EXAMPLE 1 Factor $ax^2 + bx + c$ where $c > 0$

Factor $5x^2 - 17x + 6$.

SOLUTION

You want $5x^2 - 17x + 6 = (kx + m)(lx + n)$ **where** k **and** l **are factors of** 5 **and** m **and** n **are factors of** 6**. You can assume that** k **and** l **are positive and** $k \geq l$**. Because** $mn > 0$, m **and** n **have the same sign. So,** m **and** n **must both be negative because the coefficient of** x, -17, **is negative.**

k, l	5, 1	5, 1	5, 1	**5, 1**
m, n	−6, −1	−1, −6	−3, −2	**−2, −3**
$(kx + m)(lx + n)$	$(5x - 6)(x - 1)$	$(5x - 1)(x - 6)$	$(5x - 3)(x - 2)$	$\mathbf{(5x - 2)(x - 3)}$
$ax^2 + bx + c$	$5x^2 - 11x + 6$	$5x^2 - 31x + 6$	$5x^2 - 13x + 6$	$\mathbf{5x^2 - 17x + 6}$

Factor $ax^2 + bx + c$ where $c > 0$

ANSWER

The correct factorization is $5x^2 – 17x + 6 = (5x – 2)(x – 3)$.

EXAMPLE 2 Factor $ax^2 + bx + c$ where $c < 0$

Factor $3x^2 + 20x - 7$.

SOLUTION

You want $3x^2 + 20x - 7 = (kx + m)(lx + n)$ **where** k **and** l **are factors of** 3 **and** m **and** n **are factors of** -7. **Because** $mn < 0$, m **and** n **have opposite signs.**

k, l	3, 1	**3, 1**	3, 1	3, 1
m, n	7, –1	**–1, 7**	–7, 1	1, –7
$(kx + m)(lx + n)$	$(3x + 7)(x - 1)$	$(3x - 1)(x + 7)$	$(3x - 7)(x + 1)$	$(3x + 1)(x - 7)$
$ax^2 + bx + c$	$3x^2 + 4x - 7$	$3x^2 + 20x - 7$	$3x^2 - 4x - 7$	$3x^2 - 20x - 7$

ANSWER

The correct factorization is $3x^2 + 20x - 7 = (3x - 1)(x + 7)$.

Factor the expression. If the expression cannot be factored, say so.

1. $7x^2 - 20x - 3$

SOLUTION

You want $7x^2 - 20x - 3 = (kx + m)(lx + n)$ **where** k **and** l **are factors of** 7 **and** m **and** n **are factors of** 3**. You can assume that** k **and** l **are positive and** $k \geq l$**. Because** $mn < 0$, m **and** n **have opposite signs.**

GUIDED PRACTICE for Examples 1 and 2

k, l	7, 1	7, 1	7, 1	7, 1
m, n	3, – 1	– 1, 3	– 3, 1	1, – 3
$(kx + m)(lx + n)$	$(7x + 3)(x - 1)$	$(7x - 1)(x + 3)$	$(7x - 3)(x + 1)$	$(7x + 1)(x - 3)$
$ax^2 + bx + c$	$7x^2 - 4x - 3$	$7x^2 + 20x - 3$	$7x^2 + 4x - 3$	$7x^2 - 20x - 3$

ANSWER

The correct factorization is $7x^2 - 20x - 3 = (7x + 1)(x - 3)$.

2. $5z^2 + 16z + 3$

SOLUTION

You want $5z^2 + 16z + 3 = (kx + m)(lx + n)$ **where** k **and** l **are factors of** 5 **and** m **and** n **are factors of** 3. **You can assume that** k **and** l **are positive and** $k \geq l$. **Because** $mn > 0$, m **and** n **have the same sign.**

k, l	5, 1	5, 1	5, 1	5, 1
m, n	– 3, – 1	– 1, – 3	3, 1	1, 3
$(kx + m)(lx + n)$	$(5z – 3)(z – 1)$	$(5z – 1)(z – 3)$	$(5z + 3)(z + 1)$	$(5z + 3)(z + 3)$
$ax^2 + bx + c$	$5z^2 – 8x + 3$	$5z^2 – 16x + 3$	$5z^2 + 8z – 3$	$5z^2 + 16z + 3$

ANSWER

The correct factorization is $5z^2 + 16z + 3 = (5z + 1)(z + 3)$.

GUIDED PRACTICE for Examples 1 and 2

3. $2w^2 + w + 3$

SOLUTION

You want $2w^2 + w + 3 = (kx + m)(lx + n)$ **where** k **and** l **are factors of** 2 **and** m **and** n **are factors of** 3**. Because** $mn > 0$, m **and** n **have the same sign.**

k, l	2, 1	2, 1	2, 1	2, 1
m, n	3, 1	1, 3	–3, –1	–1, –3
$(kx + m)(lx + n)$	$(2w + 3)$ $(w + 1)$	$(2w + 1)$ $(w + 3)$	$(2w – 3)$ $(w – 1)$	$(2w – 1)$ $(w – 3)$
$ax^2 + bx + c$	$2w^2 + 5w + 3$	$2w^2 + 7w + 3$	$2w^2 – 5w + 3$	$2w^2 – 7w + 3$

GUIDED PRACTICE for Examples 1 and 2

ANSWER

$2w^2 + w + 3$ **cannot be factored**

GUIDED PRACTICE for Examples 1 and 2

4. $3x^2 + 5x - 12$

SOLUTION

You want $3x^2 + 5x - 12 = (kx + m)(lx + n)$ **where** k **and** l **are factors of** 3 **and** m **and** n **are factors of** -12 **. Because** $mn < 0$, m **and** n **have opposite sign.**

k, l	3, 1	3, 1	3, 1	3, 1
m, n	12, – 1	– 1, 12	– 3, 4	– 4, 3
$(kx + m)(lx + n)$	$(3x + 12)(x - 1)$	$(3x - 1)(x + 12)$	$(3x - 3)(x + 4)$	$(3x - 4)(x + 3)$
$ax^2 + bx + c$	$3x^2 + 9x - 12$	$3x^2 + 35x - 12$	$3x^2 + 9x - 12$	$3x^2 + 5x - 12$

GUIDED PRACTICE for Examples 1 and 2

ANSWER

The correct factorization is $3x^2 + 5x - 12 = (3x - 4)(x + 3)$.

5. $4u^2 + 12u + 5$

SOLUTION

You want $4u^2 + 12u + 5 = (kx + m)\,(lx + n)$ **where** k **and** l **are factors of** 4 **and** m **and** n **are factors of** 5**. You can assume that** k **and** l **are positive and** $k \geq l$**.**

Because $mn > 0$, m **and** n **have the same sign.**

GUIDED PRACTICE for Examples 1 and 2

k, l	2, 2	2, 2	2, 2	2, 2
m, n	$-5, -1$	$-1, -5$	1, 5	5, 1
$(kx + m)(lx + n)$	$(2u - 5)(2u - 1)$	$(2u - 1)(2u - 5)$	$(2u + 1)(2u + 5)$	$(2u + 5)(2u + 1)$
$ax^2 + bx + c$	$4u^2 - 12u + 5$	$4u^2 - 12u + 5$	$4u^2 + 12u + 5$	$4u^2 + 12u + 5$

ANSWER

The correct factorization is $4u^2 + 12u + 5 = (2u + 1)(2u + 5)$.

6. $4x^2 - 9x + 2$

SOLUTION

You want $4x^2 - 9x + 2 = (kx + m)(lx + n)$ **where** k **and** l **are factors of** 4 **and** m **and** n **are factors of** 2**. You can assume that** k **and** l **are positive and** $k \geq l$**.**

Because $mn > 0$, m **and** n **have the same sign. So,** m **and** n **must both be negative because the coefficient of** $x = -9$**, is negative.**

GUIDED PRACTICE for Examples 1 and 2

k, l	4, 1	4, 1		
m, n	– 1, – 2	– 2, – 1		
$(kx + m)\ (lx + n)$	$(4x - 1)(x - 2)$	$(4x - 2)$ $(x - 1)$		
$ax^2 + bx + c$	$4x^2 - 9x + 2$	$4x^2 - 6x + 2$		

ANSWER

The correct factorization is $4x^2 - 9x + 2 = (4x - 1)\ (x - 2)$.

EXAMPLE 3

Factor with special patterns

Factor the expression.

a. $9x^2 - 64 = (3x)^2 - 8^2$

$= (3x + 8)(3x - 8)$

Difference of two squares

b. $4y^2 + 20y + 25 = (2y)^2 + 2(2y)(5) + 5^2$

$= (2y + 5)^2$

Perfect square trinomial

c. $36w^2 - 12w + 1 = (6w)^2 - 2(6w)(1) + (1)^2$

$= (6w - 1)^2$

Perfect square trinomial

Factor the expression.

7. $16x^2 - 1 = (4x)^2 - 1^2$

$= (4x + 1)(4x - 1)$

Difference of two squares

8. $9y^2 + 12y + 4 = (3y)^2 + 2(3y)(2) + (2)^2$

$= (3y + 2)^2$

Perfect square trinomial

9. $4r^2 - 28r + 49 = (2r)^2 - 2(2r)(7) + (7)^2$

$= (2r - 7)^2$

Perfect square trinomial

10. $25s^2 - 80s + 64 = (5s)^2 - 2(5s)(8) + (8)^2$

$= (5s - 8)^2$

Perfect square trinomial

GUIDED PRACTICE for Example 3

11. $49z^2 + 4z + 9 = (7z)^2 + 2(7z)(3) + (3)^2$

$= (7z + 3)^2$

Perfect square trinomial

12. $36n^2 – 9 = (3y)^2 = (6n)^2 – (3)^2$

$= (6n – 3)\ (6n + 3)$

Difference of two squares

EXAMPLE 4 Factor out monomials first

Factor the expression.

a. $5x^2 - 45 = 5(x^2 - 9)$

$= 5(x + 3)(x - 3)$

b. $6q^2 - 14q + 8 = 2(3q^2 - 7q + 4)$

$= 2(3q - 4)(q - 1)$

c. $-5z^2 + 20z = -5z(z - 4)$

d. $12p^2 - 21p + 3 = 3(4p^2 - 7p + 1)$

Factor the expression.

13. $3s^2 - 24 = 3(s^2 - 8)$

14. $8t^2 + 38t - 10 = 2(4t^2 + 19t - 5)$

$= 2(4t - 1)(t + 5)$

15. $6x^2 + 24x + 15 = 3(2x^2 + 8x + 5)$

16. $12x^2 - 28x - 24 = 4(3x^2 - 7x - 6)$

$= 4(3x + 2)(x - 3)$

17. $-16n^2 + 12n = -4n(4n - 3)$

GUIDED PRACTICE for Example 4

18. $6z^2 + 33z + 36 = 3(2z^2 + 11z + 2)$

$= 3(2z + 3)(z + 4)$

EXAMPLE 5 Solve quadratic equations

Solve (a) $3x^2 + 10x - 8 = 0$ **and** (b) $5p^2 - 16p + 15 = 4p - 5$.

a. $3x^2 + 10x - 8 = 0$ — Write original equation.

$(3x - 2)(x + 4) = 0$ — Factor.

$3x - 2 = 0$ **or** $x + 4 = 0$ — Zero product property

$x = \frac{2}{3}$ **or** $x = -4$ — Solve for x.

EXAMPLE 5 Solve quadratic equations

(b) $5p^2 - 16p + 15 = 4p - 5.$

b. $5p^2 - 16p + 15 = 4p - 5.$ **Write original equation.**

$5p^2 - 20p + 20 = 0$ **Write in standard form.**

$p^2 - 4p + 4 = 0$ **Divide each side by 5.**

$(p - 2)^2 = 0$ **Factor.**

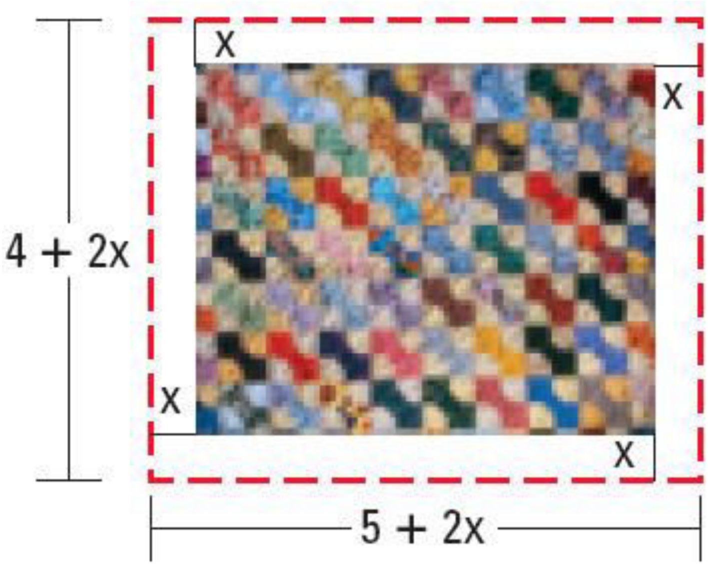
$p - 2 = 0$ **Zero product property**

$p = 2$ **Solve for p.**

EXAMPLE 6 Use a quadratic equation as a model

Quilts

You have made a rectangular quilt that is 5 **feet by** 4 **feet. You want to use the remaining** 10 square feet **of fabric to add a decorative border of uniform width to the quilt. What should the width of the quilt's border be?**



EXAMPLE 6 Use a quadratic equation as a model

SOLUTION

Write a verbal model. Then write an equation.

Area of border (square feet)	=	Area of quilt and border (square feet)	−	Area of quilt (square feet)
10	=	$(5 + 2x)(4 + 2x)$	−	$(5)(4)$

$10 = 20 + 18x + 4x^2 - 20$ — **Multiply using FOIL.**

$0 = 4x^2 + 18x - 10$ — **Write in standard form**

$0 = 2x^2 + 9x - 5$ — **Divide each side by 2.**

$0 = (2x - 1)(x + 5)$ — **Factor.**

$2x - 1 = 0$ **or** $x + 5 = 0$ — **Zero product property**

$x = \frac{1}{2}$ **or** $x = -5$ — **Solve for x.**

EXAMPLE 6 Use a quadratic equation as a model

ANSWER

Reject the negative value, -5**. The border's width should be** $\frac{1}{2}$ ft, **or** 6 in.

EXAMPLE 7 Solve a multi-step problem

Magazines

A monthly teen magazine has 28,000 **subscribers when it charges** \$10 **per annual subscription. For each** \$1 **increase in price, the magazine loses about** 2000 **subscribers. How much should the magazine charge to maximize annual revenue ? What is the maximum annual revenue ?**

EXAMPLE 7 Solve a multi-step problem

SOLUTION

STEP 1 Define the variables. Let x represent the price increase and $R(x)$ represent the annual revenue.

STEP 2 Write a verbal model. Then write and simplify a quadratic function.

EXAMPLE 7 Solve a multi-step problem

Annual revenue (dollars)	=	Number of subscribers (people)	·	Subscription price (dollars/person)
$R(x)$	=	$(28{,}000 - 2000x)$	·	$(10 + x)$

$$R(x) = (-2000x + 28{,}000)(x + 10)$$

$$R(x) = -2000(x - 14)(x + 10)$$

EXAMPLE 7 Solve a multi-step problem

STEP 3 **Identify the zeros and find their average. Find how much each subscription should cost to maximize annual revenue.**

The zeros of the revenue function are 14 and –10. The average of the zeroes is

$$\frac{14 + (-10)}{2} = 2.$$

To maximize revenue, each subscription should cost \$10 + \$2 = \$12.

STEP 4 **Find the maximum annual revenue.**

$$R(2) = -2000(2 - 14)(2 + 10) = \$288{,}000$$

EXAMPLE 7

Solve a multi-step problem

ANSWER

The magazine should charge $12 per subscription to maximize annual revenue. The maximum annual revenue is $288,000.

GUIDED PRACTICE for Examples 5, 6 and 7

Solve the equation.

19. $6x^2 - 3x - 63 = 0$

$6x^2 - 3x - 63 = 0$	**Write original equation.**
$2x^2 - x - 21 = 0$	**Divide each side by 3.**
$(2x - 7)(x + 3) = 0$	**Factor.**
$2x - 7 = 0$ **or** $x + 3 = 0$	**Zero product property**
$x = \frac{7}{2} = 3\frac{1}{2}$ **or** $x = -3$	**Solve for x.**

GUIDED PRACTICE for Examples 5, 6 and 7

20. $12x^2 + 7x + 2 = x + 8$

$12x^2 + 7x + 2 = x + 8$	**Write original equation.**
$12x^2 + 6x - 6 = 0$	**Write in standard form**
$4x^2 + 2x - 2 = 0$	**Divide each side by 3**
$(2x + 2)(2x - 1) = 0$	**Factor.**
$2x + 2 = 0$ **or** $2x - 1 = 0$	**Zero product property**
$x = -1$ **or** $x = \frac{1}{2}$	**Solve for x.**

GUIDED PRACTICE for Examples 5, 6 and 7

21. $7x^2 + 70x + 175 = 0$

$7x^2 + 70x + 175 = 0$	Write original equation.
$7x^2 + 70x + 175 = 0$	Write in standard form
$x^2 + 10x + 25 = 0$	Divide each side by 7
$(x + 5)(x - 5) = 0$	Factor.
$x + 5 = 0$ **or** $x - 5 = 0$	Zero product property
$x = -5$ **or** $x = 5$	Solve for x.

GUIDED PRACTICE for Examples 5, 6 and 7

22. What If ? In Example 7, suppose the magazine initially charges $11 per annual subscription. How much should the magazine charge to maximize annual revenue ? What is the maximum annual revenue ?

SOLUTION

STEP 1 Define the variables. Let x represent the price increase and $R(x)$ represent the annual revenue.

STEP 2 Write a verbal model. Then write and simplify a quadratic function.

GUIDED PRACTICE for Examples 5, 6 and 7

Annual Revenue = **Annual Revenue** • **Subscription price**

$$R(x) = (28{,}000 – 2000x)\ (11 + x)$$

$$R(x) = (– 2000x + 28000)\ (x + 11)$$

$$R(x) = – 2000\ (x – 14)\ \ (x + 11)$$

STEP 3 **Identify the zeros and find their average. Find how much each subscription should cost to maximize annual revenue.**

The zeros of the revenue function are 14 **and** –11**. The average of the zeroes is**

$$\frac{14 + (– 11)}{2} = \frac{3}{2}.$$

GUIDED PRACTICE for Examples 5, 6 and 7

To maximize revenue, each subscription should cost $\$11 + \frac{3}{2} = \$12.50$.

STEP 4 Find the maximum annual revenue.

$$R\left(\frac{3}{2}\right) = -2000\left(2 - 14\right)\left(\frac{3}{2} + 11\right) = \$312,500$$

ANSWER

The magazine should charge $12.50 **per subscription to maximize annual revenue. The maximum annual revenue is** $312,500.