EXAMPLE 1 Use properties of exponents

Use the properties of rational exponents to simplify the expression.

a. $7^{1/4} \cdot 7^{1/2} = 7^{(1/4 + 1/2)} = 7^{3/4}$

b. $(6^{1/2} \cdot 4^{1/3})^2 = (6^{1/2})^2 \cdot (4^{1/3})^2 = 6^{(1/2 \cdot 2)} \cdot 4^{(1/3 \cdot 2)} = 6^1 \cdot 4^{2/3} = 6 \cdot 4^{2/3}$

c. $(4^5 \cdot 3^5)^{-1/5} = [(4 \cdot 3)^5]^{-1/5} = (12^5)^{-1/5} = 12^{[5 \cdot (-1/5)]} = 12^{-1} = \frac{1}{12}$

d. $\frac{5}{5^{1/3}} = \frac{5^1}{5^{1/3}} = 5^{(1 - 1/3)} = 5^{2/3}$

e. $\left(\frac{42^{1/3}}{6^{1/3}}\right)^2 = \left[\left(\frac{42}{6}\right)^{1/3}\right]^2 = (7^{1/3})^2 = 7^{(1/3 \cdot 2)} = 7^{2/3}$

EXAMPLE 2 Apply properties of exponents

Biology

A mammal's surface area S (in square centimeters) can be approximated by the model $S = km^{2/3}$ where m is the mass (in grams) of the mammal and k is a constant. The values of k for some mammals are shown below. Approximate the surface area of a rabbit that has a mass of 3.4 kilograms $(3.4 \times 10^3$ grams$)$.

Mammal	Sheep	Rabbit	Horse	Human	Monkey	Bat
k	8.4	9.75	10.0	11.0	11.8	57.5

EXAMPLE 2 Apply properties of exponents

SOLUTION

$S = km^{2/3}$ **Write model.**

$= 9.75(3.4 \times 10^3)^{2/3}$ **Substitute** 9.75 **for** k **and** 3.4×10^3 **for** m**.**

$= 9.75(3.4)^{2/3}(10^3)^{2/3}$ **Power of a product property.**

$\approx 9.75(2.26)(10^2)$ **Power of a product property.**

≈ 2200 **Simplify.**

ANSWER **The rabbit's surface area is about** 2200 square centimeters**.**

GUIDED PRACTICE for Examples 1 and 2

Use the properties of rational exponents to simplify the expression.

1. $(5^{1/3} \cdot 7^{1/4})^3 = (5^{1/3})^3 \cdot (7^{1/4})^3 = 5^{1/3 \cdot 3} \cdot 7^{1/4 \cdot 3} = 5^1 \cdot 7^{3/4} = 5 \cdot 7^{3/4}$

2. $2^{3/4} \cdot 2^{1/2} = 2^{(3/4 + 1/2)} = 2^{5/4}$

3. $\dfrac{3}{3^{1/4}} = \dfrac{3^1}{3^{1/4}} = 3^{(1 - 1/4)} = 3^{3/4}$

4. $\left(\dfrac{20^{1/2}}{5^{1/2}}\right)^3 = \left[\left(\dfrac{20}{5}\right)^{1/2}\right]^3 = (4^{1/2})^3 = (2^2)^{3/2} = 8$

GUIDED PRACTICE for Examples 1 and 2

Biology

5. Use the information in Example 2 to approximate the surface area of a sheep that has a mass of 95 kilograms $(9.5 \times 10^4$ grams$)$.

SOLUTION

$S = km^{2/3}$ **Write model.**

$= 9.75(9.5 \times 10^4)^{2/3}$ **Substitute 9.75 for k and 9.5×10^4 for m.**

$= 9.75(9.5)^{2/3}(10^4)^{2/3}$ **Power of a product property.**

GUIDED PRACTICE for Examples 1 and 2

$\approx 9.75(95)^{10/3}$ **Power of a product property.**

$\approx 17{,}500$ **Simplify.**

ANSWER **The Sheep's surface area is about** 2200 square centimeters.

EXAMPLE 3 Use properties of radicals

Use the properties of radicals to simplify the expression.

a. $\sqrt[3]{12} \cdot \sqrt[3]{18} = \sqrt[3]{12 \cdot 8} = \sqrt[3]{216} = 6$ **Product property**

b. $\dfrac{\sqrt[4]{80}}{\sqrt[4]{5}} = \sqrt[4]{\dfrac{80}{5}} = \sqrt[4]{16} = 2$ **Quotient property**

EXAMPLE 4 Write radicals in simplest form

Write the expression in simplest form.

a. $\sqrt[3]{135} = \sqrt[3]{27 \cdot 5}$ **Factor out perfect cube.**

$= \sqrt[3]{27} \cdot \sqrt[3]{5}$ **Product property**

$= 3\sqrt[3]{5}$ **Simplify.**

EXAMPLE 4 Write radicals in simplest form

b. $\dfrac{\sqrt[5]{7}}{\sqrt[5]{8}} = \dfrac{\sqrt[5]{7}}{\sqrt[5]{8}} \cdot \dfrac{\sqrt[5]{4}}{\sqrt[5]{4}}$

Make denominator a perfect fifth power.

$= \dfrac{\sqrt[5]{28}}{\sqrt[5]{32}}$

Product property

$= \dfrac{\sqrt[5]{28}}{2}$

Simplify.

EXAMPLE 5 Add and subtract like radicals and roots

Simplify the expression.

a. $\sqrt[4]{10} + 7\sqrt[4]{10} = (1+7)\sqrt[4]{10} = 8\sqrt[4]{10}$

b. $2\,(8^{1/5}) + 10\,(8^{1/5}) = (2+10)\,(8^{1/5}) = 12\,(8^{1/5})$

c. $\sqrt[3]{54} - \sqrt[3]{2} = \sqrt[3]{27}\cdot\sqrt[3]{2} - \sqrt[3]{2} = 3\sqrt[3]{2} - \sqrt[3]{2} = (3-1)\sqrt[3]{2} = 2\sqrt[3]{2}$

GUIDED PRACTICE for Examples 3, 4, and 5

Simplify the expression.

6. $\sqrt[4]{27} \cdot \sqrt[4]{3}$

SOLUTION

$$\sqrt[4]{27} \cdot \sqrt[4]{3} = \sqrt[4]{27 \cdot 3} = \sqrt[4]{81} = 3$$

Product property

GUIDED PRACTICE for Examples 3, 4, and 5

7. $\dfrac{\sqrt[3]{250}}{\sqrt[3]{2}}$

SOLUTION

$$\frac{\sqrt[3]{250}}{\sqrt[3]{2}} = \frac{\sqrt[3]{5^3 \cdot 2}}{\sqrt[3]{2}}$$

Factor out numerator to perfect cube.

$$= \frac{\sqrt[3]{5^3} \cdot \sqrt[3]{2}}{\sqrt[3]{2}}$$

Product property

$$= 5$$

Simplify.

GUIDED PRACTICE for Examples 3, 4, and 5

8. $\sqrt[5]{\frac{3}{4}}$

SOLUTION

$$\sqrt[5]{\frac{3}{4}} = \frac{\sqrt[5]{3}}{\sqrt[5]{4}} \cdot \frac{\sqrt[5]{8}}{\sqrt[5]{8}}$$

Make denominator a perfect fifth power.

$$= \frac{\sqrt[5]{24}}{\sqrt[5]{32}}$$

Product property

$$= \frac{\sqrt[5]{24}}{2}$$

Simplify.

GUIDED PRACTICE for Examples 3, 4, and 5

9. $\sqrt[3]{5} + \sqrt[3]{40}$

SOLUTION

$$\sqrt[3]{5} + \sqrt[3]{40} = \sqrt[3]{5} + \sqrt[3]{2^3 \cdot 5}$$

$$= \sqrt[3]{5} + 2\sqrt[3]{5}$$

$$= (1+2)\sqrt[3]{5}$$

$$= 3\sqrt[3]{5}$$

EXAMPLE 6 Simplify expressions involving variables

Simplify the expression. Assume all variables are positive.

a. $\sqrt[3]{64y^6} = \sqrt[3]{4^3(y^2)^3} = \sqrt[3]{4^3} \cdot \sqrt[3]{(y^2)^3} = 4y^2$

b. $(27p^3q^{12})^{1/3} = 27^{1/3}(p^3)^{1/3}(q^{12})^{1/3} = 3p^{(3 \cdot 1/3)}q^{(12 \cdot 1/3)} = 3pq^4$

c. $\sqrt[4]{\frac{m^4}{n^8}} = \frac{\sqrt[4]{m^4}}{\sqrt[4]{n^8}} = \frac{\sqrt[4]{m^4}}{\sqrt[4]{(n^2)^4}} = \frac{m}{n^2}$

d. $\frac{14xy^{1/3}}{2x^{3/4}z^{-6}} = 7x^{(1-3/4)}y^{1/3}z^{-(-6)} = 7x^{1/4}y^{1/3}z^6$

EXAMPLE 7 Write variable expressions in simplest form

Write the expression in simplest form. Assume all variables are positive.

a. $\sqrt[5]{4a^8b^{14}c^5} = \sqrt[5]{4a^5a^3b^{10}b^4c^5}$ — **Factor out perfect fifth powers.**

$= \sqrt[5]{a^5b^{10}c^5} \cdot \sqrt[5]{4a^3b^4}$ — **Product property**

$= ab^2c\sqrt[5]{4a^3b^4}$ — **Simplify.**

b. $\sqrt[3]{\frac{x}{y^8}} = \sqrt[3]{\frac{x \cdot y}{y^8 \cdot y}}$ — **Make denominator a perfect cube.**

$= \sqrt[3]{\frac{xy}{y^9}}$ — **Simplify.**

EXAMPLE 7 Write variable expressions in simplest form

$$= \frac{\sqrt[3]{x\,y}}{\sqrt[3]{y^9}}$$

Quotient property

$$= \sqrt[3]{\frac{x\,y}{y^3}}$$

Simplify.

EXAMPLE 8 Add and subtract expressions involving variables

Perform the indicated operation. Assume all variables are positive.

a. $\frac{1}{5}\sqrt{w} + \frac{3}{5}\sqrt{w} = \left(\frac{1}{5} + \frac{3}{5}\right)\sqrt{w} = \frac{4}{5}\sqrt{w}$

b. $3xy^{1/4} - 8xy^{1/4} = (3 - 8)\,xy^{1/4} = -5xy^{1/4}$

c. $12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2} = 12z\sqrt[3]{2z^2} - 3z\sqrt[3]{2z^2}$

$= (12z - 3z)\sqrt[3]{2z^2}$

$= 9z\sqrt[3]{2z^2}$

GUIDED PRACTICE for Examples 6, 7, and 8

Simplify the expression. Assume all variables are positive.

10. $\sqrt[3]{27q^9}$

SOLUTION

$$\sqrt[3]{27q^9} = \sqrt[3]{3^3(q^3)^3} = \sqrt[3]{3^3} \cdot \sqrt[3]{(q^3)^3} = 3q^3$$

11. $\sqrt[5]{\dfrac{x^{10}}{y^5}}$

SOLUTION

$$\sqrt[5]{\frac{x^{10}}{y^5}} = \frac{\sqrt[5]{x^{10}}}{\sqrt[5]{y^5}} = \frac{\sqrt[5]{(x^2)^5}}{\sqrt[5]{y^5}} = \frac{x^2}{y}$$

GUIDED PRACTICE for Examples 6, 7, and 8

12. $\dfrac{6xy^{3/4}}{3x^{1/2}y^{1/2}}$

SOLUTION

$$\frac{6xy^{3/4}}{3x^{1/2}y^{1/2}} = 2x^{(1-1/2)}y^{(3/4-1/2)} = 2x^{1/2}y^{1/4}$$

13. $\sqrt{9w^5} - w\sqrt{w^3}$

SOLUTION

$$\begin{aligned}\sqrt{9w^5} - w\sqrt{w^3} &= \sqrt{3^2w^2 \cdot w^2 \cdot w} - w\sqrt{w^2 \cdot w} \\ &= 3w^2\sqrt{w} - w^2\sqrt{w} \\ &= 2w^2\sqrt{w}\end{aligned}$$