

EXAMPLE 1**Find a common monomial factor**

Factor the polynomial completely.

a. $x^3 + 2x^2 - 15x = x(x^2 + 2x - 15)$

$$= x(x + 5)(x - 3)$$

Factor common monomial.

Factor trinomial.

b. $2y^5 - 18y^3 = 2y^3(y^2 - 9)$

$$= 2y^3(y + 3)(y - 3)$$

Factor common monomial.

Difference of two squares

c. $4z^4 - 16z^3 + 16z^2 = 4z^2(z^2 - 4z + 4)$

$$= 4z^2(z - 2)^2$$

Factor common monomial.

Perfect square trinomial

EXAMPLE 2**Factor the sum or difference of two cubes**

Factor the polynomial completely.

a. $x^3 + 64 = x^3 + 4^3$

Sum of two cubes

$$= (x + 4)(x^2 - 4x + 16)$$

b. $16z^5 - 250z^2 = 2z^2(8z^3 - 125)$

Factor common monomial.

$$= 2z^2[(2z)^3 - 5^3]$$

Difference of two cubes

$$= 2z^2(2z - 5)(4z^2 + 10z + 25)$$

Factor the polynomial completely.

1. $x^3 - 7x^2 + 10x$

SOLUTION

$$\begin{aligned}x^3 - 7x^2 + 10x &= x^3 - 7x^2 + 10x \\ &= x(x^2 - 7x + 10) && \text{Factor common monomial.} \\ &= x(x - 5)(x - 2) && \text{Factor trinomial.}\end{aligned}$$

GUIDED PRACTICE

for Examples 1 and 2

2. $3y^5 - 75y^3$

SOLUTION

$$3y^5 - 75y^3 = 3y^3 (y^2 - 25)$$

Factor common monomial.

$$= 3y^3 (y - 5)(y + 5)$$

Difference of two squares

GUIDED PRACTICE

for Examples 1 and 2

3. $16b^5 + 686b^2$

SOLUTION

$$16b^5 + 686b^2 = 2b^2 (8b^3 + 343)$$

Factor common monomial.

$$= 2b^2 (2b + 7)(4b^2 - 14b + 49)$$

Difference of two cubes

4. $w^3 - 27$

SOLUTION

$$\begin{aligned}w^3 - 27 &= w^3 - (3)^3 \\ &= (w - 3)(w^2 + 3w + 9) \quad \text{Difference of two cubes}\end{aligned}$$

EXAMPLE 3**Factor by grouping**

Factor the polynomial $x^3 - 3x^2 - 16x + 48$ completely.

$$x^3 - 3x^2 - 16x + 48 = x^2(x - 3) - 16(x - 3) \quad \text{Factor by grouping.}$$

$$= (x^2 - 16)(x - 3) \quad \text{Distributive property}$$

$$= (x + 4)(x - 4)(x - 3) \quad \text{Difference of two squares}$$

EXAMPLE 4**Factor polynomials in quadratic form**

Factor completely: (a) $16x^4 - 81$ and (b) $2p^8 + 10p^5 + 12p^2$.

a. $16x^4 - 81 = (4x^2)^2 - 9^2$

$$= (4x^2 + 9)(4x^2 - 9)$$

$$= (4x^2 + 9)(2x + 3)(2x - 3)$$

Write as difference of two squares.

Difference of two squares

Difference of two squares

b. $2p^8 + 10p^5 + 12p^2 = 2p^2(p^6 + 5p^3 + 6)$

$$= 2p^2(p^3 + 3)(p^3 + 2)$$

Factor common monomial.

Factor trinomial in quadratic form.

Factor the polynomial completely.

5. $x^3 + 7x^2 - 9x - 63$

SOLUTION

$$x^3 + 7x^2 - 9x - 63 = x^2(x + 7) - 9(x + 3) \quad \text{Factor by grouping.}$$

$$= (x^2 - 9)(x + 7) \quad \text{Distributive property}$$

$$= (x + 3)(x - 3)(x + 7) \quad \text{Difference of two squares}$$

GUIDED PRACTICE

for Examples 3 and 4

6. $16g^4 - 625$

SOLUTION

a. $16g^4 - 625 = (4g^2)^2 - 25^2$

$$= (4g^2 + 25)(4g^2 - 25)$$

$$= (4g^2 + 25)(2g + 5)(2g - 5)$$

**Write as difference
of two squares.**

**Difference of two
squares**

**Difference of two
squares**

$$7. \quad 4t^6 - 20t^4 + 24t^2$$

SOLUTION

$$\begin{aligned} 4t^6 - 20t^4 + 24t^2 &= 4t^2(t^4 - 5t^2 - 6t) \\ &= 4t^2(t^2 - 3)(t^2 - 2) \end{aligned}$$

Factor common monomial.

Factor trinomial in quadratic form.

EXAMPLE 5**Standardized Test Practice**

What are the real-number solutions of the equation $3x^5 + 15x = 18x^3$?

Ⓐ 0, 1, 3, 5

Ⓑ -1, 0, 1

Ⓒ 0, 1, $\sqrt{5}$

Ⓓ $-\sqrt{5}$, -1, 0, 1, $\sqrt{5}$

SOLUTION

$$3x^5 + 15x = 18x^3$$

Write original equation.

$$3x^5 - 18x^3 + 15x = 0$$

Write in standard form.

$$3x(x^4 - 6x^2 + 5) = 0$$

Factor common monomial.

EXAMPLE 5**Standardized Test Practice**

$$3x(x^2 - 1)(x^2 - 5) = 0$$

Factor trinomial.

$$3x(x + 1)(x - 1)(x^2 - 5) = 0$$

Difference of two squares

$$x = 0, x = -1, x = 1, x = \sqrt{5}, \text{ or } x = -\sqrt{5}$$

Zero product property**ANSWER** The correct answer is *D*.**A****B****C****D**

Find the real-number solutions of the equation.

8. $4x^5 - 40x^3 + 36x = 0$

$$4x^5 - 40x^3 + 36x = 0$$

Write original equation.

$$4x(x^4 - 10x^2 + 9) = 0$$

Factor common monomial.

$$4x(x^2 - 1)(x^2 - 9) = 0$$

Factor trinomial.

$$4x(x + 1)(x - 1)(x - 3)(x + 3) = 0$$

Difference of two squares

$x = 0, x = -1, x = 1, x = 3$, or $x = -3$ **Zero product property**

ANSWER $-3, 3, -1, 1, 0$

GUIDED PRACTICE**for Example 5**

9. $2x^5 + 24x = 14x^3$

$$2x^5 + 24x = 14x^3$$

Write original equation.

$$2x^5 - 14x^3 + 24x = 0$$

Write in standard form.

$$2x(x^4 - 7x^2 + 12) = 0$$

Factor common monomial.

$$2x(x^2 - 4)(x^2 - 3) = 0$$

Factor trinomial.

$$2x(x + 2)(x - 2)(x^2 - 3) = 0$$

Difference of two squares

ANSWER

$$-\sqrt{3}, \sqrt{3}, 2, 0, -2$$

GUIDED PRACTICE**for Example 5**

10. $-27x^3 + 15x^2 = -6x^4$

$-27x^3 + 15x^2 = -6x^4$

Write original equation.

$6x^4 - 27x^3 - 15x^2 = 0$

Write in standard form.

$3x^2(2x^2 - 9x - 5) = 0$

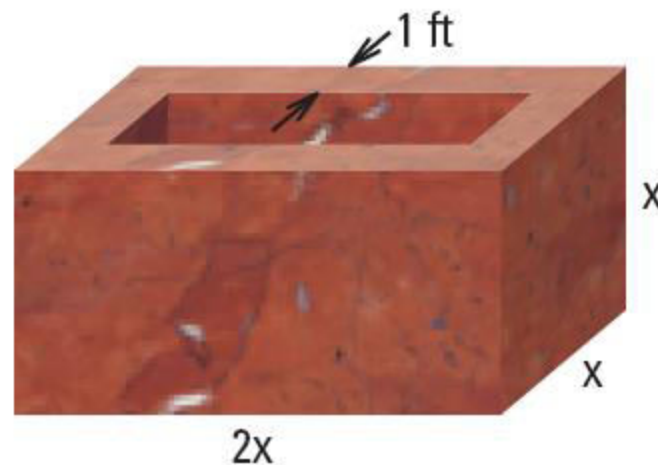
Factor common monomial.**ANSWER**

$0, \frac{9 \pm \sqrt{41}}{4}$

EXAMPLE 6**Solve a polynomial equation****City Park**

You are designing a marble basin that will hold a fountain for a city park. The basin's sides and bottom should be 1 foot thick. Its outer length should be twice its outer width and outer height.

What should the outer dimensions of the basin be if it is to hold 36 cubic feet of water?



EXAMPLE 6**Solve a polynomial equation****SOLUTION**

Volume (cubic feet)	=	Interior length (feet)	•	Interior width (feet)	•	Interior height (feet)
↓		↓		↓		↓
36	=	$(2x - 2)$	•	$(x - 2)$	•	$(x - 1)$

$$36 = (2x - 2)(x - 2)(x - 1)$$

Write equation.

$$0 = 2x^3 - 8x^2 + 10x - 40$$

Write in standard form.

$$0 = 2x^2(x - 4) + 10(x - 4)$$

Factor by grouping.

$$0 = (2x^2 + 10)(x - 4)$$

Distributive property

ANSWER

The only real solution is $x = 4$. The basin is 8 ft long, 4 ft wide, and 4 ft high.

11. **What if ?** In Example 6, what should the basin's dimensions be if it is to hold 128 cubic feet of water and have outer length $6x$, width $3x$, and height x ?

ANSWER

length: about 15.66 ft;
width: about 7.836 ft;
height: about 2.61 ft