

EXAMPLE 1**Find *n*th roots**

Find the indicated real *n*th root(s) of *a*.

a. $n = 3, a = -216$

b. $n = 4, a = 81$

SOLUTION

a. **Because $n = 3$ is odd and $a = -216 < 0$, -216 has one real cube root. Because $(-6)^3 = -216$, you can write $= \sqrt[3]{-216} = -6$ or $(-216)^{1/3} = -6$.**

b. **Because $n = 4$ is even and $a = 81 > 0$, 81 has two real fourth roots. Because $3^4 = 81$ and $(-3)^4 = 81$, you can write $\pm \sqrt[4]{81} = \pm 3$**

EXAMPLE 2**Evaluate expressions with rational exponents**

Evaluate (a) $16^{3/2}$ and (b) $32^{-3/5}$.

SOLUTION**Rational Exponent Form**

$$\text{a. } 16^{3/2} = (16^{1/2})^3 = 4^3 = 64$$

$$\begin{aligned} \text{b. } 32^{-3/5} &= \frac{1}{32^{3/5}} = \frac{1}{(32^{1/5})^3} \\ &= \frac{1}{2^3} = \frac{1}{8} \end{aligned}$$

Radical Form

$$16^{3/2} = (\sqrt{16})^3 = 4^3 = 64$$

$$\begin{aligned} 32^{-3/5} &= \frac{1}{32^{3/5}} = \frac{1}{(\sqrt[5]{32})^3} \\ &= \frac{1}{2^3} = \frac{1}{8} \end{aligned}$$

EXAMPLE 3**Approximate roots with a calculator****Expression****Keystrokes****Display**

a. $9^{1/5}$

9  (  5  

1.551845574

b. $12^{3/8}$

12  (  8  

2.539176951

c. $(\sqrt[4]{7})^3 = 7^{3/4}$

7  (  4  

4.303517071

GUIDED PRACTICE**for Examples 1, 2 and 3**

Find the indicated real n th root(s) of a .

1. $n = 4, a = 625$

SOLUTION

Because $n = 4$ is even and $a = 625 > 0$, 625 has two real n th roots. Because $5^4 = 625$ and $(-5)^4 = 625$, you can write ${}^4\sqrt{625} = \pm 5$ or $\pm(625)^{1/4} = \pm 5$.

2. $n = 6, a = 64$

SOLUTION

Because $n = 6$ is even and $a = 64 > 0$, 64 has two real n th roots. Because $2^6 = 64$ and $(-2)^6 = 64$, you can write ${}^6\sqrt{64} = \pm 2$ or $\pm(64)^{1/6} = \pm 2$.

GUIDED PRACTICE**for Examples 1, 2 and 3**

Find the indicated real *n*th root(s) of *a*.

3. $n = 3, a = -64.$

SOLUTION

Because $n = 3$ is odd and $a = -64 < 0$, -64 has one real cube root. Because $(-4)^3 = -64$, you can write ${}^3\sqrt{-64} = -4$ or $(-64)^{1/3} = -4.$

4. $n = 5, a = 243$

SOLUTION

Because $n = 5$ is odd and $a = 243 > 0$, 243 has one real *n*th root. Because $(3)^5 = 243$, you can write ${}^5\sqrt{243} = 3$ or $(243)^{1/5} = 3.$

GUIDED PRACTICE**for Examples 1, 2 and 3**

Evaluate expressions without using a calculator.

5. $4^{5/2}$

SOLUTION

$$4^{5/2} = (4^{1/2})^5 = 2^5 = 32$$

6. $9^{-1/2}$

SOLUTION

$$9^{-1/2} = (9^{1/2})^{-1} = 3^{-1} = \frac{1}{3}$$

GUIDED PRACTICE**for Examples 1, 2 and 3****Evaluate expressions without using a calculator.**

7. $81^{3/4}$

SOLUTION

$$81^{3/4} = (81^{1/4})^3 = 3^3 = 27$$




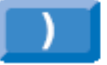




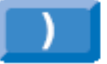











8. $1^{7/8}$

SOLUTION

$$1^{7/8} = (1^{1/8})^7 = (1)^7 = 1$$

GUIDED PRACTICE**for Examples 1, 2 and 3**

Evaluate the expression using a calculator. Round the result to two decimal places when appropriate.

Expression	Keystrokes	Display
9. $4^{2/5}$	4  (  5  	1.74
10. $64^{-2/3}$	$\frac{1}{64}$  ( 2  3  	0.06
11. $(\sqrt[4]{16})^5$	16  (  4  	32
12. $(\sqrt[3]{-30})^2$	-30  (  3  	9.65

EXAMPLE 4**Solve equations using *nth* roots**

Solve the equation.

a. $4x^5 = 128$

$$x^5 = 32$$

Divide each side by 4.

$$x = \sqrt[5]{32}$$

Take fifth root of each side.

$$x = 2$$

Simplify.

EXAMPLE 4**Solve equations using *nth* roots**

b. $(x - 3)^4 = 21$

$$x - 3 = \pm \sqrt[4]{21}$$

Take fourth roots of each side.

$$x = \pm \sqrt[4]{21} + 3$$

Add 3 to each side.

$$x = \sqrt[4]{21} + 3 \quad \mathbf{or} \quad x = -\sqrt[4]{21} + 3$$

Write solutions separately.

$$x \approx 5.14 \quad \mathbf{or} \quad x \approx 0.86$$

Use a calculator.

EXAMPLE 5

Use n th roots in problem solving

Biology

A study determined that the weight w (in grams) of coral cod near Palawan Island, Philippines, can be approximated using the model

$$w = 0.0167\ell^3$$

where ℓ is the coral cod's length (in centimeters). Estimate the length of a coral cod that weighs 200 grams.



EXAMPLE 5**Use n th roots in problem solving****SOLUTION**

$$w = 0.0167 l^3$$

Write model for weight.

$$200 = 0.0167 l^3$$

Substitute 200 for w .

$$11,976 \approx l^3$$

Divide each side by 0.0167.

$$\sqrt[3]{11,976} \approx l$$

Take cube root of each side.

$$22.9 \approx l$$

Use a calculator.

ANSWER

A coral cod that weighs 200 grams is about 23 centimeters long.

GUIDED PRACTICE**for Examples 4 and 5**

Solve the equation. Round the result to two decimal places when appropriate.

13. $x^3 = 64$

SOLUTION

$$x^3 = 64$$

$$x = \sqrt[3]{64}$$

$$x = 4$$

take 3rd root of each side.

Simplify.

GUIDED PRACTICE**for Examples 4 and 5**

$$14. \quad \frac{1}{2} x^5 = 512$$

SOLUTION

$$\frac{1}{2} x^5 = 512$$

$$x^5 = 1024$$

$$x = \sqrt[5]{1024}$$

$$x = 4$$

Multiply each side by 2.

take 5th root of each side.

Simplify.

GUIDED PRACTICE

Evaluate nth Roots and Use Rational Exponents

for Examples 4 and 5

15. $3x^2 = 108$

SOLUTION

$$3x^2 = 108$$

$$x^2 = 36$$

$$x = \sqrt[2]{36}$$

$$x = \pm 6$$

Divide each side by 3.

take 2nd root of each side.

Simplify.

GUIDED PRACTICE**for Examples 4 and 5**

$$16. \quad \frac{1}{4}x^3 = 2$$

SOLUTION

$$\frac{1}{4}x^3 = 2$$

$$x^3 = 8$$

$$x = \sqrt[3]{8}$$

$$x = 2$$

Multiply each side by 4.

take 3rd root of each side.

Simplify.

GUIDED PRACTICE**for Examples 4 and 5**

$$17. (x - 2)^3 = -14$$

SOLUTION

$$(x - 2)^3 = -14$$

$$(x - 2) = \sqrt[3]{-14}$$

$$x = \sqrt[3]{-14} + 2$$

$$x = \sqrt[3]{-14} + 2$$

$$x = -0.41$$

take 3rd root of each side.

add 2 to both sides.

Write solution.

Use a calculator.

GUIDED PRACTICE**for Examples 4 and 5**

$$18. \quad (x + 5)^4 = 16$$

SOLUTION

$$(x + 5)^4 = 16$$

$$(x + 5) = \pm \sqrt[4]{16}$$

$$x = \pm \sqrt[4]{16} - 5$$

$$x = 2 - 5 \quad \text{or} \quad x = -2 - 5$$

$$x = -3 \quad \text{or} \quad x = -7$$

take 4th root of each side.

add 5 to each side.

Write solutions separately.

Use a calculator.

GUIDED PRACTICE**for Examples 4 and 5**

19. WHAT IF? Use the information from Example 5 to estimate the length of a coral cod that has the given weight.

a. 275 grams

SOLUTION

$$w = 0.0167l^3$$

Write model for weight.

$$275 = 0.0167l^3$$

Substitute 275 for w .

$$16467 = l^3$$

Divide each side by 0.0167.

$$\sqrt[3]{16467} = l$$

Take cube root of each side.

$$25 = l$$

Use a calculator.

GUIDED PRACTICE

Evaluate nth Roots and Use Rational Exponents

for Examples 4 and 5

ANSWER

A coral cod that has the 275 grams is about 25 *cm* long.

GUIDED PRACTICE

Evaluate nth Roots and Use Rational Exponents

for Examples 4 and 5

b. 340 grams

SOLUTION

$$w = 0.0167l^3$$

$$340 = 0.0167l^3$$

$$20360 = l^3$$

$$\sqrt[3]{20360} = l$$

$$27 = l$$

Write model for weight.

Substitute 340 for w .

Divide each side by 0.0167

Take cube root of each side

Use a calculator

ANSWER

A coral cod that has the 340 grams is about 27 cm long.

GUIDED PRACTICE**for Examples 4 and 5**

c. 450 grams

SOLUTION

$$w = 0.0167l^3$$

Write model for weight.

$$450 = 0.0167l^3$$

Substitute 450 for w .

$$26946 = l^3$$

Divide each side by 0.0167

$$\sqrt[3]{26946} = l$$

Take cube root of each side

$$30 = l$$

Use a calculator**ANSWER**

A coral cod that has the 450 grams is about 30 *cm* long.