EXAMPLE 1 Factor trinomials of the form $x^2 + bx + c$

Factor the expression.

a. $x^2 - 9x + 20$

b. $x^2 + 3x - 12$

SOLUTION

a. **You want** $x^2 - 9x + 20 = (x + m)(x + n)$ **where** $mn = 20$ **and** $m + n = -9$.

Factors of 20: m, n	1, 20	−1, −20	2, 10	−2, −10	4, 5	**−4, −5**
Sum of factors: $m + n$	21	−21	12	−12	9	**−9**

ANSWER

Notice that $m = -4$ **and** $n = -5$.

So, $x^2 - 9x + 20 = (x - 4)(x - 5)$.

EXAMPLE 1 Factor trinomials of the form $x^2 + bx + c$

b. **You want** $x^2 + 3x - 12 = (x + m)(x + n)$ **where** $mn = -12$ **and** $m + n = 3$.

Factors of −12: m, n	−1, 12	1, −12	−2, 6	2, −6	−3, 4	3, −4
Sum of factors: $m + n$	11	−11	4	−4	1	−1

ANSWER

Notice that there are no factors m **and** n **such that** $m + n = 3$**. So,** $x^2 + 3x - 12$ **cannot be factored.**

GUIDED PRACTICE for Example 1

Factor the expression. If the expression cannot be factored, say so.

1. $x^2 - 3x - 18$

SOLUTION

You want $x^2 - 3x - 18 = (x + m)(x + n)$ **where** $mn = -18$ **and** $m + n = -3$.

Factor of -18 : m, n	$1, -18$	$-1, 18$	$-3, 6$	$-2, 9$	$2, -9$	$-6, 3$
Sum of factors: $m + n$	-17	17	3	7	-7	-3

GUIDED PRACTICE for Example 1

ANSWER

Notice $m = -6$ **and** $n = 3$ **so** $x^2 - 3x - 18 = (x - 6)(x + 3)$

GUIDED PRACTICE for Example 1

2. $n^2 - 3n + 9$

SOLUTION

You want $n^2 - 3n + 9 = (x + m)(x + n)$ **where** $mn = 9$ **and** $m + n = -3$.

Factor of 9 : m, n	1, 9	– 1, – 9	3, 3	– 3, – 3
Sum of factors: $m + n$	10	–10	6	– 6

GUIDED PRACTICE for Example 1

ANSWER

Notice that there are no factors m and n such that $m + n = -3$. So, $n^2 - 3x + 9$ cannot be factored.

GUIDED PRACTICE for Example 1

3. $r^2 + 2r - 63$

SOLUTION

You want $r^2 + 2r - 63 = (x + m)(x + n)$ **where** $mn = -63$ **and** $m + n = 2$.

Factor of -63 : m, n	$-1, 63$	$1, -63$	$21, -3$	$-21, -3$	$9, -7$
Sum of factors: $m + n$	-17	-17	11	7	2

GUIDED PRACTICE for Example 1

ANSWER

Notice that $m = 9$ and $n = -7$ **. So,** $r^2 + 2r - 63 = (r + 9)(r - 7)$

EXAMPLE 2 Factor with special patterns

Factor the expression.

a. $x^2 - 49 = x^2 - 7^2$

$= (x + 7)(x - 7)$

Difference of two squares

b. $d^2 + 12d + 36 = d^2 + 2(d)(6) + 6^2$

$= (d + 6)^2$

Perfect square trinomial

c. $z^2 - 26z + 169 = z^2 - 2(z)(13) + 13^2$

$= (z - 13)^2$

Perfect square trinomial

GUIDED PRACTICE for Example 2

Factor the expression.

4. $x^2 - 9 = x^2 - 3^2$

$= (x - 3)(x + 3)$

Difference of two squares

5. $q^2 - 100 = q^2 - 10^2$

$= (q - 10)(q + 10)$

Difference of two squares

6. $y^2 + 16y + 64 = y^2 + 2(y)\,8 + 8^2$

$= (y + 8)^2$

Perfect square trinomial

GUIDED PRACTICE for Example 2

7. $w^2 - 18w + 81 = w^2 - 2(w) + 9^2$

$= (w - 9)^2$

Perfect square trinomial

EXAMPLE 3 Standardized Test Practice

What are the roots of the equation $x^2 - 5x - 36 = 0$?

(A) $-4, -9$ (B) $4, -9$ (C) $-4, 9$ (D) $4, 9$

SOLUTION

$x^2 - 5x - 36 = 0$ Write original equation.

$(x - 9)(x + 4) = 0$ Factor.

$x - 9 = 0$ **or** $x + 4 = 0$ Zero product property

$x = 9$ **or** $x = -4$ Solve for x.

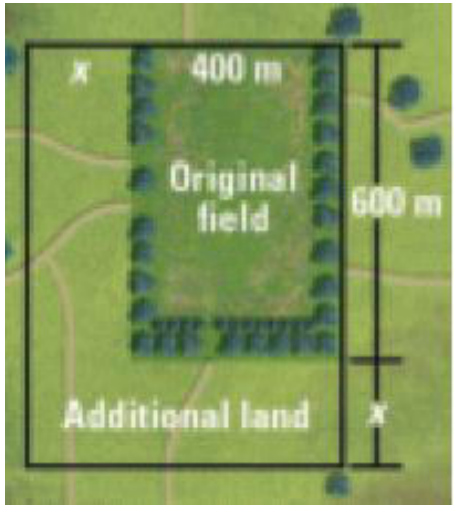
ANSWER

The correct answer is C. (A) (B) (C) (D)

EXAMPLE 4 Use a quadratic equation as a model

Nature Preserve

A town has a nature preserve with a rectangular field that measures 600 meters by 400 meters. The town wants to double the area of the field by adding land as shown. Find the new dimensions of the field.



EXAMPLE 4 Use a quadratic equation as a model

SOLUTION

New area (square meters)	=	New length (meters)	·	New width (meters)
$2(600)(400)$	=	$(600 + x)$	·	$(400 + x)$

$$480{,}000 = 240{,}000 + 1000x + x^2$$ **Multiply using FOIL.**

$$0 = x^2 + 1000x - 240{,}000$$ **Write in standard form.**

$$0 = (x - 200)(x + 1200)$$ **Factor.**

$$x - 200 = 0 \quad \textbf{or} \quad x + 1200 = 0$$ **Zero product property**

$$x = 200 \quad \textbf{or} \quad x = -1200$$ **Solve for x.**

EXAMPLE 4 Use a quadratic equation as a model

ANSWER

Reject the negative value, -1200. The field's length and width should each be increased by 200 meters. The new dimensions are 800 meters by 600 meters.

GUIDED PRACTICE for Examples 3 and 4

8. Solve the equation $x^2 - x - 42 = 0.$

SOLUTION

$x^2 - x - 42 = 0$			**Write original equation.**
$(x + 6)(x - 7) = 0$			**Factor.**
$x + 6 = 0$	**or**	$x - 7 = 0$	**Zero product property**
$x = -6$	**or**	$x = 7$	**Solve for** x**.**

GUIDED PRACTICE for Examples 3 and 4

9. What If ? In Example 4, suppose the field initially measures 1000 meters by 300 meters. Find the new dimensions of the field.

SOLUTION

New Area	=	**New Length (meters)**	•	**New width (meters)**
$2(1000)(300)$	=	$(1000 + x)$	•	$(300 + x)$

$600000 = 300000 + 1000x + 300x + x^2$ **Multiply using FOIL.**

$0 = x^2 + 1300x - 300000$ **Write in standard form.**

$0 = (x - 200)(x + 1500)$ **Factor.**

$x - 200 = 0$ **or** $x + 1500 = 0$ **Zero product property**

GUIDED PRACTICE for Examples 3 and 4

$x = 200$ **or** $x = -1500$ **Solve for** x**.**

ANSWER

Reject the negative value, -1200**. The field's length and width should each be increased by** 200 **meters. The new dimensions are** 1200 **meters by** 500 **meters.**

EXAMPLE 5 Find the zeros of quadratic functions.

Find the zeros of the function by rewriting the function in intercept form.

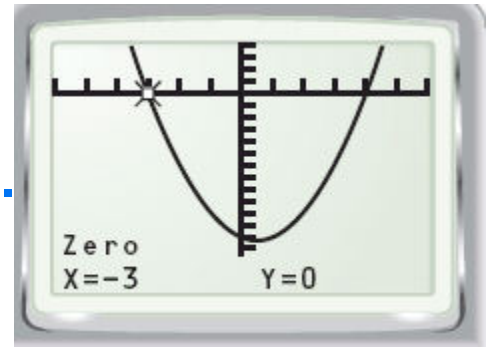
a. $y = x^2 - x - 12$

b. $y = x^2 + 12x + 36$

SOLUTION

a. $y = x^2 - x - 12$ **Write original function.**

$= (x + 3)(x - 4)$ **Factor.**



The zeros of the function are -3 and 4.

Check Graph $y = x^2 - x - 12$. The graph passes through $(-3, 0)$ and $(4, 0)$.

EXAMPLE 5 Find the zeros of quadratic functions.

Find the zeros of the function by rewriting the function in intercept form.

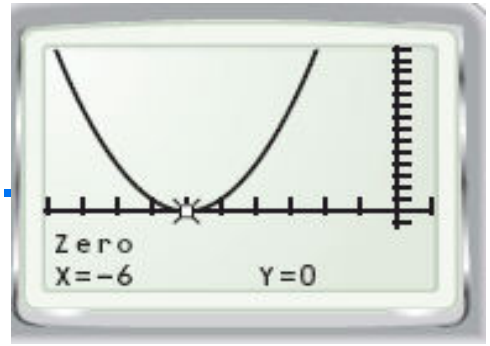
a. $y = x^2 - x - 12$

b. $y = x^2 + 12x + 36$

SOLUTION

b. $y = x^2 + 12x + 36$ **Write original function.**

$= (x + 6)(x + 6)$ **Factor.**



The zeros of the function is -6

Check Graph $y = x^2 + 12x + 36$**. The graph passes through** $(-6, 0)$.

GUIDED PRACTICE for Example 5

Find the zeros of the function by rewriting the function in intercept form.

10. $y = x^2 + 5x - 14$

SOLUTION

$y = x^2 + 5x - 14$ **Write original function.**

$= (x + 7)(x - 2)$ **Factor.**

The zeros of the function is -7 **and** 2

Check Graph $y = x^2 + 5x - 14$**. The graph passes through** $(-7, 0)$ **and** $(2, 0)$.

GUIDED PRACTICE for Example 5

11. $y = x^2 - 7x - 30$

SOLUTION

$y = x^2 - 7x - 30$ **Write original function.**

$= (x + 3)(x - 10)$ **Factor.**

The zeros of the function is -3 **and** 10

Check Graph $y = x^2 - 7x - 30$**. The graph passes through** $(-3, 0)$ **and** $(10, 0)$.

GUIDED PRACTICE for Example 5

12. $f(x) = x^2 - 10x + 25$

SOLUTION

$f(x) = x^2 - 10x + 25$ **Write original function.**

$= (x - 5)(x - 5)$ **Factor.**

The zeros of the function is 5

Check Graph $f(x) = x^2 - 10x + 25$**. The graph passes through** $(5, 0)$.