EXAMPLE 1 Graph a function of the form $y = ax^2$

Graph $y = 2x^2$. Compare the graph with the graph of $y = x^2$.

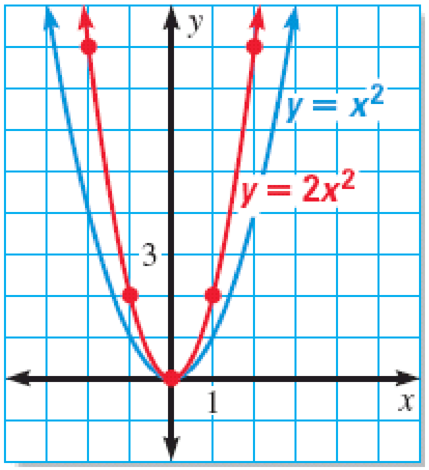
SOLUTION

STEP 1 Make a table of values for $y = 2x^2$.

x	–2	–1	0	1	2
y	8	2	0	2	8

STEP 2 Plot the points from the table.

STEP 3 Draw a smooth curve through the points.



EXAMPLE 1 Graph a function of the form $y = ax^2$

STEP 4 Compare the graphs of $y = 2x^2$ and $y = x^2$. Both open up and have the same vertex and axis of symmetry. The graph of $y = 2x^2$ is narrower than the graph of $y = x^2$.

EXAMPLE 2 Graph a function of the form $y = ax^2 + c$

Graph $y = -\frac{1}{2}x^2 + 3$ Compare the graph with the graph of $y = x^2$

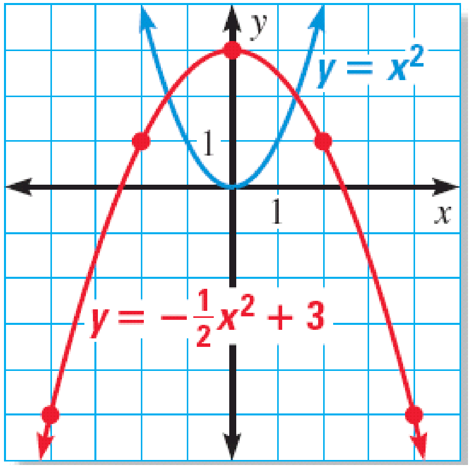
SOLUTION

STEP 1 Make a table of values for $y = -\frac{1}{2}x^2 + 3$

x	−4	−2	0	2	4
y	−5	1	3	1	−5

STEP 2 Plot the points from the table.

STEP 3 Draw a smooth curve through the points.



EXAMPLE 2 Graph a function of the form $y = ax^2$

STEP 4 **Compare the graphs of** $y = -\frac{1}{2}x^2 + 3$ **and** $y = x^2$.

Both graphs have the same axis of symmetry. However, the graph of $y = -\frac{1}{2}x^2 + 3$ **opens down and is wider than the graph of** $y = x^2$. **Also, its vertex is** 3 **units higher.**

GUIDED PRACTICE for Examples 1 and 2

Graph the function. Compare the graph with the graph of $y = x^2$.

1. $y = -4x^2$

SOLUTION

STEP 1 **Make a table of values for $y = -4x^2$.**

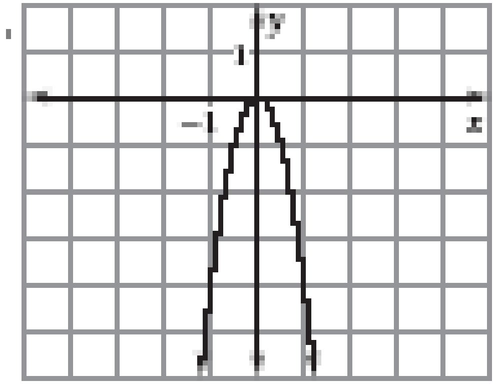
X	– 2	– 1	0	2	– 1
Y	– 16	– 4	0	– 16	– 4

GUIDED PRACTICE for Examples 1 and 2

STEP 2 Plot the points from the table.

STEP 3 Draw a smooth curve through the points.

STEP 4 Compare the graphs of $y = -4x^2$ and $y = x^2$.



ANSWER

Same axis of symmetry and vertex, opens down, and is narrower

GUIDED PRACTICE for Examples 1 and 2

2. $y = -x^2 - 5$

SOLUTION

STEP 1 Make a table of values for $y = -x^2 - 5$.

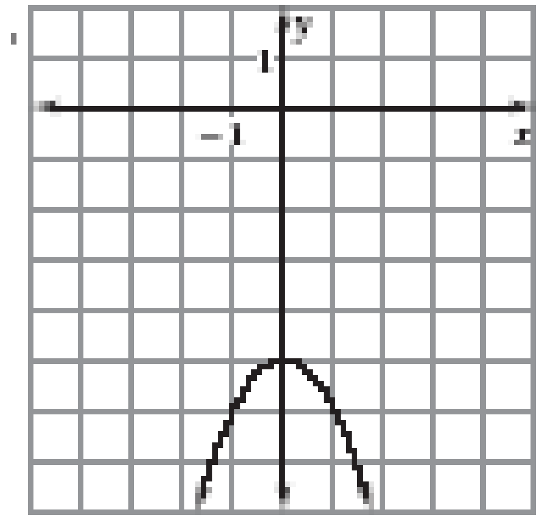
X	– 2	– 1	0	2	– 1
Y	– 9	– 6	– 5	– 9	– 6

STEP 2 Plot the points from the table.

STEP 3 Draw a smooth curve through the points.

STEP 4 Compare the graphs of $y = -x^2 - 5$ and $y = x^2$.

GUIDED PRACTICE for Examples 1 and 2



ANSWER

Same axis of symmetry, vertex is shifted down 5 units, and opens down

GUIDED PRACTICE for Examples 1 and 2

3. $f(x) = \frac{1}{4}x^2 + 2$

SOLUTION

STEP 1 Make a table of values for $f(x) = \frac{1}{4}x^2 + 2$

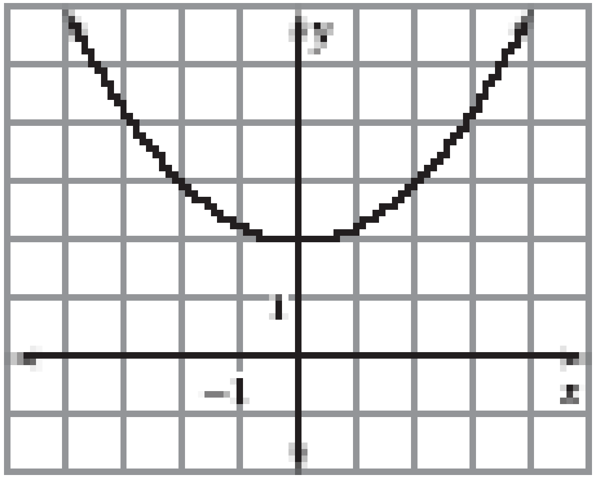
X	– 4	– 2	0	– 4	2
Y	– 2	4	2	6	4

STEP 2 Plot the points from the table.

STEP 3 Draw a smooth curve through the points.

STEP 4 Compare the graphs of $f(x) = \frac{1}{4}x^2 + 2$ and $y = x^2$.

GUIDED PRACTICE for Examples 1 and 2



ANSWER

Same axis of symmetry, vertex is shifted up 2 units, opens up, and is wider

EXAMPLE 3 Graph a function of the form $y = ax^2 + bx + c$

Graph $y = 2x^2 - 8x + 6$.

SOLUTION

STEP 1 Identify the coefficients of the function. The coefficients are $a = 2$, $b = -8$, and $c = 6$. Because $a > 0$, the parabola opens up.

STEP 2 Find the vertex. Calculate the x - coordinate.

$$x = -\frac{b}{2a} = -\frac{(-8)}{2(2)} = 2$$

Then find the y - coordinate of the vertex.

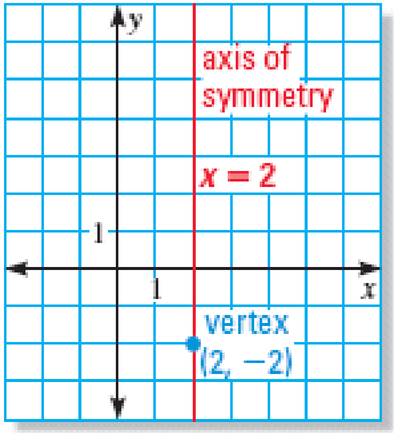
$$y = 2(2)^2 - 8(2) + 6 = -2$$

So, the vertex is $(2, -2)$. Plot this point.

EXAMPLE 3 Graph a function of the form $y = ax^2 + bx + c$

STEP 3 **Draw the axis of symmetry** $x = 2$.

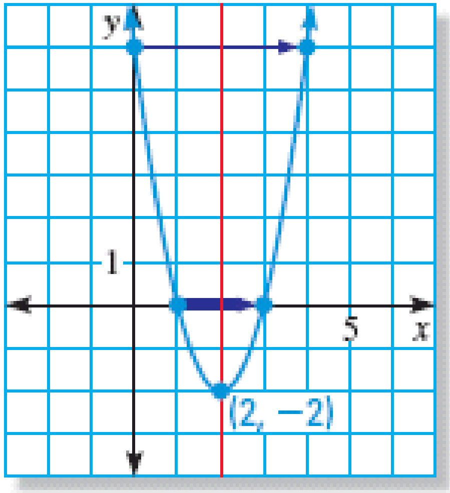
STEP 4 **Identify the** y **- intercept** c**, which is** 6**. Plot the point** (0, 6)**. Then reflect this point in the axis of symmetry to plot another point,** (4, 6)**.**



STEP 5 **Evaluate the function for another value of** x**, such as** $x = 1$.

$$y = 2(1)^2 - 8(1) + 6 = 0$$

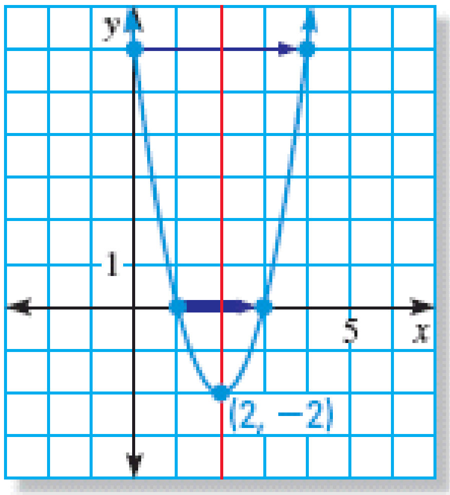
Plot the point (1, 0) **and its reflection** (3, 0) **in the axis of symmetry.**



http://www.classzone.com/cz/books/algebra_2_2011_na/book_home.htm

EXAMPLE 3 Graph a function of the form $y = ax^2 + bx + c$

STEP 6 Draw a parabola through the plotted points.



Graph the function. Label the vertex and axis of symmetry.

4. $y = x^2 - 2x - 1$

SOLUTION

STEP 1 **Identify the coefficients of the function. The coefficients are $a = 1$, $b = -2$, and $c = -1$. Because $a > 0$, the parabola opens up.**

STEP 2 **Find the vertex. Calculate the x - coordinate.**

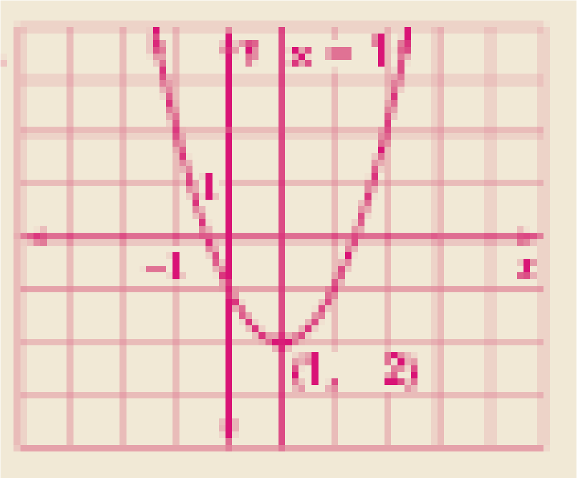
$$x = \frac{b}{2a} = -\frac{(-2)}{2(1)} = 1$$

Then find the y - coordinate of the vertex.

$$y = 1^2 - 2 \cdot 1 + 1 = -2$$

So, the vertex is $(1, -2)$. Plot this point.

STEP 3 Draw the axis of symmetry $x = 1$.



GUIDED PRACTICE for Example 3

5. $y = 2x^2 + 6x + 3$

SOLUTION

STEP 1 **Identify the coefficients of the function. The coefficients are $a = 2$, $b = 6$, and $c = 3$. Because $a > 0$, the parabola opens up.**

STEP 2 **Find the vertex. Calculate the x - coordinate.**

$$x = \frac{-b}{2a} = \frac{-6}{2 \cdot 2} = \frac{-3}{2}$$

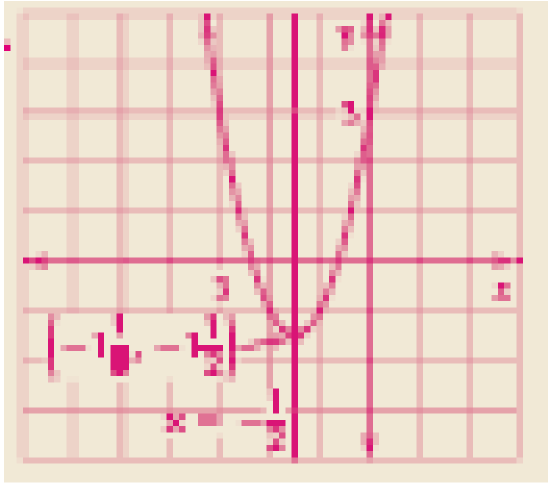
Then find the y - coordinate of the vertex.

$$y = 2 \cdot \left(\frac{-3}{2}\right) + 6 \cdot \left(\frac{-3}{2}\right) + 3 = -9$$

So, the vertex is $\frac{-3}{2}, -9$. Plot this point.

GUIDED PRACTICE for Example 3

STEP 3 **Draw the axis of symmetry** $x = \frac{-3}{2}$



GUIDED PRACTICE for Example 3

6. $f(x) = -\frac{1}{3}x^2 - 5x + 2$

SOLUTION

STEP 1 **Identify the coefficients of the function. The coefficients are** $a = -\frac{1}{3}$, $b = -5$, **and** $c = 2$ **.**
Because $a > 0$**, the parabola opens up.**

STEP 2 **Find the vertex. Calculate the** x **- coordinate.**

$$x = \frac{-b}{2a} = \frac{(-5)}{2 \cdot \left(\frac{-3}{2}\right)} = \frac{15}{2}$$

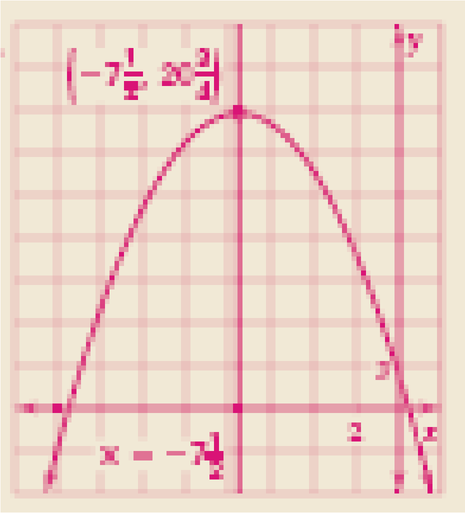
Then find the y **- coordinate of the vertex.**

GUIDED PRACTICE for Example 3

$$y = \frac{-3}{2}\left(\frac{15}{2}\right) - 5\left(\frac{15}{2}\right) + 2 = \frac{-76}{2}$$

So, the vertex is $\frac{15}{2}, \frac{-76}{2}$ **. Plot this point.**

STEP 3 **Draw the axis of symmetry** $x = \frac{15}{2}$



EXAMPLE 4 Find the minimum or maximum value

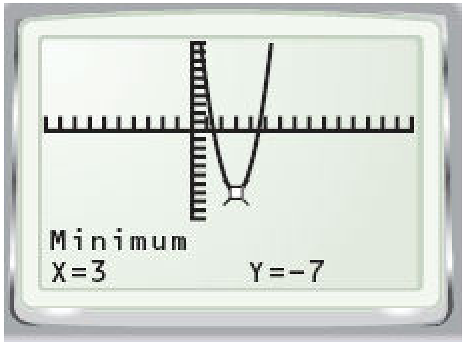
Tell whether the function $y = 3x^2 - 18x + 20$ has a *minimum value* or a *maximum value*. Then find the minimum or maximum value.

SOLUTION

Because $a > 0$, the function has a minimum value. To find it, calculate the coordinates of the vertex.

$$x = -\frac{b}{2a} = -\frac{(-18)}{2a} = 3$$

$$y = 3(3)^2 - 18(3) + 20 = -7$$



ANSWER

The minimum value is $y = -7$. You can check the answer on a graphing calculator.

EXAMPLE 5 Solve a multi-step problem

Go - Carts

A go-cart track has about 380 racers per week and charges each racer \$35 to race. The owner estimates that there will be 20 more racers per week for every \$1 reduction in the price per racer. How can the owner of the go-cart track maximize weekly revenue ?

EXAMPLE 5 Solve a multi-step problem

SOLUTION

STEP 1 Define the variables. Let x represent the price reduction and $R(x)$ represent the weekly revenue.

STEP 2 Write a verbal model. Then write and simplify a quadratic function.

Revenue (dollars)	=	Price (dollars/racer)	•	Attendance (racers)
$R(x)$	=	$(35 - x)$	•	$(380 + 20x)$

$$R(x) = 13{,}300 + 700x - 380x - 20x^2$$

$$R(x) = -20x^2 + 320x + 13{,}300$$

EXAMPLE 5 Solve a multi-step problem

STEP 3 **Find the coordinates $(x, R(x))$ of the vertex.**

$$x = -\frac{b}{2a} = -\frac{320}{2(-20)} = 8$$

Find x - coordinate.

$$R(8) = -20(8)^2 + 320(8) + 13{,}300 = 14{,}580$$

Evaluate $R(8)$.

ANSWER

The vertex is (8, 14,580), which means the owner should reduce the price per racer by \$8 to increase the weekly revenue to \$14,580.

GUIDED PRACTICE for Examples 4 and 5

7. Find the minimum value of $y = 4x^2 + 16x - 3$.

SOLUTION

Because $a > 0$**, the function has a minimum value. To find it, calculate the coordinates of the vertex.**

$$x = -\frac{b}{2a} = -\frac{16}{2a} = -2$$

$$y = 4(-2)^2 + 16(-2) - 3 = -19$$

ANSWER

The minimum value is $y = -19$**. You can check the answer on a graphing calculator.**

GUIDED PRACTICE for Examples 4 and 5

8. **What If ? In Example 5, suppose each $1 reduction in the price per racer brings in 40 more racers per week. How can weekly revenue be maximized?**

SOLUTION

STEP 1 **Define the variables. Let x represent the price reduction and $R(x)$ represent the weekly revenue.**

GUIDED PRACTICE for Examples 4 and 5

STEP 2 **Write a verbal model. Then write and simplify a quadratic function.**

Revenue (dollars) = Price (dollars/racer) · Attendance (racers)

$$R(x) = -20x^2 + 1020x + 13{,}300$$

GUIDED PRACTICE for Examples 4 and 5

STEP 3 **Find the coordinates $(x, R(x))$ of the vertex.**

$$x = -\frac{b}{2a} = -\frac{1020x}{2(-40)} = 12.5$$

Find x - coordinate.

Evaluate $R(12.75)$.

$$R(12.75) = -40(12.75) + 1020(12.75) + 13{,}300 = 19802.5$$

ANSWER

The vertex is (12.75, 19,802.5), which means the owner should reduce the price per racer by \$12.75 to increase the weekly revenue to \$19,802.50.