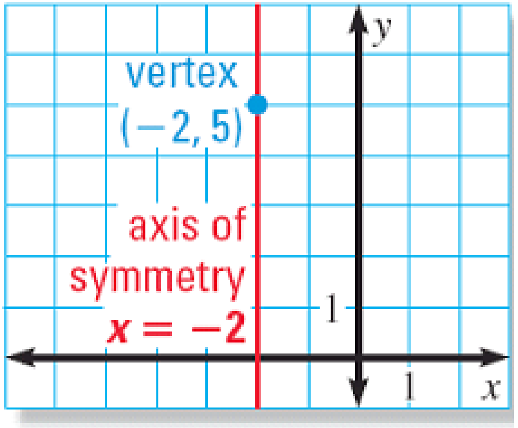
EXAMPLE 1

Graph a quadratic function in vertex form

Graph $y = -\frac{1}{4}(x + 2)^2 + 5$.

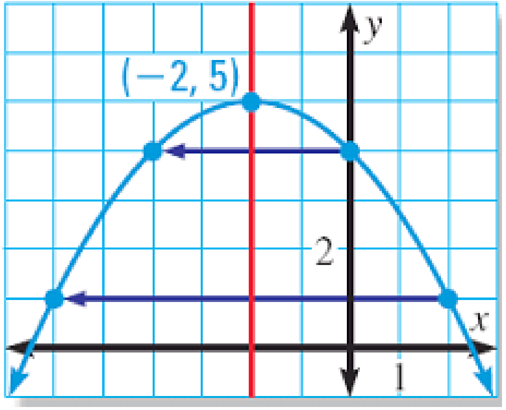


SOLUTION

STEP 1 **Identify the constants** $a = -\frac{1}{4}$, $h = -2$, **and** $k = 5$.

Because $a < 0$, **the parabola opens down.**

STEP 2 **Plot the vertex** $(h, k) = (-2, 5)$ **and draw the axis of symmetry** $x = -2$.



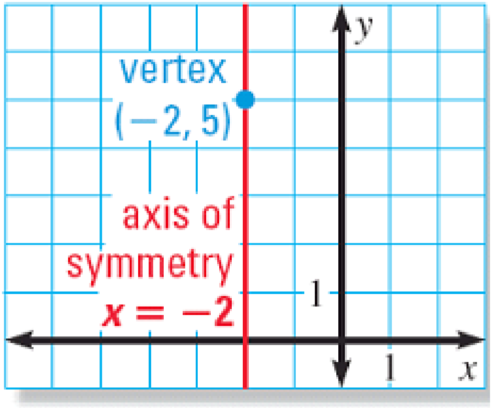
Graph a quadratic function in vertex form

STEP 3 **Evaluate the function for two values of x.**

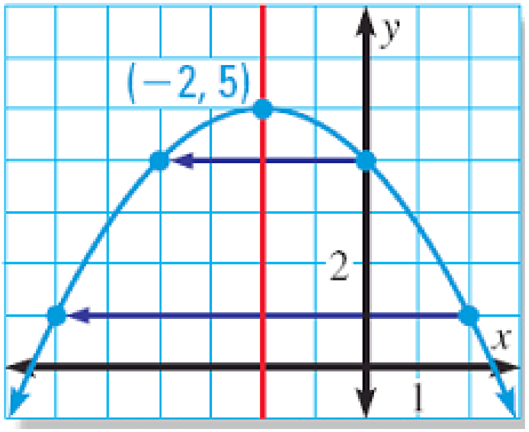
$$x = 0:\ y = -\frac{1}{4}(0 + 2)^2 + 5 = 4$$

$$x = 2:\ y = -\frac{1}{4}(2 + 2)^2 + 5 = 1$$

Plot the points $(0, 4)$ and $(2, 1)$ and their reflections in the axis of symmetry.



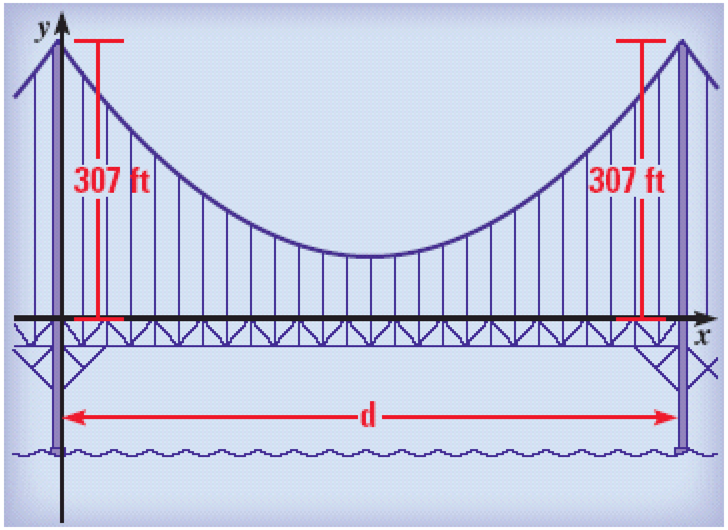
STEP 4 **Draw a parabola through the plotted points.**



EXAMPLE 2 Use a quadratic model in vertex form

Civil Engineering

The Tacoma Narrows Bridge in Washington has two towers that each rise 307 feet above the roadway and are connected by suspension cables as shown. Each cable can be modeled by the function.



$$y = \frac{1}{7000}(x - 1400)^2 + 27$$

where x and y are measured in feet. What is the distance d between the two towers ?

EXAMPLE 2 Use a quadratic model in vertex form

SOLUTION

The vertex of the parabola is $(1400, 27)$. So, a cable's lowest point is 1400 feet from the left tower shown above. Because the heights of the two towers are the same, the symmetry of the parabola implies that the vertex is also 1400 feet from the right tower. So, the distance between the two towers is $d = 2\,(1400) = 2800$ feet.

GUIDED PRACTICE for Examples 1 and 2

Graph the function. Label the vertex and axis of symmetry.

1. $y = (x + 2)^2 - 3$

SOLUTION

STEP 1 **Identify the constants** $a = 1$, $h = -2$, **and** $k = -3$. **Because** $a > 0$, **the parabola opens up.**

STEP 2 **Plot the vertex** $(h, k) = (-2, -3)$ **and draw the axis of symmetry** $x = -2$.

GUIDED PRACTICE for Examples 1 and 2

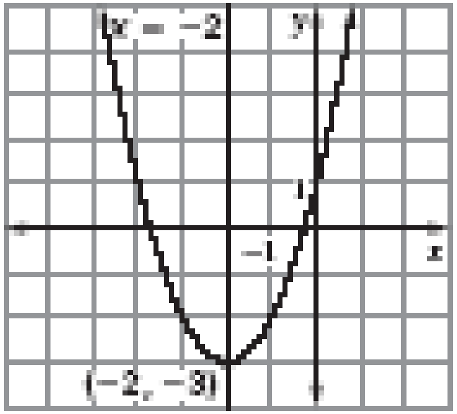
STEP 3 **Evaluate the function for two values of x.**

$x = 0$: $y = (0 + 2)^2 + - 3 = 1$

$x = 2$: $y = (2 + 2)^2 - 3 = 13$

Plot the points (0, 4) **and** (2, 1) **and their reflections in the axis of symmetry.**

STEP 4 **Draw a parabola through the plotted points.**



GUIDED PRACTICE for Examples 1 and 2

2. $y = -(x + 1)^2 + 5$

SOLUTION

STEP 1 Identify the constants $a = 1$, $h = -2$, and $k = -3$. Because $a < 0$, the parabola opens down.

STEP 2 Plot the vertex $(h, k) = (-1, 1)$ and draw the axis of symmetry $x = -1$.

STEP 3 Evaluate the function for two values of x.

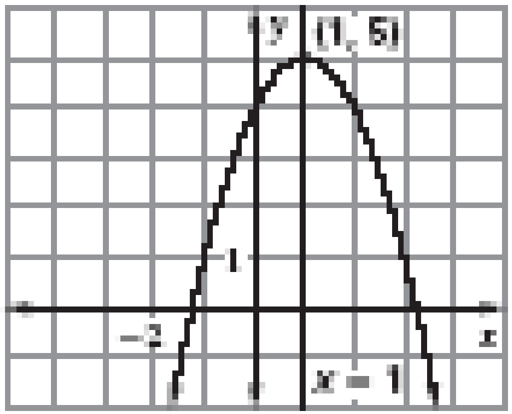
$$x = 0:\ y = -(0 + 2)^2 + 5 = 4$$

$$x = 2:\ y = -(0 - 2)^2 + 5 = 1$$

Plot the points $(0, 4)$ and $(2, 1)$ and their reflections in the axis of symmetry.

GUIDED PRACTICE for Examples 1 and 2

STEP 4 **Draw a parabola through the plotted points.**



GUIDED PRACTICE for Examples 1 and 2

3. $f(x) = \frac{1}{2}(x-3)^2 - 4$

SOLUTION

STEP 1 **Identify the constants** $a = \frac{1}{2}$, $h = -3$, **and** $h = -4$.

Because $a > 0$, **the parabola opens up.**

STEP 2 **Plot the vertex** $(h, k) = (-3, -4)$ **and draw the axis of symmetry** $x = -3$.

STEP 3 **Evaluate the function for two values of** x**.**

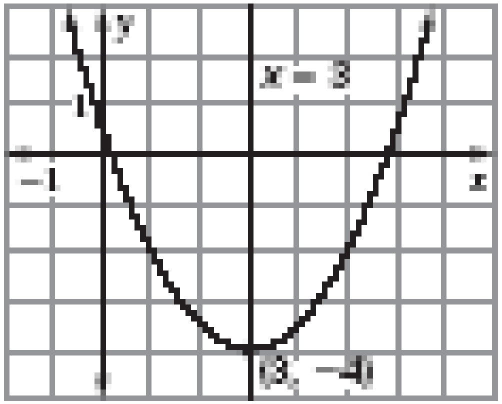
$$x = 0:\ f(x) = \frac{1}{2}(0-3)^2 - 4 = \frac{5}{2}$$

$$x = 0:\ f(x) = \frac{1}{2}(2-3)^2 - 4 = \frac{-3}{2}$$

GUIDED PRACTICE for Examples 1 and 2

Plot the points $(0, 4)$ and $(2, 1)$ and their reflections in the axis of symmetry.

STEP 4 **Draw a parabola through the plotted points.**



GUIDED PRACTICE for Examples 1 and 2

4. WHAT IF? Suppose an architect designs a bridge with cables that can be modeled by $y = \frac{1}{6500}(x - 1400)^2 + 27$ **where *x* and *y* are measured in feet. Compare this function's graph to the graph of the function in Example 2.**

SOLUTION

This graph is slightly steeper than the graph in Example 2. They both have the same vertex and axis of symmetry, and both open up.

Solution missing

EXAMPLE 3 Graph a quadratic function in intercept form

Graph $y = 2(x + 3)(x - 1)$.

SOLUTION

STEP 1 **Identify the** x **- intercepts.**
Because $p = -3$ **and** $q = 1$**, the** x **- intercepts occur at the points** $(-3, 0)$ **and** $(1, 0)$.

STEP 2 **Find the coordinates of the vertex.**

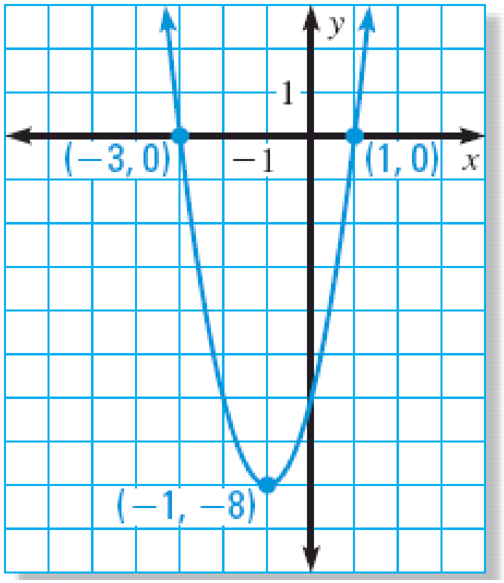
$$x = \frac{p + q}{2} = \frac{-3 + 1}{2} = -1$$

$$y = 2(-1 + 3)(-1 - 1) = -8$$

EXAMPLE 3

Graph a quadratic function in intercept form

STEP 3 **Draw a parabola through the vertex and the points where the x - intercepts occur.**

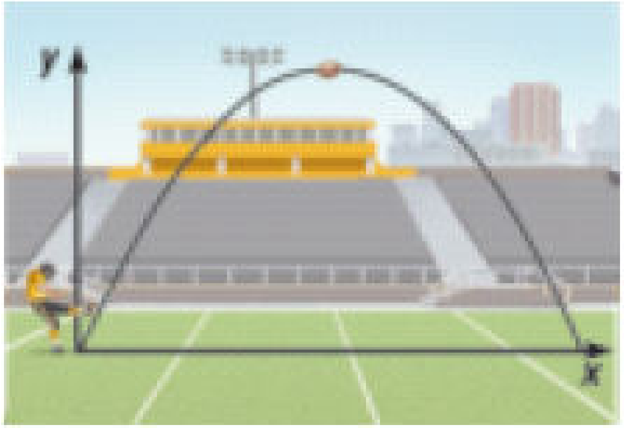


EXAMPLE 4

Use a quadratic function in intercept form

Football

The path of a placekicked football can be modeled by the function $y = -0.026x(x - 46)$ where x is the horizontal distance (in yards) and y is the corresponding height (in yards).



a. How far is the football kicked ?

b. What is the football's maximum height ?

EXAMPLE 4 Use a quadratic function in intercept form

SOLUTION

a. **Rewrite the function as** $y = -0.026(x - 0)(x - 46)$**. Because** $p = 0$ **and** $q = 46$**, you know the** x **- intercepts are** 0 **and** 46**. So, you can conclude that the football is kicked a distance of** 46 **yards.**

b. **To find the football's maximum height, calculate the coordinates of the vertex.**

$$x = \frac{p + q}{2} = \frac{0 + 46}{2} = 23$$

$$y = -0.026(23)(23 - 46) \approx 13.8$$

The maximum height is the ***y*****-coordinate of the vertex, or about** 13.8 **yards.**

GUIDED PRACTICE **for Examples 3 and 4**

Graph the function. Label the vertex, axis of symmetry, and x - intercepts.

5. $y = (x - 3)(x - 7)$

SOLUTION

STEP 1 **Identify the x - intercepts. Because $p = 3$ and $q = 7$, the x - intercepts occur at the points $(3, 0)$ and $(7, 0)$.**

STEP 2 **Find the coordinates of the vertex.**

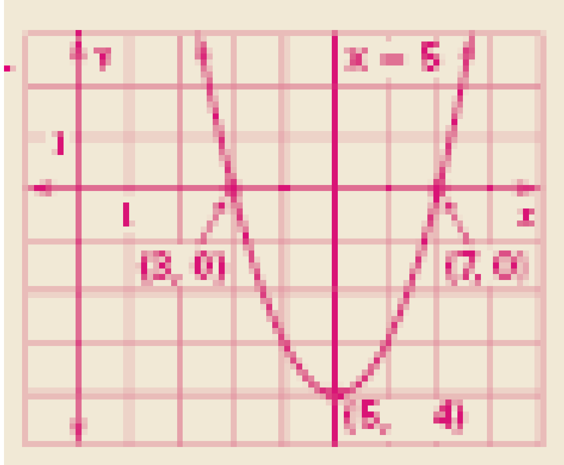
$$x = \frac{p + q}{2} = \frac{3 + 1}{2} = -5$$

$$y = (5 - 3)(5 - 1) = -4$$

So the vertex is $(5, -4)$

GUIDED PRACTICE for Examples 3 and 4

STEP 3 **Draw a parabola through the vertex and the points where the x - intercepts occur.**



GUIDED PRACTICE for Examples 3 and 4

6. $f(x) = 2(x - 4)(x + 1)$

SOLUTION

STEP 1 Identify the x - intercepts. Because $p = 4$ and $q = -1$, the x - intercepts occur at the points $(4, 0)$ and $(-1, 0)$.

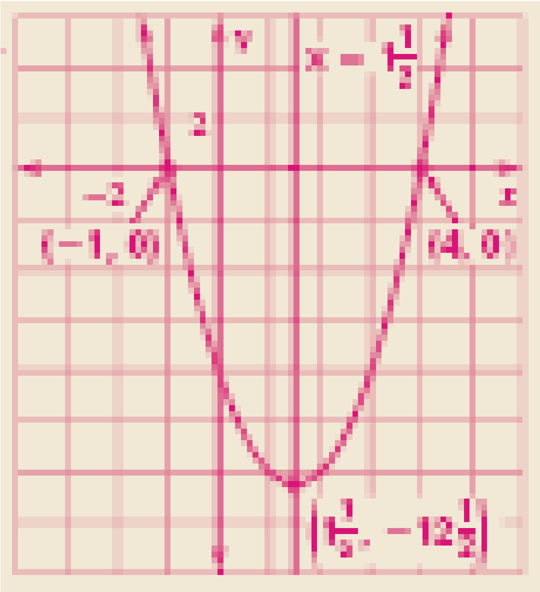
STEP 2 Find the coordinates of the vertex.

$$x = \frac{p+q}{2} = \frac{4+(-1)}{2} = \frac{3}{2}$$

$$y = 2\left(\frac{3}{2} - 4\right)\left(\frac{3}{2} + 1\right) = -\frac{25}{2}$$

So the vertex is $\frac{3}{2}$, $\frac{25}{2}$

STEP 3 **Draw a parabola through the vertex and the points where the x - intercepts occur.**



GUIDED PRACTICE for Examples 3 and 4

7. $y = -(x + 1)(x - 5)$

SOLUTION

STEP 1 **Identify the x - intercepts. Because $p = -1$ and $q = 5$, the x - intercepts occur at the points $(-1, 0)$ and $(5, 0)$.**

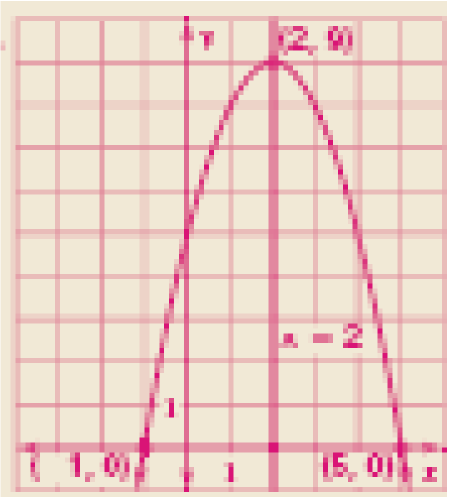
STEP 2 **Find the coordinates of the vertex.**

$$x = \frac{p + q}{2} = \frac{-1 + 5}{2} = -2$$

$$y = -(2 + 3)(2 - 1) = 9$$

So the vertex is $(2, 9)$

STEP 3 **Draw a parabola through the vertex and the points where the x - intercepts occur.**



GUIDED PRACTICE for Examples 3 and 4

8. WHAT IF? In Example 4, what is the maximum height of the football if the football's path can be modeled by the function $y = -0.025x(x - 50)$?

SOLUTION

a. Rewrite the function as $y = -0.025(x - 0)(x - 50)$. Because $p = 0$ and $q = 50$, you know the x - intercepts are 0 and 50. So, you can conclude that the football is kicked a distance of 50 yards.

b. To find the football's maximum height, calculate the coordinates of the vertex.

$$x = \frac{p + q}{2} = \frac{0 + 50}{2} = 25$$

$$y = 20.025(25)(25 - 46) \approx 15.625$$

GUIDED PRACTICE for Examples 3 and 4

The maximum height is the *y*-coordinate of the vertex, or about 15.625 **yards.**

EXAMPLE 5 Change from intercept form to standard form

Write $y = -2(x + 5)(x - 8)$ in standard form.

$y = -2(x + 5)(x - 8)$	Write original function.
$= -2(x^2 - 8x + 5x - 40)$	Multiply using FOIL.
$= -2(x^2 - 3x - 40)$	Combine like terms.
$= -2x^2 + 6x + 80$	Distributive property

EXAMPLE 6 Change from vertex form to standard form

Write $f(x) = 4(x - 1)^2 + 9$ in standard form.

$f(x) = 4(x - 1)^2 + 9$	**Write original function.**
$= 4(x - 1)(x - 1) + 9$	**Rewrite $(x - 1)^2$.**
$= 4(x^2 - x - x + 1) + 9$	**Multiply using FOIL.**
$= 4(x^2 - 2x + 1) + 9$	**Combine like terms.**
$= 4x^2 - 8x + 4 + 9$	**Distributive property**
$= 4x^2 - 8x + 13$	**Combine like terms.**

GUIDED PRACTICE for Examples 5 and 6

Write the quadratic function in standard form.

9. $y = -(x - 2)(x - 7)$

$y = -(x - 2)(x - 7)$ — Write original function.

$= -(x^2 - 7x - 2x + 14)$ — Multiply using FOIL.

$= -(x^2 - 9x + 14)$ — Combine like terms.

$= -x^2 + 9x - 14$ — Distributive property

GUIDED PRACTICE for Examples 5 and 6

10. $y = -4(x - 1)(x + 3)$

$y = -4(x - 1)(x + 3)$	Write original function.
$= -4(x^2 + 3x - x - 3)$	Multiply using FOIL.
$= -4(x^2 + 2x - 3)$	Combine like terms.
$= -4x^2 - 8x + 12$	Distributive property

11. $f(x) = 2(x + 5)(x + 4)$

$f(x) = 2(x + 5)(x + 4)$	Write original function.
$= 2(x^2 + 4x + 5x + 20)$	Multiply using FOIL.
$= 2(x^2 + 9x + 20)$	Combine like terms.
$= 2x^2 + 18x + 40$	Distributive property

GUIDED PRACTICE for Examples 5 and 6

12. $y = -7(x - 6)(x + 1)$

$y = -7(x - 6)(x + 1)$ — **Write original function.**

$= -7(x^2 + x - 6x - 6)$ — **Multiply using FOIL.**

$= -7(x^2 - 5x - 6)$ — **Combine like terms.**

$= -7x^2 + 35x + 42$ — **Distributive property**

GUIDED PRACTICE for Examples 5 and 6

13. $y = -3(x+5)^2 - 1$

$y = -3(x+5)^2 - 1$	**Write original function.**
$= -3(x+5)(x+5) - 1$	**Rewrite $(x+5)^2$.**
$= -3(x^2 + 5x + 5x + 25) - 1$	**Multiply using FOIL.**
$= -3(x^2 + 10x + 25) - 1$	**Combine like terms.**
$= -3x^2 - 30x - 75 - 1$	**Distributive property**
$= -3x^2 - 30x - 76$	**Combine like terms.**

GUIDED PRACTICE for Examples 5 and 6

14. $g(x) = 6(x - 4)^2 - 10$

$g(x) = 6(x - 4)^2 - 10$	**Write original function.**
$= 6(x - 4)(x - 4) - 10$	**Rewrite $(x - 4)^2$.**
$= 6(x^2 - 4x - 4x + 16) - 10$	**Multiply using FOIL.**
$= 6(x^2 - 8x + 16) - 10$	**Combine like terms.**
$= 6x^2 - 48x + 96 - 10$	**Distributive property**
$= 6x^2 - 48x + 86$	**Combine like terms.**

GUIDED PRACTICE for Examples 5 and 6

15. $f(x) = -(x+2)^2 + 4$

$f(x) = -(x+2)^2 + 4$	**Write original function.**
$= -(x+2)(x+2) + 4$	**Rewrite $(x+2)^2$.**
$= -(x^2 + 2x + 2x + 4) + 4$	**Multiply using FOIL.**
$= -(x^2 + 4x + 4) + 4$	**Combine like terms.**
$= -x^2 - 4x - 4 + 4$	**Distributive property**
$= -x^2 - 4x$	**Combine like terms.**

GUIDED PRACTICE for Examples 5 and 6

16. $y = 2(x - 3)^2 + 9$

$y = 2(x - 3)^2 + 9$	**Write original function.**
$= 2(x - 3)(x - 3) + 9$	**Rewrite $(x - 3)^2$.**
$= 2(x^2 - 3x - 3x + 9) + 9$	**Multiply using FOIL.**
$= 2(x^2 - 6x + 9) + 9$	**Combine like terms.**
$= 2x^2 - 12x + 18 + 9$	**Distributive property**
$= 2x^2 - 12x + 27$	**Combine like terms.**