EXAMPLE 1 Add polynomials vertically and horizontally

a. Add $2x^3 - 5x^2 + 3x - 9$ and $x^3 + 6x^2 + 11$ in a vertical format.

SOLUTION

a.

$$\begin{array}{rl} & 2x^3 - 5x^2 + 3x - 9 \\ + & x^3 + 6x^2 \qquad\quad + 11 \\ \hline & 3x^3 + x^2 + 3x + 2 \end{array}$$

EXAMPLE 1 Add polynomials vertically and horizontally

b. Add $3y^3 - 2y^2 - 7y$ and $-4y^2 + 2y - 5$ in a horizontal format.

$$(3y^3 - 2y^2 - 7y) + (-4y^2 + 2y - 5)$$

$$= 3y^3 - 2y^2 - 4y^2 - 7y + 2y - 5$$

$$= 3y^3 - 6y^2 - 5y - 5$$

EXAMPLE 2 Subtract polynomials vertically and horizontally

a. Subtract $3x^3 + 2x^2 - x + 7$ from $8x^3 - x^2 - 5x + 1$ in a vertical format.

SOLUTION

a. Align like terms, then add the opposite of the subtracted polynomial.

$$\begin{array}{r} 8x^3 - x^2 - 5x + 1 \\ -\ (3x^3 + 2x^2 - x + 7) \\ \hline \end{array}$$

$$\begin{array}{r} 8x^3 - \ x^2 - 5x + 1 \\ +\ -3x^3 - 2x^2 + \ x - 7 \\ \hline 5x^3 - 3x^2 - \ 4x - 6 \end{array}$$

EXAMPLE 2 Subtract polynomials vertically and horizontally

b. Subtract $5z^2 - z + 3$ from $4z^2 + 9z - 12$ in a horizontal format.

Write the opposite of the subtracted polynomial, then add like terms.

$$(4z^2 + 9z - 12) - (5z^2 - z + 3) = 4z^2 + 9z - 12 - 5z^2 + z - 3$$

$$= 4z^2 - 5z^2 + 9z + z - 12 - 3$$

$$= -z^2 + 10z - 15$$

GUIDED PRACTICE for Examples 1 and 2

Find the sum or difference.

1. $(t^2 - 6t + 2) + (5t^2 - t - 8)$

SOLUTION

$$\begin{array}{r} t^2 - 6t + 2 \\ +\ 5t^2 - t - 8 \\ \hline 6t^2 - 7t - 6 \end{array}$$

GUIDED PRACTICE for Examples 1 and 2

2. $(8d - 3 + 9d^3) - (d^3 - 13d^2 - 4)$

SOLUTION

$= (8d - 3 + 9d^3) - (d^3 - 13d^2 - 4)$

$= (8d - 3 + 9d^3) - d^3 + 13d^2 + 4)$

$= 9d^3 - 3\ d^3 + 13d^2 + 8d - 3 + 4$

$= 8d^3 + 13d^2 + 8d + 1$

EXAMPLE 3 Multiply polynomials vertically and horizontally

a. **Multiply** $-2y^2 + 3y - 6$ **and** $y - 2$ **in a vertical format.**

b. **Multiply** $x + 3$ **and** $3x^2 - 2x + 4$ **in a horizontal format.**

SOLUTION

a.

$$\begin{array}{r} -2y^2 + 3y - 6 \\ \times \qquad\quad y - 2 \\ \hline 4y^2 - 6y + 12 \\ -2y^3 + 3y^2 - 6y \\ \hline -2y^3 + 7y^2 - 12y + 12 \end{array}$$

Multiply $-2y^2 + 3y - 6$ **by** -2 **.**

Multiply $-2y^2 + 3y - 6$ **by** y

Combine like terms.

EXAMPLE 3 Multiply polynomials vertically and horizontally

b. $(x + 3)(3x^2 - 2x + 4) = (x + 3)3x^2 - (x + 3)2x + (x + 3)4$

$= 3x^3 + 9x^2 - 2x^2 - 6x + 4x + 12$

$= 3x^3 + 7x^2 - 2x + 12$

EXAMPLE 4 Multiply three binomials

Multiply $x - 5$, $x + 1$, and $x + 3$ in a horizontal format.

$$(x-5)(x+1)(x+3) = (x^2-4x-5)(x+3)$$

$$= (x^2-4x-5)x + (x^2-4x-5)3$$

$$= x^3 - 4x^2 - 5x + 3x^2 - 12x - 15$$

$$= x^3 - x^2 - 17x - 15$$

EXAMPLE 5 Use special product patterns

a. $(3t + 4)(3t - 4) = (3t)^2 - 4^2$

$= 9t^2 - 16$

Sum and difference

b. $(8x - 3)^2 = (8x)^2 - 2(8x)(3) + 3^2$

$= 64x^2 - 48x + 9$

Square of a binomial

c. $(pq + 5)^3 = (pq)^3 + 3(pq)^2(5) + 3(pq)(5)^2 + 5^3$

$= p^3q^3 + 15p^2q^2 + 75pq + 125$

Cube of a binomial

GUIDED PRACTICE for Examples 3, 4 and 5

Find the product.

3. $(x + 2)(3x^2 - x - 5)$

SOLUTION

$$\begin{array}{r} 3x^2 - x - 5 \\ x + 2 \\ \hline 6x^2 - 2x - 10 \\ 3x^3 - x^2 - 5x \\ \hline 3x^3 + 5x^2 - 7x - 10 \end{array}$$

Multiply $3x^2 - x - 5$ **by** 2 **.**

Multiply $3x^2 - x - 5$ **by** x **.**

Combine like terms.

GUIDED PRACTICE for Examples 3, 4 and 5

4. $(a-5)(a+2)(a+6)$

SOLUTION

$$(a-5)(a+2)(a+6) = (a^2-3a-10)(a+6)$$

$$= (a^2-3a-10)a + (a^2-3a-10)6$$

$$= (a^3-3a^2-10a+6a^2-18a-60)$$

$$= (a^3+3a^2-28a-60)$$

GUIDED PRACTICE for Examples 3, 4 and 5

5. $(xy - 4)^3$

SOLUTION

$$(xy - 4)^3 = (xy)^3 - 3(xy)^2 + 3(xy)(4)^2 - (4)^3$$

$$= x^3y^3 - 12x^2y^2 + 48xy - 64$$

EXAMPLE 6 Use polynomial models

Petroleum

Since 1980, the number W (in thousands) of United States wells producing crude oil and the average daily oil output per well O (in barrels) can be modeled by

Oil refinery in Long Beach, California

$$W = -0.575t^2 + 10.9t + 548 \quad \textbf{and} \quad O = -0.249t + 15.4$$

where t is the number of years since 1980. Write a model for the average total amount T of crude oil produced per day. What was the average total amount of crude oil produced per day in 2000?

EXAMPLE 6 Use polynomial models

SOLUTION

To find a model for T, multiply the two given models.

$$
\begin{array}{rrrr}
 & -0.575t^2 + & 10.9t + & 548 \\
\times & & -0.249t + & 15.4 \\
\hline
 & -8.855t^2 + & 167.86t + & 8439.2 \\
0.143175t^3 - & 2.7141t^2 - & 136.452t & \\
\hline
0.143175t^3 - & 11.5691t^2 + & 31.408t + & 8439.2
\end{array}
$$

EXAMPLE 6 Use polynomial models

ANSWER

Total daily oil output can be modeled by $T = 0.143t^3 - 11.6t^2 + 31.4t + 8440$ **where** T **is measured in thousands of barrels. By substituting** $t = 20$ **into the model, you can estimate that the average total amount of crude oil produced per day in** 2000 **was about** 5570 **thousand barrels, or** $5{,}570{,}000$ **barrels.**

GUIDED PRACTICE for Example 6

Industry

6. **The models below give the average depth D (in feet) of new wells drilled and the average cost per foot C (in dollars) of drilling a new well. In both models, t represents the number of years since 1980. Write a model for the average *total* cost T of drilling a new well.**

$$D = 109t + 4010$$

$$C = 0.542t^2 - 7.16t + 79.4$$

GUIDED PRACTICE for Example 6

SOLUTION

To find a model for T, multiply the two given models.

$$\begin{array}{rr} & 0.542t^2 - 7.16t + 79.4 \\ \times & 109t + 4010 \\ \hline & 2173.68t^2 + 28711.6t + 318394 \\ 59.078t^3 - & 780.44t^2 - 8654.6t \\ \hline 59.078t^3 + & 1392.98t^2 - 20057t + 318394 \end{array}$$

GUIDED PRACTICE for Example 6

ANSWER **Total daily oil output can be modeled by**

$$T = 59.078t^3 + 1392.98t^2 - 20{,}057t + 318394.$$