group like terms. Combine like terms.

Simplify the expression.

GUIDED PRACTICE

9. 15*m* – 9*m*



15m - 9m = 6m Combine like terms.

for Example 5

10. 2n-1+6n+5

SOLUTION

$$2n-1+6n+5 = 2n+6n+5-1$$
 Group like terms.
= $8n+4$ Combine like terms.

11. $3p^3 + 5p^2 - p^3$

GUIDED PRACTICE

SOLUTION

$$3p^3 + 5p^2 - p^3 = 3p^3 - p^3 + 5p^2$$
 Group like terms.
= $2p^3 + 5p^3$ Combine like terms.

for Example 5

12.
$$2q^2 + q - 7q - 5q^2$$

SOLUTION

$$2q^2 + q - 7q - 5q^2 = 2q^2 - 5q^2 - 7q + q$$
 Group like terms.
= $-3q^2 - 6q$ Combine like terms.

13. 8(x-3) - 2(x+6)

GUIDED PRACTICE

SOLUTION

$$8(x-3) - 2(x+6) = 8x - 24 - 2x - 12$$

Distributive property
$$= 8x - 2x - 24 - 12$$

Group like terms.
$$= 6x - 36$$

Combine like terms.

for Example 5

14.
$$-4y - x + 10x + y$$

SOLUTION

$$-4y - x + 10x + y = -4y + y - x + 10x$$
 Group like terms.
= $9x - 3y$ Combine like terms.

Graph Exponential Growth and Decay Functions

Simplify a mathematical model

Digital Photo Printing

EXAMPLE 5

You send 15 digital images to a printing service that charges \$.80 per print in large format and \$.20 per print in small format. Write and simplify an expression that represents the total cost if n of the 15 prints are in large format. Then find the total cost if 5 of the 15 prints are in large format.



Simplify a mathematical model

SOLUTION

EXAMPLE 5

Write a verbal model. Then write an algebraic expression.



An expression for the total cost is 0.8n + 0.2(15 - n).

$$0.8n + 0.2(15 - n) = 0.8n + 3 - 0.2n$$
 Distributive property

= (0.8n - 0.2n) + 3 Group like terms.

Graph Exponential Growth and Decay Functions

EXAMPLE 5 Simplify a mathematical model

= 0.6*n* + 3

Combine like terms.

ANSWER

When n = 5, the total cost is 0.6(5) + 3 = 3 + 3 = \$6.

15. What If? In Example 5, write and simplify an expression for the total cost if the price of a large print is \$.75 and the price of a small print is \$.25.

for Example 5

SOLUTION

GUIDED PRACTICE

Write a verbal model. Then write an algebraic expression.



An expression for the total cost is 0.75n + 0.25(15 - n).

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0.75n + 0.25(15 - n) = 0.75n + 3.75 - 0.2n Distributive property.

for Example 5

= 7.5n - 0.25n + 3.75 Group like terms.

= 0.5n + 3.75 Combine like terms.