EXAMPLE 1 Identify polynomial functions

Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

a. $h(x) = x^4 - \frac{1}{4}x^2 + 3$

SOLUTION

a. **The function is a polynomial function that is already written in standard form. It has degree** 4 **(quartic) and a leading coefficient of** 1**.**

EXAMPLE 1 Identify polynomial functions

Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

b. $g(x) = 7x - \sqrt{3} + \pi x^2$

SOLUTION

b. **The function is a polynomial function written as** $g(x) = \pi x^2 + 7x - \sqrt{3}$ **in standard form. It has degree** 2 **(quadratic) and a leading coefficient of** π**.**

EXAMPLE 1 Identify polynomial functions

Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

c. $f(x) = 5x^2 + 3x^{-1} - x$

SOLUTION

c. **The function is not a polynomial function because the term** $3x^{-1}$ **has an exponent that is not a whole number.**

EXAMPLE 1 Identify polynomial functions

Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

d. $k(x) = x + 2^x - 0.6x^5$

SOLUTION

d. **The function is not a polynomial function because the term 2^x does not have a variable base and an exponent that is a whole number.**

EXAMPLE 2 **Evaluate by direct substitution**

Use direct substitution to evaluate
$f(x) = 2x^4 - 5x^3 - 4x + 8$ **when** $x = 3$**.**

$f(x) = 2x^4 - 5x^3 - 4x + 8$ — **Write original function.**

$f(3) = 2(3)^4 - 5(3)^3 - 4(3) + 8$ — **Substitute** 3 **for** x**.**

$= 162 - 135 - 12 + 8$ — **Evaluate powers and multiply.**

$= 23$ — **Simplify**

GUIDED PRACTICE for Examples 1 and 2

Decide whether the function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

1. $f(x) = 13 - 2x$

SOLUTION

$f(x) = -2x + 13$

It is a polynomial function.

Standard form: $-2x + 13$

Degree: 1

Type: linear

Leading coefficient of -2**.**

GUIDED PRACTICE for Examples 1 and 2

2. $p(x) = 9x^4 - 5x^{-2} + 4$

SOLUTION

$p(x) = 9x^4 - 5x^{-2} + 4$

The function is not a polynomial function.

GUIDED PRACTICE for Examples 1 and 2

3. $h(x) = 6x^2 + \pi - 3x$

SOLUTION

$h(x) = 6x^2 - 3x + \pi$

The function is a polynomial function that is already written in standard form will be $6x^2 - 3x + \pi$ **.** **It has degree** 2 (linear) **and a leading coefficient of** 6**.**

It is a polynomial function.
Standard form: $6x2 - 3x + \pi$
Degree: 2
Type: quadratic
Leading coefficient of 6

GUIDED PRACTICE for Examples 1 and 2

Use direct substitution to evaluate the polynomial function for the given value of x.

4. $f(x) = x^4 + 2x^3 + 3x^2 - 7;\ x = -2$

SOLUTION

$f(x) = x^4 + 2x^3 + 3x^2 - 7;\ x = -2$ **Write original function.**

$f(-2) = (-2)^4 + 2(-2)^3 + 3(-2)^2 - 7$ **Substitute -2 for x.**

$= 16 - 16 + 12 - 7$ **Evaluate powers and multiply.**

$= 5$ **Simplify**

GUIDED PRACTICE for Examples 1 and 2

5. $g(x) = x^3 - 5x^2 + 6x + 1;\ x = 4$

SOLUTION

$g(x) = x^3 - 5x^2 + 6x + 1;\ x = 4$ — **Write original function.**

$g(x) = 4^3 - 5(4)^2 + 6(4) + 1$ — **Substitute** 4 **for** x**.**

$= 64 - 80 + 24 + 1$ — **Evaluate powers and multiply.**

$= 9$ — **Simplify**

EXAMPLE 3 Evaluate by synthetic substitution

Use synthetic substitution to evaluate $f(x)$ from Example 2 when $x = 3$.

$f(x) = 2x4 – 5x3 – 4x + 8$

SOLUTION

STEP 1 **Write the coefficients of $f(x)$ in order of descending exponents. Write the value at which $f(x)$ is being evaluated to the left.**

x-value → $3 \;|\; 2 \quad -5 \quad 0 \quad -4 \quad 8$ ← coefficients

EXAMPLE 3 Evaluate by synthetic substitution

STEP 2 **Bring down** **the leading coefficient.** **Multiply** **the leading coefficient by the** x**-value. Write the product under the second coefficient.** **Add.**

$$
\begin{array}{r|rrrrr}
3 & 2 & -5 & 0 & -4 & 8 \\
 & & 6 & & & \\
\hline
 & 2 & 1 & & &
\end{array}
$$

STEP 3 **Multiply** **the previous sum by the** x**-value. Write the product under the third coefficient.** **Add.** **Repeat for all of the remaining coefficients. The final sum is the value of** $f(x)$ **at the given** x**-value.**

EXAMPLE 3 Evaluate by synthetic substitution

3	2	−5	0	−4	8
		6	3	9	15
	2	1	3	5	**23**

ANSWER **Synthetic substitution gives $f(3) = 23$, which matches the result in Example 2.**

EXAMPLE 4 Standardized Test Practice

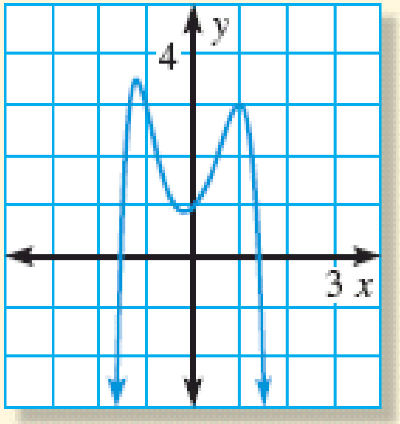
What is true about the degree and leading coefficient of the polynomial function whose graph is shown?

Ⓐ Degree is odd; leading coefficient is positive

Ⓑ Degree is odd; leading coefficient is negative

Ⓒ Degree is even; leading coefficient is positive

Ⓓ Degree is even; leading coefficient is negative



From the graph, $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$. So, the degree is even and the leading coefficient is negative.

ANSWER **The correct answer is D.** Ⓐ Ⓑ Ⓒ Ⓓ

GUIDED PRACTICE for Examples 3 and 4

Use synthetic substitution to evaluate the polynomial function for the given value of x.

6. $f(x) = 5x^3 + 3x^2 - x + 7;\ x = 2$

STEP 1 **Write the coefficients of $f(x)$ in order of descending exponents. Write the value at which $f(x)$ is being evaluated to the left.**

$$x\text{-value} \rightarrow \quad 2 \;\big|\; 5 \quad 3 \quad {-1} \quad 7 \quad \leftarrow \text{coefficients}$$

GUIDED PRACTICE for Examples 3 and 4

STEP 2 **Bring down the leading coefficient. Multiply the leading coefficient by the x-value. Write the product under the second coefficient. Add.**

2	5	3	-1	7
		10		
	5	13		

STEP 3 **Multiply the previous sum by the x-value. Write the product under the third coefficient. Add. Repeat for all of the remaining coefficients. The final sum is the value of $f(x)$ at the given x-value.**

GUIDED PRACTICE for Examples 3 and 4

$$
\begin{array}{r|rrrr}
2 & 5 & 3 & -1 & 7 \\
  &   & 10 & 26 & 50 \\
\hline
  & 5 & 13 & 25 & 57
\end{array}
$$

ANSWER **Synthetic substitution gives** $f(2) = 57$

GUIDED PRACTICE for Examples 3 and 4

7. $g(x) = -2x^4 - x^3 + 4x - 5;\ x = -1$

STEP 1 **Write the coefficients of $g(x)$ in order of descending exponents. Write the value at which $g(x)$ is being evaluated to the left.**

$$\begin{array}{r|rrrrr} -1 & -2 & -1 & 0 & 4 & -5 \\ & & & & & \\ \hline \end{array}$$

x-value → -1 ← coefficients

GUIDED PRACTICE for Examples 3 and 4

STEP 2 **Bring down** the leading coefficient. **Multiply** the leading coefficient by the x-value. Write the product under the second coefficient. **Add**.

$$\begin{array}{r|rrrrr} -1 & -2 & -1 & 0 & 4 & -5 \\ & & & & & \\ \hline & -2 & 1 & & & \end{array}$$

STEP 3 **Multiply** the previous sum by the x-value. Write the product under the third coefficient. **Add**. Repeat for all of the remaining coefficients. The final sum is the value of $f(x)$ at the given x-value.

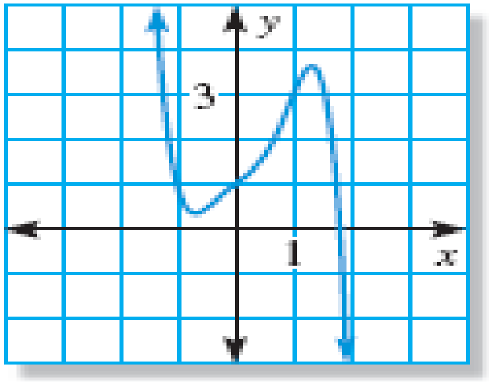
GUIDED PRACTICE for Examples 3 and 4

$$
\begin{array}{r|rrrrr}
-1 & -2 & -1 & 0 & 4 & -5 \\
 & & 2 & -1 & 1 & -10 \\
\hline
 & -2 & 1 & -1 & 5 & -10
\end{array}
$$

ANSWER **Synthetic substitution gives** $f(-1) = -10$

GUIDED PRACTICE for Examples 3 and 4

8. **Describe the degree and leading coefficient of the polynomial function whose graph is shown.**



ANSWER

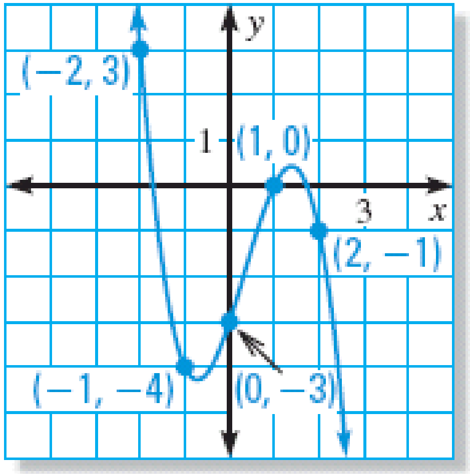
degree: odd, leading coefficient: negative

EXAMPLE 5 Graph polynomial functions

Graph $(a)\ f(x) = -x^3 + x^2 + 3x - 3$ **and**
$(b)\ f(x) = 5x^4 - x^3 - 4x^2 + 4$.

SOLUTION

a. **To graph the function, make a table of values and plot the corresponding points. Connect the points with a smooth curve and check the end behavior.**

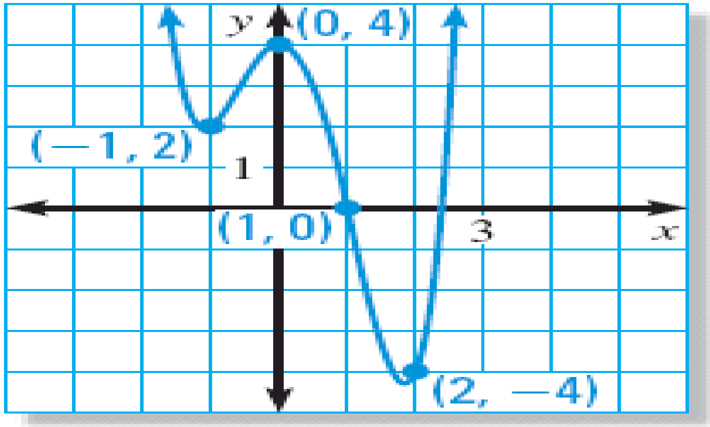


x	−3	−2	−1	0	1	2	3
y	24	3	−4	−3	0	−1	−12

The degree is odd and leading coefficient is negative. So, $f(x) \to +\infty$ **as** $x \to -\infty$ **and** $f(x) \to -\infty$ **as** $x \to +\infty$.

EXAMPLE 5 Graph polynomial functions

b. To graph the function, make a table of values and plot the corresponding points. Connect the points with a smooth curve and check the end behavior.



x	−3	−2	−1	0	1	2	3
y	76	12	2	4	0	−4	22

The degree is even and leading coefficient is positive. So, $f(x) \rightarrow \infty$ as $x \rightarrow -\infty$ and $f(x) \rightarrow \infty$ as $x \rightarrow +\infty$.

EXAMPLE 6 Solve a multi-step problem

Physical Science

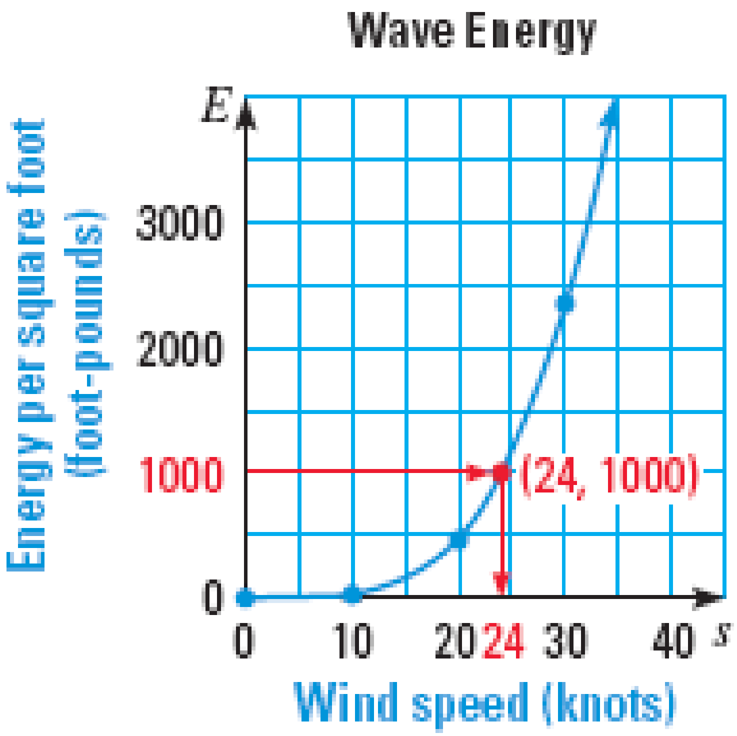
The energy E (in foot-pounds) in each square foot of a wave is given by the model $E = 0.0029s^4$ where s is the wind speed (in knots). Graph the model. Use the graph to estimate the wind speed needed to generate a wave with 1000 foot-pounds of energy per square foot.

EXAMPLE 6 Solve a multi-step problem

SOLUTION

STEP 1 **Make a table of values. The model only deals with positive values of s**

s	0	10	20	30	40
E	0	29	464	2349	7424



EXAMPLE 6 Solve a multi-step problem

STEP 2 Plot the points and connect them with a smooth curve. Because the leading coefficient is positive and the degree is even, the graph rises to the right.

STEP 3 Examine the graph to see that $s \approx 24$ when $E = 1000$.

ANSWER The wind speed needed to generate the wave is about 24 knots.

GUIDED PRACTICE for Examples 5 and 6

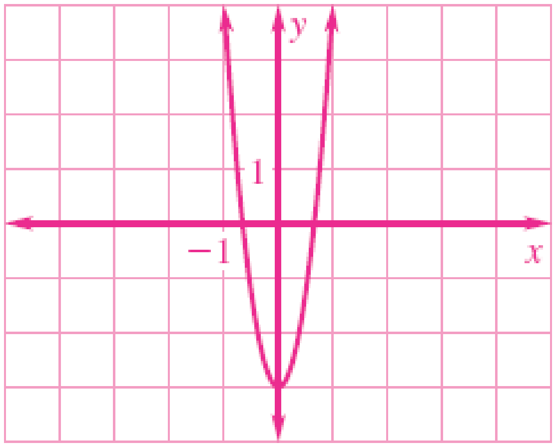
Graph the polynomial function.

9. $f(x) = x^4 + 6x^2 - 3$

SOLUTION

To graph the function, make a table of values and plot the corresponding points. Connect the points with a smooth curve and check the end behavior.

x	-2	-1	0	1	2
y	37	4	-3	4	37

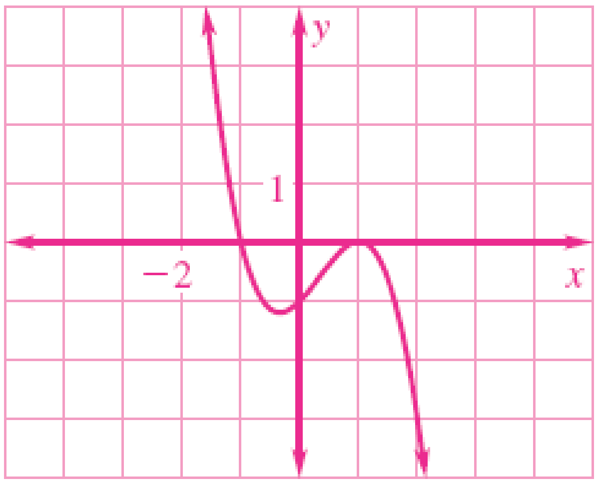


GUIDED PRACTICE for Examples 5 and 6

10. $f(x) = 2x^3 + x^2 + x - 1$

SOLUTION

To graph the function, make a table of values and plot the corresponding points. Connect the points with a smooth curve and check the end behavior.



x	– 3	–2	–1	0	1	2
y	40	9	0	–1.7	–3	2

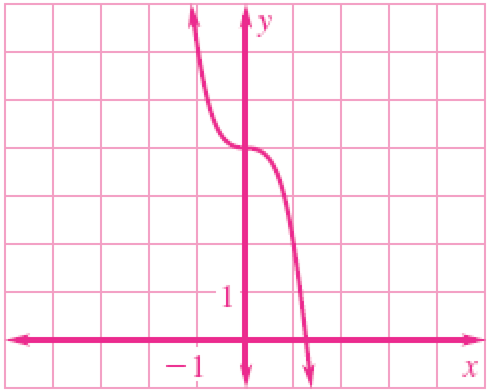
GUIDED PRACTICE for Examples 5 and 6

11. $f(x) = 4 - 2x^3$

SOLUTION

a. **To graph the function, make a table of values and plot the corresponding points. Connect the points with a smooth curve and check the end behavior.**

x	-2	-1	0	1	2
y	20	6	4	-12	2



GUIDED PRACTICE for Examples 5 and 6

12. **WHAT IF? If wind speed is measured in miles per hour, the model in Example 6 becomes $E = 0.0051s^4$. Graph this model. What wind speed is needed to generate a wave with 2000 foot-pounds of energy per square foot?**

ANSWER

about 25 mi/h.