EXAMPLE 1 Find *nth* roots

Find the indicated real *nth* root(s) of a.

a. $n = 3, a = -216$ **b.** $n = 4, a = 81$

SOLUTION

a. **Because $n = 3$ is odd and $a = -216 < 0$, -216 has one real cube root. Because $(-6)^3 = -216$, you can write $= \sqrt[3]{-216} = -6$ or $(-216)^{1/3} = -6$.**

b. **Because $n = 4$ is even and $a = 81 > 0$, 81 has two real fourth roots. Because $3^4 = 81$ and $(-3)^4 = 81$, you can write $\pm\sqrt[4]{81} = \pm 3$**

EXAMPLE 2 Evaluate expressions with rational exponents

Evaluate (a) $16^{3/2}$ **and** (b) $32^{-3/5}$.

SOLUTION

Rational Exponent Form

a. $16^{3/2} = (16^{1/2})^3 = 4^3 = 64$

b. $32^{-3/5} = \frac{1}{32^{3/5}} = \frac{1}{(32^{1/5})^3}$

$= \frac{1}{2^3} = \frac{1}{8}$

Radical Form

$16^{3/2} = (\sqrt{16})^3 = 4^3 = 64$

$32^{-3/5} = \frac{1}{32^{3/5}} = \frac{1}{(\sqrt[5]{32})^3}$

$= \frac{1}{2^3} = \frac{1}{8}$

EXAMPLE 3 Approximate roots with a calculator

Expression	Keystrokes	Display
a. $9^{1/5}$	9 ^ (1 ÷ 5) ENTER	1.551845574
b. $12^{3/8}$	12 ^ (3 ÷ 8) ENTER	2.539176951
c. $(\sqrt[4]{7})^3 = 7^{3/4}$	7 ^ (3 ÷ 4) ENTER	4.303517071

GUIDED PRACTICE for Examples 1, 2 and 3

Find the indicated real *nth* root(s) of *a*.

1. $n = 4, a = 625$

SOLUTION

Because $n = 4$ **is even and** $a = 625 > 0$, 625 **has two real *nth* roots. Because** $5^4 = 625$ **and** $(-5)^4 = 625$, **you can write** $\sqrt[4]{625} = \pm 5$ or $\pm(625)1/4 = \pm 5$.

2. $n = 6, a = 64$

SOLUTION

Because $n = 6$ **is even and** $a = 64 > 0$, 64 **has two real *nth* roots. Because** $2^6 = 64$ **and** $(-2)^6 = 64$, **you can write** $\sqrt[6]{64} = \pm 2$ **or** $\pm(64)^{1/6} = \pm 2$.

GUIDED PRACTICE for Examples 1, 2 and 3

Find the indicated real *nth* root(s) of a.

3. $n = 3, a = -64$.

SOLUTION

Because $n = 3$ is odd and $a = -64 < 0$, -64 has one real cube root. Because $(-4)^3 = -64$, you can write $\sqrt[3]{-64} = -4$ or $(-64)^{1/3} = -4$.

4. $n = 5, a = 243$

SOLUTION

Because $n = 5$ is odd and $a = 243 > 0$, 243 has one real *nth* root. Because $(3)^5 = 243$, you can write $\sqrt[5]{243} = 3$ or $(243)^{1/5} = 3$.

GUIDED PRACTICE for Examples 1, 2 and 3

Evaluate expressions without using a calculator.

5. $4^{5/2}$

SOLUTION

$$4^{5/2} = (4^{1/2})^5 = 2^5 = 32$$

6. $9^{-1/2}$

SOLUTION

$$9^{-1/2} = (9^{1/2})^{-1} = 3^{-1} = \frac{1}{3}$$

GUIDED PRACTICE for Examples 1, 2 and 3

Evaluate expressions without using a calculator.

7. $81^{3/4}$

SOLUTION

$$81^{3/4} = (81^{1/4})^3 = 3^3 = 27$$

8. $1^{7/8}$

SOLUTION

$$1^{7/8} = (1^{1/8})^7 = (1)^7 = 1$$

GUIDED PRACTICE for Examples 1, 2 and 3

Evaluate the expression using a calculator. Round the result to two decimal places when appropriate.

	Expression	Keystrokes	Display
9.	$4^{2/5}$	4 [^] [(] 2 [÷] 5 [)] [ENTER]	1.74
10.	$64^{-2/3}$	$\frac{1}{64}$ [^] [(] - 2 [÷] 3 [)] [ENTER]	0.06
11.	$(\sqrt[4]{16})^5$	16 [^] [(] 5 [÷] 4 [)] [ENTER]	32
12.	$(\sqrt[3]{-30})^2$	–30 [^] [(] 2 [÷] 3 [)] [ENTER]	9.65

EXAMPLE 4 Solve equations using *nth* roots

Solve the equation.

a. $4x^5 = 128$

$x^5 = 32$ **Divide each side by 4.**

$x = \sqrt[5]{32}$ **Take fifth root of each side.**

$x = 2$ **Simplify.**

EXAMPLE 4 Solve equations using *nth* roots

b. $(x-3)^4 = 21$

$x - 3 = \pm \sqrt[4]{21}$ — Take fourth roots of each side.

$x = \pm \sqrt[4]{21} + 3$ — Add 3 to each side.

$x = \sqrt[4]{21} + 3$ **or** $x = -\sqrt[4]{21} + 3$ — Write solutions separately.

$x \approx 5.14$ **or** $x \approx 0.86$ — Use a calculator.

EXAMPLE 5 Use *nth* roots in problem solving

Biology

A study determined that the weight w **(in** grams**) of coral cod near Palawan Island, Philippines, can be approximated using the model**

$$w = 0.0167\ell^{3}$$

where l **is the coral cod's length (in** centimeters**). Estimate the length of a coral cod that weighs** 200 grams**.**

http://www.classzone.com/cz/books/algebra_2_2011_na/book_home.htm

EXAMPLE 5 Use *nth* roots in problem solving

SOLUTION

$w = 0.0167\, l^3$	**Write model for weight.**
$200 = 0.0167\, l^3$	**Substitute** 200 **for** w**.**
$11{,}976 \approx l^3$	**Divide each side by** 0.0167**.**
$\sqrt[3]{11{,}976} \approx l$	**Take cube root of each side.**
$22.9 \approx l$	**Use a calculator.**

ANSWER

A coral cod that weighs 200 grams **is about** 23 centimeters **long.**

GUIDED PRACTICE for Examples 4 and 5

Solve the equation. Round the result to two decimal places when appropriate.

13. $x^3 = 64$

SOLUTION

$$x^3 = 64$$

$$x = \sqrt[3]{64}$$ take $3rd$ root of each side.

$$x = 4$$ Simplify.

GUIDED PRACTICE for Examples 4 and 5

14. $\frac{1}{2}x^5 = 512$

SOLUTION

$$\frac{1}{2}x^5 = 512$$

$$x^5 = 1024 \qquad \text{Multiply each side by } 2.$$

$$x = \sqrt[5]{1024} \qquad \text{take } 5th \text{ root of each side.}$$

$$x = 4 \qquad \text{Simplify.}$$

GUIDED PRACTICE for Examples 4 and 5

15. $3x^2 = 108$

SOLUTION

$$3x^2 = 108$$

$$x^2 = 36 \qquad \text{Divide each side by } 3.$$

$$x = \sqrt[2]{36} \qquad \text{take } 2nd \text{ root of each side.}$$

$$x = \pm 6 \qquad \text{Simplify.}$$

GUIDED PRACTICE for Examples 4 and 5

16. $\frac{1}{4}x^3 = 2$

SOLUTION

$$\frac{1}{4}x^3 = 2$$

$$x^3 = 8 \qquad \text{Multiply each side by } 4.$$

$$x = \sqrt[3]{8} \qquad \text{take } 3rd \text{ root of each side.}$$

$$x = 2 \qquad \text{Simplify.}$$

GUIDED PRACTICE for Examples 4 and 5

17. $(x-2)^3 = -14$

SOLUTION

$$(x-2)^3 = -14$$

$$(x-2) = \sqrt[3]{-14}$$ take $3rd$ root of each side.

$$x = \sqrt[3]{-14} + 2$$ add 2 to both sides.

$$x = \sqrt[3]{-14} + 2$$ Write solution.

$$x = -0.41$$ Use a calculator.

GUIDED PRACTICE for Examples 4 and 5

18. $(x + 5)^4 = 16$

SOLUTION

$$(x + 5)^4 = 16$$

$$(x + 5) = \pm \sqrt[4]{16}$$ **take $4th$ root of each side.**

$$x = \pm \sqrt[4]{16} - 5$$ **add 5 to each side.**

$x = 2 - 5$ **or** $x = -2 - 5$ **Write solutions separately.**

$x = -3$ **or** $x = -7$ **Use a calculator.**

GUIDED PRACTICE for Examples 4 and 5

19. WHAT IF? Use the information from Example 5 to estimate the length of a coral cod that has the given weight.

a. 275 grams

SOLUTION

$w = 0.0167l^3$ **Write model for weight.**

$275 = 0.0167l^3$ **Substitute 275 for w.**

$16467 = l^3$ **Divide each side by 0.0167.**

$\sqrt[3]{16467} = l$ **Take cube root of each side.**

$25 = l$ **Use a calculator.**

GUIDED PRACTICE for Examples 4 and 5

ANSWER

A coral cod that has the 275 grams **is about** 25 *cm* **long.**

GUIDED PRACTICE for Examples 4 and 5

b. 340 grams

SOLUTION

$w = 0.0167l^3$ **Write model for weight.**

$340 = 0.0167l^3$ **Substitute** 340 **for** w**.**

$20360 = l^3$ **Divide each side by** 0.0167

$\sqrt[3]{20360} = l$ **Take cube root of each side**

$27 = l$ **Use a calculator**

ANSWER **A coral cod that has the** 340 grams **is about** 27 *cm* **long.**

GUIDED PRACTICE for Examples 4 and 5

c. 450 grams

SOLUTION

$w = 0.0167l^3$ **Write model for weight.**

$450 = 0.0167l^3$ **Substitute** 450 **for** w**.**

$26946 = l^3$ **Divide each side by** 0.0167

$\sqrt[3]{26946} = l$ **Take cube root of each side**

$30 = l$ **Use a calculator**

ANSWER **A coral cod that has the** 450 grams **is about** 30 *cm* **long.**