EXAMPLE 1 Solve a quadratic equation

Solve $2x^2 + 11 = -37$.

$2x^2 + 11 = -37$	Write original equation.
$2x^2 = -48$	Subtract 11 from each side.
$x^2 = -24$	Divide each side by 2.
$x = \pm\sqrt{-24}$	Take square roots of each side.
$x = \pm i\sqrt{24}$	Write in terms of i.
$x = \pm 2i\sqrt{6}$	Simplify radical.

ANSWER

The solutions are $2i\sqrt{6}$ **and** $-2i\sqrt{6}$.

GUIDED PRACTICE for Example 1

Solve the equation.

1. $x^2 = -13.$

$x^2 = -13.$	Write original equation.
$x = \pm\sqrt{-13}$	Take square roots of each side.
$x = \pm i\sqrt{13}$	Write in terms of i.
$x = \pm i\sqrt{13}$	Simplify radical.

ANSWER

The solutions are $x = i\sqrt{13}$ **and** $-i\sqrt{13}$**.**

GUIDED PRACTICE for Example 1

Solve the equation.

2. $x^2 = -38.$

$x^2 = -38.$	Write original equation.
$x = \pm\sqrt{-38}$	Take square roots of each side.
$x = \pm i\sqrt{38}$	Write in terms of i.
$x = \pm i\sqrt{38}$	Simplify radical.

ANSWER

The solutions are $x = i\sqrt{38}$ **and** $-i\sqrt{38}$ **.**

Solve the equation.

3. $x^2 + 11 = 3.$

$x^2 + 11 = 3.$ **Write original equation.**

$x^2 = -8.$ **Subtract 11 from each side.**

$x = \pm\sqrt{-8}$ **Take square roots of each side.**

$x = \pm i\sqrt{8}$ **Write in terms of i.**

$x = \pm 2i\sqrt{2}$ **Simplify radical.**

ANSWER

The solutions are $2i\sqrt{2}$ and $-2i\sqrt{2}$.

GUIDED PRACTICE for Example 1

Solve the equation.

4. $x^2 - 8 = -36$.

$x^2 - 8 = -36$.	Write original equation.
$x^2 = -28.$	Add 8 to each side.
$x = \pm\sqrt{-28}$	Take square roots of each side.
$x = \pm i\sqrt{28}$	Write in terms of i.
$x = \pm 2i\sqrt{7}$	Simplify radical.

ANSWER

The solutions are $2i\sqrt{7}$ and $-2i\sqrt{7}$.

Solve the equation.

5. $3x^2 - 7 = -31$.

$3x^2 - 7 = -31$.	Write original equation.
$3x^2 = -24$.	Add 7 to each side.
$x^2 = -8$.	Divided each side by 3
$x = \pm\sqrt{-8}$	Take square roots of each side.
$x = \pm\, i\sqrt{8}$	Write in terms of i.
$x = \pm\, 2i\sqrt{2}$	Simplify radical.

ANSWER

The solutions are $2i\sqrt{2}$ **and** $-2i\sqrt{2}$ **.**

GUIDED PRACTICE for Example 1

Solve the equation.

6. $5x^2 + 33 = 3$.

$5x^2 + 33 = 3$.	**Write original equation.**
$5x^2 = -30$.	**Add 7 to each side.**
$x^2 = -6.$	**Divided each side by 3**
$x = \pm\sqrt{-6}$	**Take square roots of each side.**
$x = \pm i\sqrt{6}$	**Write in terms of i.**
$x = \pm i\sqrt{6}$	**Simplify radical.**

ANSWER

The solutions are $i\sqrt{6}$ and $-i\sqrt{6}$.

EXAMPLE 2 Add and subtract complex numbers

Write the expression as a complex number in standard form.

a. $(8 - i) + (5 + 4i)$ **b.** $(7 - 6i) - (3 - 6i)$ **c.** $10 - (6 + 7i) + 4i$

SOLUTION

a. $(8 - i) + (5 + 4i) =$ **Definition of complex addition**

$(8 + 5) + (-1 + 4)i$

$= 13 + 3i$ **Write in standard form.**

b. $(7 - 6i) - (3 - 6i) =$ **Definition of complex subtraction**

$(7 - 3) + (-6 + 6)i$

$= 4 + 0i$ **Simplify.**

$= 4$ **Write in standard form.**

EXAMPLE 2

Add and subtract complex numbers

c. $10 - (6 + 7i) + 4i =$ — Definition of complex subtraction

$[(10 - 6) - 7i] + 4i$

$= (4 - 7i) + 4i$ — Simplify.

$= 4 + (-7 + 4)i$ — Definition of complex addition

$= 4 - 3i$ — Write in standard form.

Write the expression as a complex number in standard form.

7. $(9 - i) + (-6 + 7i)$

$= (9 - i) + (-6 + 7i)$

$= (9 - 6) + (-1 + 7)i$ **Definition of complex addition**

$= 3 + 6i$ **Write in standard form.**

GUIDED PRACTICE for Example 2

Write the expression as a complex number in standard form.

8. $(3 + 7i) - (8 - 2i)$

$= (3 + 7i) - (8 - 2i)$

$= (3 - 8) + (7 + 2)i$ **Definition of complex subtraction**

$= -5 + 9i$ **Write in standard form.**

GUIDED PRACTICE for Example 2

Write the expression as a complex number in standard form.

9. $-4-(1+i)-(5+9i)$

$= -4-(1+i)-(5+9i)$

$= [(-4-1-5)-i]-9i$ **Definition of complex subtraction**

$= (-10-i)-9i$ **Simplify.**

$= -10+(-1-9)i$ **Definition of complex addition**

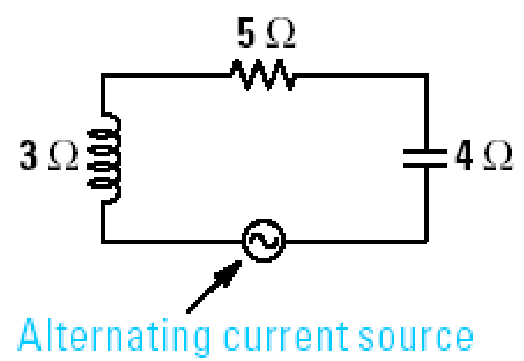
$= -10-10i$ **Write in standard form.**

Use addition of complex numbers in real life

Electricity

Circuit components such as resistors,inductors, and capacitors all oppose the flow of current. This opposition is called *resistance* for resistors and *reactance* for inductors and capacitors. A circuit's total opposition to current flow is *impedance*. All of these quantities are measured in ohms ( ). Ω

Component and symbol	Resistor	Inductor	Capacitor
Resistance or reactance	R	L	C
Impedance	R	Li	$-Ci$



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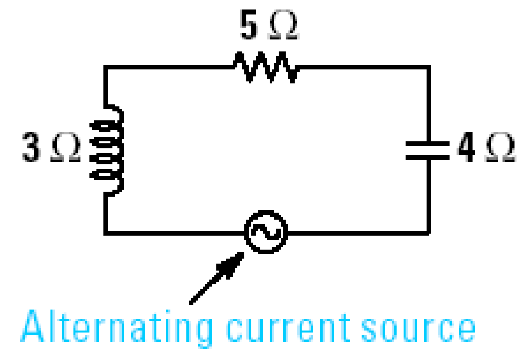
EXAMPLE 3 Use addition of complex numbers in real life

The table shows the relationship between a component's resistance or reactance and its contribution to impedance. A *series circuit* is also shown with the resistance or reactance of each component labeled.

The impedance for a series circuit is the sum of the impedances for the individual components. Find the impedance of the circuit shown above.

EXAMPLE 3 Use addition of complex numbers in real life

Component and symbol	Resistor	Inductor	Capacitor
Resistance or reactance	R	L	C
Impedance	R	Li	$-Ci$



SOLUTION

The resistor has a resistance of 5 *ohms*, **so its impedance is** 5 *ohms*. **The inductor has a reactance of** 3 *ohms*, **so its impedance is** $3i$ *ohms*. **The capacitor has a reactance of** 4 *ohms*, **so its impedance is** $-4i$ *ohms*.

Impedance of circuit

$= 5 + 3i + (-4i)$ **Add the individual impedances.**

$= 5 - i$ **Simplify.**

EXAMPLE 3 Use addition of complex numbers in real life

ANSWER

The impedance of the circuit is $= 5 - i\ ohms$**.**

EXAMPLE 4 Multiply complex numbers

Write the expression as a complex number in standard form.

a. $4i(-6 + i)$

b. $(9 - 2i)(-4 + 7i)$

SOLUTION

a. $4i(-6 + i) = -24i + 4i^2$ — **Distributive property**

$= -24i + 4(-1)$ — **Use $i^2 = -1$.**

$= -24i - 4$ — **Simplify.**

$= -4 - 24i$ — **Write in standard form.**

EXAMPLE 4 Multiply complex numbers

b. $(9 - 2i)(-4 + 7i)$

$= -36 + 63i + 8i - 14i^2$	**Multiply using FOIL.**
$= -36 + 71i - 14(-1)$	**Simplify and use** $i^2 = -1$ **.**
$= -36 + 71i + 14$	**Simplify.**
$= -22 + 71i$	**Write in standard form.**

EXAMPLE 5 Divide complex numbers

Write the quotient $\frac{7+5i}{1-4i}$ in standard form.

$$\frac{7+5i}{1-4i} = \frac{7+5i}{1-4i} \cdot \frac{1+4i}{1+4i}$$

Multiply numerator and denominator by $1+4i$, the complex conjugate of $1-4i$.

$$= \frac{7+28i+5i+20i^2}{1+4i-4i-16i^2}$$

Multiply using FOIL.

$$= \frac{7+33i+20(-1)}{1-16(-1)}$$

Simplify and use $i^2 = 1$.

$$= \frac{-13+33i}{17}$$

Simplify.

EXAMPLE 5 Divide complex numbers

$$= -\frac{13}{17} + \frac{33}{17}i$$

Write in standard form.

GUIDED PRACTICE for Examples 3, 4 and 5

10. **WHAT IF? In Example 3, what is the impedance of the circuit if the given capacitor is replaced with one having a reactance of 7 ohms?**

SOLUTION

The resistor has a resistance of 5 *ohms*, **so its impedance is** 5 *ohms*. **The inductor has a reactance of** 3 *ohms*, **so its impedance is** $3i$ *ohms*. **The capacitor has a reactance of** 7 *ohms*, **so its impedance is** $-7i$ *ohms*.

Impedance of circuit

$= 5 + 3i + (-7i)$ **Add the individual impedances.**

$= 5 - 4i$ **Simplify.**

ANSWER

The impedance of the circuit is $= 5 - 4i$ *ohms*.

GUIDED PRACTICE for Examples 3, 4 and 5

11. $i(9 - i)$

SOLUTION

$i(9 - i)$	$= 9i - i^2$	**Distributive property**
	$= 9i + (-1)^2$	**Use $i^2 = -1$.**
	$= 9i + 1$	**Simplify.**
	$= 1 + 9i$	**Write in standard form.**

GUIDED PRACTICE for Examples 3, 4 and 5

12. $(3 + i)(5 - i)$

$= 15 - 3i + 5i - i^2$ **Multiply using FOIL.**

$= 15 - 3i + 5i - (1)^2$ **Simplify and use $i^2 = -1$.**

$= 15 - 3i + 5i + 1$ **Simplify.**

$= 16 + 2i$ **Write in standard form.**

GUIDED PRACTICE for Examples 3, 4 and 5

13. $\dfrac{5}{1+i}$

$$\frac{5}{1+i} = \frac{5}{1+i} \cdot \frac{1-i}{1-i}$$

Multiply numerator and denominator by $1 - i$, the complex conjugate of $1 + i$.

$$= \frac{5 - 5i}{1 - i + i - i^2}$$

Multiply using FOIL.

$$= \frac{5 - 5i}{1 + 1}$$

Simplify and use $i^2 = 1$.

$$= \frac{5 - 5i}{2}$$

Simplify.

GUIDED PRACTICE for Examples 3, 4 and 5

$$= -\frac{5}{2} - \frac{5}{2}i$$

Write in standard form.

GUIDED PRACTICE for Examples 3, 4 and 5

14. $\dfrac{5 + 2i}{3 - 2i}$

$$\frac{5 + 2i}{3 - 2i} = \frac{5 + 2i}{3 - 2i} \cdot \frac{3 + 2i}{3 + 2i}$$

Multiply numerator and denominator $3 + 2i$, the complex conjugate of $3 - 2i$.

$$= \frac{15 + 10i + 6i + 4i^2}{9 + 6i - 6i - 4i^2}$$

Multiply using FOIL.

$$= \frac{15 + 16i + 4(-1)}{9 - 4(-1)^2}$$

Simplify and use $i^2 = 1$.

$$= \frac{11 + 16i}{13}$$

Simplify.

GUIDED PRACTICE for Examples 3, 4 and 5

$$= -\frac{11}{13} + \frac{16}{13}i$$

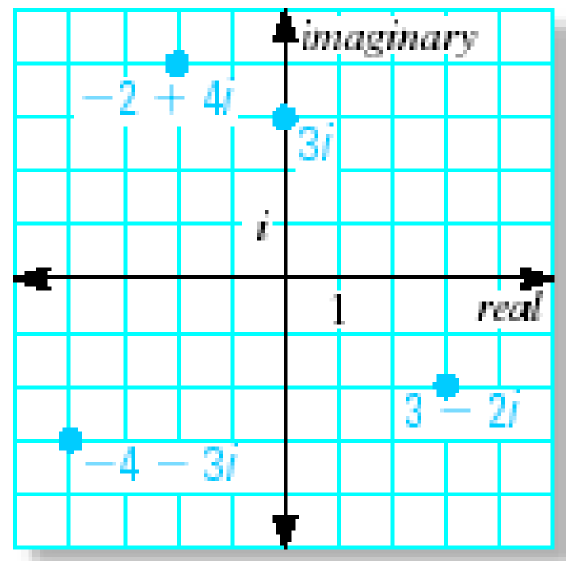
Write in standard form.

EXAMPLE 6 Plot complex numbers

Plot the complex numbers in the same complex plane.

a. $3 – 2i$ **b.** $– 2 + 4i$ **c.** $3i$ **d.** 24 2 $3i$

SOLUTION



a. **To plot $3 – 2i$, start at the origin, move 3 units to the right, and then move 2 units down.**

b. **To plot $– 2 + 4i$, start at the origin, move 2 units to the left, and then move 4 units up.**

c. **To plot $3i$, start at the origin and move 3 units up.**

d. **To plot $– 4 – 3i$, start at the origin, move 4 units to the left, and then move 3 units down.**

EXAMPLE 7 Find absolute values of complex numbers

Find the absolute value of (a) $-4 + 3i$ **and** (b) $-3i$**.**

a. $|-4 + 3i| = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5$

b. $|-3i| = |0 + (-3i)| = \sqrt{0^2 + (-3)^2} = \sqrt{9} = 3$

GUIDED PRACTICE for Examples 6 and 7

15. $4 - i$

SOLUTION

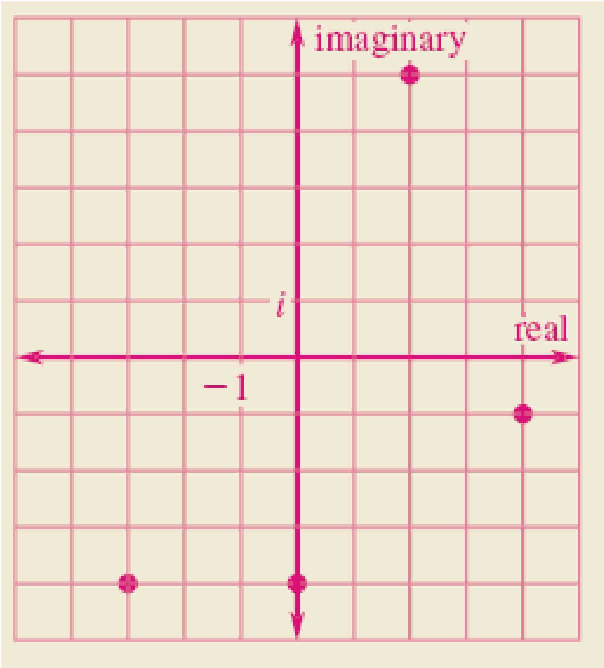
To plot $4 - i$, start at the origin, move 3 units to the right, and then move 1 units down.

$$\left| -4 + i \right|$$

$$= \sqrt{(4)^2 + (i)^2}$$

$$= \sqrt{16 + 1}$$

$$= \sqrt{17}$$



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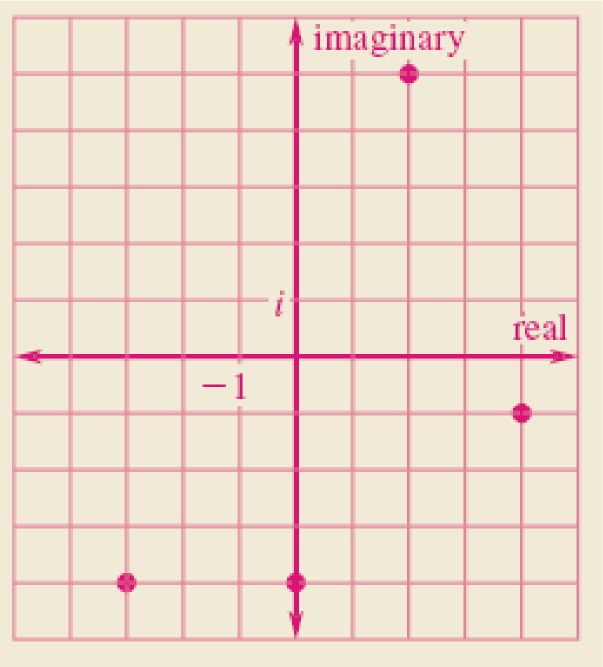
GUIDED PRACTICE for Examples 6 and 7

16. $-3-4i$

SOLUTION

To plot $-3-4i$**, start at the origin, move** 3 **units to the right, and then move** 4 **units down.**

$$|-3-4i|$$
$$= \sqrt{(-3)^2+(-4)^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$= 5$$



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GUIDED PRACTICE for Examples 6 and 7

17. $2 + 5i$

SOLUTION

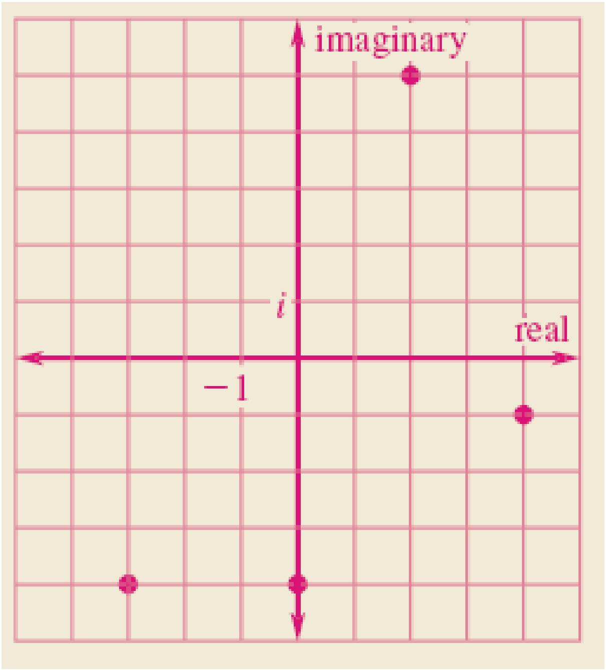
To plot $2 + 5i$, start at the origin, move 2 units to the right, and then move 5 units down.

$$|2 + 5i|$$

$$= \sqrt{(2)^2 + (5)^2}$$

$$= \sqrt{4 + 25}$$

$$= \sqrt{29}$$



GUIDED PRACTICE for Examples 6 and 7

18. $-4i$

SOLUTION

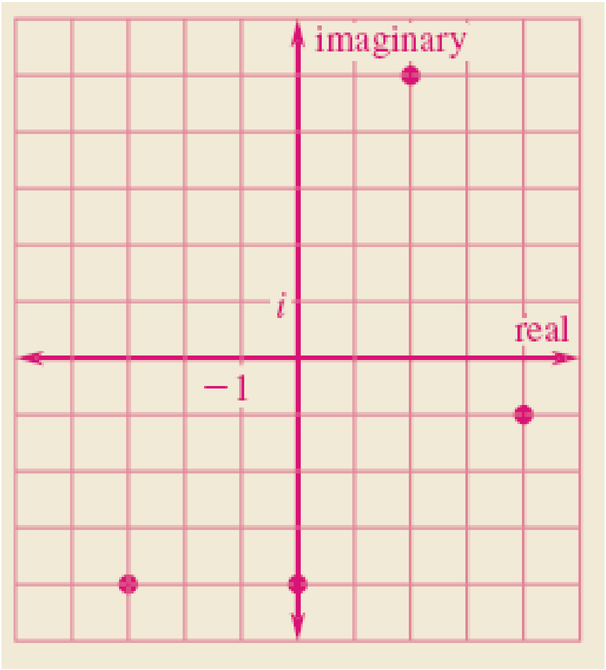
To plot $-4\,i$**, start at the origin, move** 4 **units down.**

$$\left|\ 4\,i\ \right|$$

$$= \sqrt{(4)^2}$$

$$= \sqrt{16}$$

$$= 4$$



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