EXAMPLE 1 Solve an equation with two real solutions

Solve $x^2 + 3x = 2$.

$x^2 + 3x = 2$ — **Write original equation.**

$x^2 + 3x - 2 = 0$ — **Write in standard form.**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ — **Quadratic formula**

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-2)}}{2(1)}$$ $a = 1, b = 3, c = -2$

$$x = \frac{-3 \pm \sqrt{17}}{2}$$ — **Simplify.**

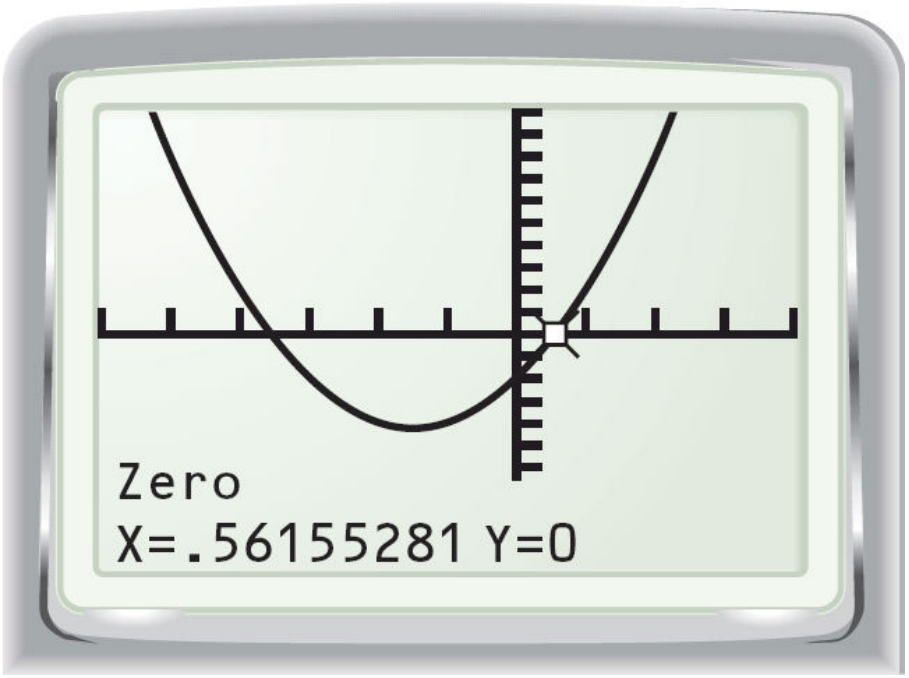
ANSWER

The solutions are $x = \frac{-3 + \sqrt{17}}{2} \approx 0.56$ **and** $x = \frac{-3 - \sqrt{17}}{2} \approx -3.56$.

EXAMPLE 1 Solve an equation with two real solutions

CHECK

Graph $y = x^2 + 3x - 2$ **and note that the** x**-intercepts are about** 0.56 **and about** -3.56**.** ✓



EXAMPLE 2 Solve an equation with one real solutions

Solve $25x^2 - 18x = 12x - 9.$

$25x^2 - 18x = 12x - 9.$ **Write original equation.**

$25x^2 - 30x + 9 = 0.$ **Write in standard form.**

$$x = \frac{30 \pm \sqrt{(-30)^2 - 4(25)(9)}}{2(25)}$$ $a = 25, b = -30, c = 9$

$$x = \frac{30 \pm \sqrt{0}}{50}$$ **Simplify.**

$$x = \frac{3}{5}$$ **Simplify.**

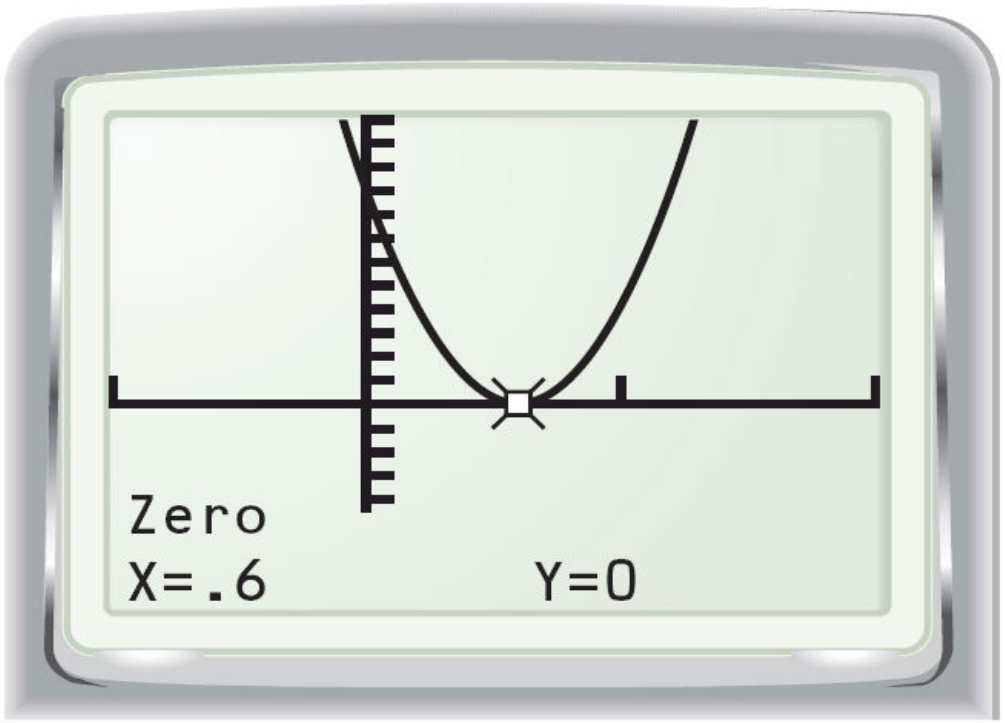
ANSWER

The solution is $\frac{3}{5}$

EXAMPLE 2 Solve an equation with one real solutions

CHECK

Graph $y = -5x^2 - 30x + 9$ **and note that the only** x**-intercept is** $0.6 = \frac{3}{5}$**.** ✓



EXAMPLE 3 Solve an equation with imaginary solutions

Solve $-x^2 + 4x = 5.$

$$-x^2 + 4x = 5$$ **Write original equation.**

$$-x^2 + 4x - 5 = 0.$$ **Write in standard form.**

$$x = \frac{-4 \pm \sqrt{-4^2 - 4(-1)(-5)}}{2(-1)}$$ $a = -1,\ b = 4,\ c = -5$

$$x = \frac{-4 \pm \sqrt{-4}}{-2}$$ **Simplify.**

$$x = \frac{-4 \pm 2i}{-2}$$ **Rewrite using the imaginary unit i.**

$$x = 2 \pm i$$ **Simplify.**

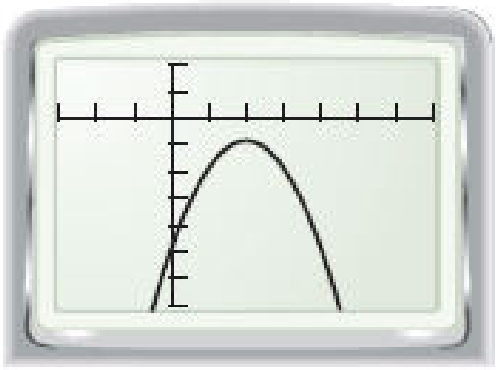
ANSWER

The solution is $2 + i$ and $2 - i$.

EXAMPLE 3 Solve an equation with imaginary solutions

CHECK

Graph $y = 2x^2 + 4x - 5$. **There are no** x**-intercepts. So, the original equation has no real solutions. The algebraic check for the imaginary solution** $2 + i$ **is shown.**



$$-(2 + i)^2 + 4(2 + i) \stackrel{?}{=} 5$$

$$-3 - 4i + 8 + 4i \stackrel{?}{=} 5$$

$$5 = 5 \checkmark$$

GUIDED PRACTICE for Examples 1, 2, and 3

Use the quadratic formula to solve the equation.

1. $x^2 = 6x - 4$

SOLUTION

$$x^2 = 6x - 4$$ **Write original equation.**

$$x^2 - 6x + 4 = 0$$ **Write in standard form.**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ **Quadratic formula**

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$$ $a = 1,\ b = -6,\ c = 4$

$$x = \frac{+3 \pm \sqrt{20}}{2}$$ **Simplify.**

GUIDED PRACTICE for Examples 1, 2, and 3

ANSWER

The solutions are $x = \dfrac{3 + \sqrt{20}}{2} = 3 + \sqrt{5}$ **and**

$x = \dfrac{3 - \sqrt{20}}{2} = 3 - \sqrt{5}$

GUIDED PRACTICE for Examples 1, 2, and 3

Use the quadratic formula to solve the equation.

2. $4x^2 - 10x = 2x - 9$

SOLUTION

$4x^2 - 10x = 2x - 9$ **Write original equation.**

$4x^2 - 12x + 9 = 0$ **Write in standard form.**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ **Quadratic formula**

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(9)}}{2(4)}$$ $a = 4,\ b = -12,\ c = 9$

$$x = \frac{12 \pm \sqrt{0}}{8}$$ **Simplify.**

GUIDED PRACTICE for Examples 1, 2, and 3

ANSWER

The solution is $\frac{3}{2} = 1\frac{1}{2}$ **.**

GUIDED PRACTICE for Examples 1, 2, and 3

Use the quadratic formula to solve the equation.

3. $7x - 5x^2 - 4 = 2x + 3$

SOLUTION

$7x - 5x^2 - 4 = 2x + 3$ **Write original equation.**

$-5x^2 + 5x - 7 = 0$ **Write in standard form.**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ **Quadratic formula**

$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(-5)(-7)}}{2(-5)}$$ $a = -5,\ b = 5,\ c = -7$

$$x = \frac{-5 \pm \sqrt{-115}}{-10}$$ **Simplify.**

GUIDED PRACTICE for Examples 1, 2, and 3

$$x = \frac{-5 \pm i\sqrt{115}}{-10}$$

Rewrite using the imaginary unit i.

$$x = \frac{5 \pm i\sqrt{115}}{10}$$

Simplify.

ANSWER

The solutions are $\frac{5 + i\sqrt{115}}{10}$ and $\frac{5 - i\sqrt{115}}{10}$.

EXAMPLE 4 Use the discriminant

Find the discriminant of the quadratic equation and give the number and type of solutions of the equation.

a. $x^2 - 8x + 17 = 0$ **b.** $x^2 - 8x + 16 = 0$ **c.** $x^2 - 8x + 15 = 0$

SOLUTION

Equation	Discriminant	Solution(s)
$ax^2 + bx + c = 0$	$b^2 - 4ac$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2ac}$
a. $x^2 - 8x + 17 = 0$	$(-8)^2 - 4(1)(17) = -4$	**Two imaginary:** $4 \pm i$
b. $x^2 - 8x + 16 = 0$	$(-8)^2 - 4(1)(16) = 0$	**One real:** 4
b. $x^2 - 8x + 15 = 0$	$(-8)^2 - 4(1)(15) = 0$	**Two real:** 3,5

GUIDED PRACTICE for Example 4

Find the discriminant of the quadratic equation and give the number and type of solutions of the equation.

4. $2x^2 + 4x - 4 = 0$

SOLUTION

Equation	Discriminant	Solution(s)
$ax^2 + bx + c = 0$	$b^2 - 4ac$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2ac}$
$2x^2 + 4x - 4 = 0$	$4^2 - 4(2)(-4)$	**Two real solutions**
	$= 48$	

GUIDED PRACTICE for Example 4

5. $3x^2 + 12x + 12 = 0$

SOLUTION

Equation	Discriminant	Solution(s)
$ax^2 + bx + c = 0$	$b^2 - 4ac$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2ac}$
$3x^2 + 12x + 12 = 0$	$12^2 - 4(12)(3) = 0$	**One real solution**

GUIDED PRACTICE for Example 4

6. $8x^2 = 9x - 11$

SOLUTION

Equation	Discriminant	Solution(s)
$ax^2 + bx + c = 0$	$b^2 - 4ac$	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2ac}$
$8x^2 - 9x + 11 = 0$	$(-9)^2 - 4(8)(11)$	
	$= -271$	**Two imaginary solutions**

GUIDED PRACTICE for Example 4

7. $7x^2 - 2x = 5$

SOLUTION

Equation	Discriminant	Solution(s)
$ax^2 + bx + c = 0$	$b^2 - 4ac$	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2ac}$
$7x^2 - 2x - 5 = 0$	$(-2)^2 - 4(7)(-5)$	**Two real solutions**
	$= 144$	

GUIDED PRACTICE for Example 4

8. $4x^2 + 3x + 12 = 3 - 3x$

SOLUTION

Equation	Discriminant	Solution(s)
$ax^2 + bx + c = 0$	$b^2 - 4ac$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2ac}$
$4x^2 + 6x + 9 = 0$	$(6)^2 - 4(4)(9)$	**Two imaginary solutions**
	$= -108$	

GUIDED PRACTICE for Example 4

9. $3x - 5x^2 + 1 = 6 - 7x$

SOLUTION

Equation	Discriminant	Solution(s)
$ax^2 + bx + c = 0$	$b^2 - 4ac$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2ac}$
$-5x^2 + 4x - 5 = 0$	$(4)^2 - 4(-5)(-5)$	
	$= 0$	**One real solution**

EXAMPLE 5 Solve a vertical motion problem

Juggling

A juggler tosses a ball into the air. The ball leaves the juggler's hand 4 feet **above the ground and has an initial vertical velocity of** 40 feet **per second. The juggler catches the ball when it falls back to a height of** 3 feet**. How long is the ball in the air?**

SOLUTION

Because the ball is thrown, use the model $h = -16t^2 + v_0t + h_0$**. To find how long the ball is in the air, solve for** t **when** $h = 3$**.**

EXAMPLE 5 Solve a vertical motion problem

$h = -16t^2 + v_0t + h_0$ **Write height model.**

$3 = -16t^2 + 40t + 4$ **Substitute** 3 **for** h**,** 40 **for** v_0**, and** 4 **for** h_0**.**

$0 = -16t^2 + 40t + 1$ **Write in standard form.**

$t = \dfrac{-40 \pm \sqrt{40^2 - 4(-16)(1)}}{2(-16)}$ **Quadratic formula**

$t = \dfrac{-40 \pm \sqrt{1664}}{-32}$ **Simplify.**

$t \approx -0.025$ or ≈ 2.5 **Use a calculator.**

EXAMPLE 5 **Solve a vertical motion problem**

ANSWER

Reject the solution -0.025 **because the ball's time in the air cannot be negative. So, the ball is in the air for about** 2.5 seconds**.**

GUIDED PRACTICE for Example 5

10. **What If? In Example 5, suppose the ball leaves the juggler's hand with an initial vertical velocity of 50 feet per second. How long is the ball in the air?**

SOLUTION

Because the ball is thrown, use the model $h = -16t^2 + v_0t + h_0$. To find how long the ball is in the air, solve for t when $h = 3$.

GUIDED PRACTICE for Example 5

$h = -16t^2 + v_0t + h_0$ **Write height model.**

$3 = -16t^2 + 50t + 4$ **Substitute** 3 **for** h**,** 50 **for** v_0**, and** 4 **for** h_0**.**

$0 = -16t^2 + 40t + 1$ **Write in standard form.**

$t = \dfrac{-50 \pm \sqrt{50^2 - 4(-16)(1)}}{2(-16)}$ **Quadratic formula**

$t = \dfrac{-50 \pm \sqrt{2564}}{-32}$ **Simplify.**

$t \approx -0.01$ **or** $t \approx 3.1$ **Use a calculator.**

GUIDED PRACTICE for Example 5

ANSWER

Reject the solution -0.01 **because the ball's time in the air cannot be negative. So, the ball is in the air for about** 2.5 seconds**.**