EXAMPLE 1 Solve a quadratic equation by finding square roots

Solve $x^2 - 8x + 16 = 25$.

$x^2 - 8x + 16 = 25$	Write original equation.
$(x - 4)^2 = 25$	Write left side as a binomial squared.
$x - 4 = \pm 5$	Take square roots of each side.
$x = 4 \pm 5$	Solve for x.

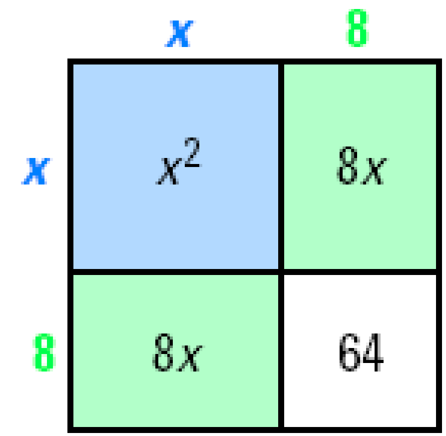
ANSWER

The solutions are $4 + 5 = 9$ and $4 - 5 = -1$.

EXAMPLE 2 Make a perfect square trinomial

Find the value of c that makes $x^2 + 16x + c$ a perfect square trinomial. Then write the expression as the square of a binomial.

SOLUTION



STEP 1

Find half the coefficient of x. $\frac{16}{2} = 8$

STEP 2

Square the result of Step 1. $8^2 = 64$

STEP 3

Replace c with the result of Step 2. $x^2 + 16x + 64$

EXAMPLE 2 Make a perfect square trinomial

ANSWER

The trinomial $x^2 + 16x + c$ **is a perfect square when** $c = 64$. **Then** $x^2 + 16x + 64 = (x + 8)(x + 8) = (x + 8)^2$.

GUIDED PRACTICE for Examples 1 and 2

Solve the equation by finding square roots.

1. $x^2 + 6x + 9 = 36$.

$x^2 + 6x + 9 = 36$ — **Write original equation.**

$(x + 3)^2 = 36$ — **Write left side as a binomial squared.**

$x + 3 = \underline{+}\, 6$ — **Take square roots of each side.**

$x = -3 \,\underline{+}\, 6$ — **Solve for x.**

ANSWER

The solutions are $-3 + 6 = 3$ **and** $-3 - 6 = -9$.

GUIDED PRACTICE for Examples 1 and 2

2. $x^2 - 10x + 25 = 1$.

$x^2 - 10x + 25 = 1$	**Write original equation.**
$(x - 5)^2 = 1$	**Write left side as a binomial squared.**
$x - 5 = \pm 1$	**Take square roots of each side.**
$x = 5 \pm 1$	**Solve for x.**

ANSWER

The solutions are $5 - 1 = 4$ **and** $5 + 1 = 6$.

GUIDED PRACTICE for Examples 1 and 2

3. $x^2 - 24x + 144 = 100.$

$x^2 - 24x + 144 = 100$	**Write original equation.**
$(x - 12)^2 = 100$	**Write left side as a binomial squared.**
$x - 12 = \pm 10$	**Take square roots of each side.**
$x = 12 \pm 10$	**Solve for x.**

ANSWER

The solutions are $12 - 10 = 2$ and $12 + 10 = 22$.

GUIDED PRACTICE for Examples 1 and 2

4. $x^2 + 14x + c$

Find the value of *c* that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

	x	7
x	x^2	$7x$
7	$7x$	49

SOLUTION

STEP 1

Find half the coefficient of x. $\frac{14}{2} = 7$

STEP 2

Square the result of Step 1. $7^2 = 49$

STEP 3

Replace c with the result of Step 2. $x^2 + 14x + 49$

GUIDED PRACTICE for Examples 1 and 2

ANSWER

The trinomial $x^2 + 14x + c$ **is a perfect square when** $c = 49$.

Then $x^2 + 14x + 49 = (x + 7)(x + 7) = (x + 7)^2$.

GUIDED PRACTICE for Examples 1 and 2

5. $x^2 + 22x + c$

	x	11
x	x^2	$11x$
11	$11x$	121

SOLUTION

STEP 1

Find half the coefficient of x. $\frac{22}{2} = 22$

STEP 2

Square the result of Step 1. $11^2 = 121$

STEP 3

Replace c with the result of Step 2. $x^2 + 22x + 121$

ANSWER

The trinomial $x^2 + 22x + c$ **is a perfect square when** $c = 144$.

Then $x^2 + 22x + 144 = (x + 11)(x + 11) = (x + 11)^2$.

6. $x^2 - 9x + c$

	x	$-\frac{9}{2}$
x	x^2	$-\frac{9}{2}x$
$-\frac{9}{2}$	$-\frac{9}{2}x$	$\frac{81}{4}$

SOLUTION

STEP 1

Find half the coefficient of x**.** $\frac{-9}{2} = -\frac{9}{2}$

STEP 2

Square the result of Step 1**.** $\left(-\frac{9}{2}\right) = \frac{81}{4}$

STEP 3

Replace c **with the result of Step** 2**.** $x^2 - 9x + \frac{81}{4}$

GUIDED PRACTICE for Examples 1 and 2

ANSWER

The trinomial $x^2 - 9x + c$ **is a perfect square when** $c = \frac{81}{4}$.

Then $x^2 - 9x + \frac{81}{4} = (x - \frac{9}{2})(x - \frac{9}{2}) = (x - \frac{9}{2})^2$.

EXAMPLE 3 Solve $ax^2 + bx + c = 0$ when $a = 1$

Solve $x^2 - 12x + 4 = 0$ by completing the square.

$x^2 - 12x + 4 = 0$ — **Write original equation.**

$x^2 - 12x = -4$ — **Write left side in the form $x^2 + bx$.**

$x^2 - 12x + 36 = -4 + 36$ — **Add $\left(\frac{-12}{2}\right)^2 = (-6)^2 = 36$ to each side.**

$(x - 6)^2 = 32$ — **Write left side as a binomial squared.**

$x - 6 = \pm\sqrt{32}$ — **Take square roots of each side.**

$x = 6 \pm \sqrt{32}$ — **Solve for x.**

$x = 6 \pm 4\sqrt{2}$ — **Simplify:** $\sqrt{32} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$

ANSWER

The solutions are $6 + 4\sqrt{2}$ and $6 - 4\sqrt{2}$

Solve $ax^2 + bx + c = 0$ when $a = 1$

CHECK

You can use algebra or a graph.

Algebra Substitute each solution in the original equation to verify that it is correct.

Graph Use a graphing calculator to graph

$y = x^2 - 12x + 4$. **The x-intercepts are about** $0.34 \approx 6 - 4\sqrt{2}$ **and** $11.66 \approx 6 + 4\sqrt{2}$

EXAMPLE 4 Solve $ax^2 + bx + c = 0$ when $a \neq 1$

Solve $2x^2 + 8x + 14 = 0$ by completing the square.

$2x^2 + 8x + 14 = 0$	Write original equation.
$x^2 + 4x + 7 = 0$	Divide each side by the coefficient of x^2.
$x^2 + 4x = -7$	Write left side in the form $x^2 + bx$.
$x^2 - 4x + 4 = -7 + 4$	Add $\left(\frac{4}{2}\right)^2 = 2^2 = 4$ to each side.
$(x + 2)^2 = -3$	Write left side as a binomial squared.
$x + 2 = \pm\sqrt{-3}$	Take square roots of each side.
$x = -2 \pm \sqrt{-3}$	Solve for x.
$x = -2 \pm i\sqrt{3}$	Write in terms of the imaginary unit i.

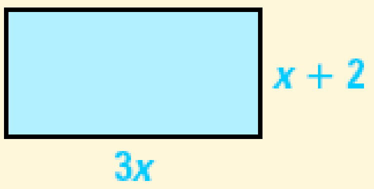
EXAMPLE 4 Solve $ax^2 + bx + c = 0$ when $a \neq 1$

ANSWER

The solutions are $-2 + i\sqrt{3}$ **and** $-2 - i\sqrt{3}$.

EXAMPLE 5 Standardized Test Practice

The area of the rectangle shown is 72 square units. What is the value of x?



(A) -6

(B) 4

(C) 8.48

(D) -6 or 4

SOLUTION

Use the formula for the area of a rectangle to write an equation.

EXAMPLE 5 Standardized Test Practice

$3x(x + 2) = 72$	**Length $\times$ Width = Area**
$3x^2 + 6x = 72$	**Distributive property**
$x^2 + 2x = 24$	**Divide each side by the coefficient of x^2.**
$x^2 - 2x + 1 = 24 + 1$	**Add $\left(\frac{2}{2}\right)^2 = 1^2 = 1$ to each side.**
$(x + 1)^2 = 25$	**Write left side as a binomial squared.**
$x + 1 = \pm 5$	**Take square roots of each side.**
$x = -1 \pm 5$	**Solve for x.**

EXAMPLE 5 Standardized Test Practice

So, $x = -1 + 5 = 4$ or $x = -1 - 5 = -6$. **You can reject** $x = -6$ **because the side lengths would be** -18 **and** -4, **and side lengths cannot be negative.**

ANSWER

The value of x **is** 4. **The correct answer is** B. Ⓐ Ⓑ Ⓒ Ⓓ

GUIDED PRACTICE for Examples 3, 4 and 5

7. Solve $x^2 + 6x + 4 = 0$ by completing the square.

$x^2 + 6x + 4 = 0$	Write original equation.
$x^2 + 6x = -4$	Write left side in the form $x^2 + bx$.
$x^2 + 6x + 9 = -4 + 9$	Add $\left(\frac{6}{2}\right)^2 = (3)^2 = 9$ to each side.
$(x + 3)^2 = 5$	Write left side as a binomial squared.
$x + 3 = \pm\sqrt{5}$	Take square roots of each side.
$x = -3 \pm \sqrt{5}$	Solve for x.

ANSWER

The solutions are $-3+\sqrt{2}$ **and** $-3-\sqrt{5}$

GUIDED PRACTICE for Examples 3, 4 and 5

8. Solve $x^2 - 10x + 8 = 0$ by completing the square.

$x^2 - 10x + 8 = 0$ Write original equation.

$x^2 - 10x = -8$ Write left side in the form $x^2 + bx$.

$x^2 - 10x + 25 = -8 + 25$ Add $\left(\frac{10}{2}\right)^2 = (5)^2 = 25$ to each side.

$(x - 5)^2 = 17$ Write left side as a binomial squared.

$x - 5 = \pm\sqrt{17}$ Take square roots of each side.

$x = 5 \pm\sqrt{17}$ Solve for x.

ANSWER

The solutions are $5 + \sqrt{17}$ and $5 - \sqrt{17}$

GUIDED PRACTICE for Examples 3, 4 and 5

9. Solve $2n^2 - 4n - 14 = 0$ by completing the square.

$2n^2 - 4n - 14 = 0$ — Write original equation.

$2n^2 - 4n = 14$ — Write left side in the form $x^2 + bx$.

$n^2 - 2n = 7$ — Divided each side by 2.

$n^2 - 2n + 1 = 7 + 1$ — Add $\left(\frac{2}{1}\right)^2 = (1)^2 = 1$ to each side.

$(n - 1)^2 = 8$ — Write left side as a binomial squared.

$n - 1 = \pm\sqrt{8}$ — Take square roots of each side.

$n = 1 \pm 2\sqrt{2}$ — Solve for n.

ANSWER

The solutions are $1 + 2\sqrt{2}$ and $1 - 2\sqrt{2}$

GUIDED PRACTICE for Examples 3, 4 and 5

10. Solve $3x^2 + 12n - 18 = 0$ by completing the square.

$3x^2 + 12n - 18 = 0$	Write original equation.
$x^2 + 4n - 6 = 0$	Divided each side by the coefficient of x^2.
$x^2 + 4n = 6$	Write left side in the form $x^2 + bx$.
$x^2 + 4x + 4 = 6 + 4$	Add $\left(\frac{4}{2}\right)^2 = (2)^2 = 4$ to each side.
$(x + 2)^2 = 10$	Write left side as a binomial squared.
$x + 2 = \pm\sqrt{10}$	Take square roots of each side.
$x = -2 \pm \sqrt{10}$	Solve for x.

GUIDED PRACTICE for Examples 3, 4 and 5

ANSWER

The solutions are $-2+\sqrt{10}$ **and** $-2-\sqrt{10}$

GUIDED PRACTICE for Examples 3, 4 and 5

11. $6x(x + 8) = 12$

$6x(x + 8) = 12$	Write original equation
$6x^2 + 48x = 12$	Distributive property
$x^2 + 8x = 2$	Divide each side by the coefficient of x^2.
$x^2 + 8x + 16 = 2 + 16$	Add $\left(\frac{8}{2}\right)^2 = 4^2 = 16$ to each side.
$(x + 4)^2 = 18$	Write left side as a binomial squared.
$x + 4 = \pm 3\sqrt{2}$	Take square roots of each side.
$x = -4 \pm 3\sqrt{2}$	Solve for x.

GUIDED PRACTICE for Examples 3, 4 and 5

ANSWER

The solutions are $-4 + 3\sqrt{2}$ **and** $-4 - 3\sqrt{2}$

GUIDED PRACTICE for Examples 3, 4 and 5

11. $4p(p-2) = 100$

$4p(p-2) = 100$	Write original equation
$4p^2 - 8p = 100$	Distributive property
$p^2 - 2p = 25$	Divide each side by the coefficient of p^2.
$p^2 - 2x + 1 = 25 + 1$	Add $\left(\frac{8}{2}\right)^2 = 4^2 = 16$ to each side.
$(p-1)^2 = 26$	Write left side as a binomial squared.
$p - 1 = \sqrt{26}$	Take square roots of each side.
$p = 1 \pm \sqrt{26}$	Solve for x.

GUIDED PRACTICE for Examples 3, 4 and 5

ANSWER

The solutions are $1 + \sqrt{26}$ **and** $1 - \sqrt{26}$

EXAMPLE 6 Write a quadratic function in vertex form

Write $y = x^2 - 10x + 22$ in vertex form. Then identify the vertex.

$y = x^2 - 10x + 22$	**Write original function.**
$y + ? = (x^2 - 10x + ?) + 22$	**Prepare to complete the square.**
$y + 25 = (x^2 - 10x + 25) + 22$	**Add $\left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$ to each side.**
$y + 25 = (x - 5)^2 + 22$	**Write $x^2 - 10x + 25$ as a binomial squared.**
$y = (x - 5)^2 - 3$	**Solve for y.**

ANSWER

The vertex form of the function is $y = (x - 5)^2 - 3$. The vertex is $(5, -3)$.

EXAMPLE 7 Find the maximum value of a quadratic function

Baseball

The height y (in feet) of a baseball t seconds after it is hit is given by this function:

$$y = -16t^2 + 96t + 3$$

Find the maximum height of the baseball.

SOLUTION

The maximum height of the baseball is the y-coordinate of the vertex of the parabola with the given equation.

EXAMPLE 7 Find the maximum value of a quadratic function

$y = -16t^2 + 96t + 3$ Write original function.

$y = -16(t^2 - 6t) + 3$ Factor -16 from first two terms.

$y + (-16)(?) = -16(t^2 - 6t + ?) + 3$ Prepare to complete the square.

$y + (-16)(9) = -16(t^2 - 6t + 9) + 3$ Add $(-16)(9)$ to each side.

$y - 144 = -16(t - 3)^2 + 3$ Write $t^2 - 6t + 9$ as a binomial squared.

$y = -16(t - 3)^2 + 147$ Solve for y.

ANSWER

The vertex is (3, 147), so the maximum height of the baseball is 147 feet.

GUIDED PRACTICE for Examples 6 and 7

13. **Write $y = x^2 - 8x + 17$ in vertex form. Then identify the vertex.**

$y = x^2 - 8x + 17$	**Write original function.**
$y + ? = (x^2 - 8x + ?) + 17$	**Prepare to complete the square.**
$y + 16 = (x^2 - 8x + 16) + 17$	**Add** $\left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$ **to each side.**
$y + 16 = (x - 4)^2 + 17$	**Write** $x^2 - 8x + 16$ **as a binomial squared.**
$y = (x - 4)^2 + 1$	**Solve for** y**.**

ANSWER

The vertex form of the function is $y = (x - 4)^2 + 1$**.**
The vertex is $(4, 1)$.

GUIDED PRACTICE for Examples 6 and 7

14. **Write $y = x^2 + 6x + 3$ in vertex form. Then identify the vertex.**

$y = x^2 + 6x + 3$	Write original function.
$y + ? = (x^2 + 6x + ?) + 3$	Prepare to complete the square.
$y + 9 = (x^2 + 6x + 9) + 3$	Add $\left(\frac{6}{2}\right)^2 = (3)^2 = 9$ to each side.
$y + 9 = (x + 3)^2 + 3$	Write $x^2 + 6x + 9$ as a binomial squared.
$y = (x + 3)^2 - 6$	Solve for y.

ANSWER

The vertex form of the function is $y = (x + 3)^2 - 6$. The vertex is $(-3, -6)$.

GUIDED PRACTICE for Examples 6 and 7

15. **Write $f(x) = x^2 - 4x - 4$ in vertex form. Then identify the vertex.**

$f(x) = x^2 - 4x - 4$ — Write original function.

$y + ? = (x^2 - 4x + ?) - 4$ — Prepare to complete the square.

$y + 4 = (x^2 - 4x + 4) - 4$ — Add $\left(\frac{-4}{2}\right)^2 = (-2)^2 = 4$ to each side.

$y + 4 = (x - 2)^2 - 4$ — Write $x^2 - 4x + 4$ as a binomial squared.

$y = (x - 2)^2 - 8$ — Solve for y.

ANSWER

The vertex form of the function is $y = (x - 2)^2 - 8$.
The vertex is $(2, -8)$.

GUIDED PRACTICE for Examples 6 and 7

16. **What if ? In example 7, suppose the height of the baseball is given by $y = -16t^2 + 80t + 2$. Find the maximum height of the baseball.**

SOLUTION

The maximum height of the baseball is the y-coordinate of the vertex of the parabola with the given equation.

$y = -16t^2 + 80t + 2$	**Write original function.**
$y = -4((2t)^2 - 20t) + 2$	**Factor -4 from first two terms.**
$y + (-4)(?) = -4((2t)^2 - 20t + ?) + 2$	**Prepare to complete the square.**

GUIDED PRACTICE for Examples 6 and 7

$y + (-4)(25) = -4((2t)^2 - 20t + 25) + 2$ **Add $(-4)(25)$ to each side.**

$y - 100 = -4(2t - 5)^2 + 2$ **Write $2t^2 - 20 + 25$ as a binomial squared.**

$y = -4(2t - 5)^2 + 102$ **Solve for y.**

ANSWER

The vertex is $(5, 102)$, so the maximum height of the baseball is 102 feet.