EXAMPLE 1 Add and subtract functions

Let $f(x) = 4x^{1/2}$ and $g(x) = -9x^{1/2}$. Find the following.

a. $f(x) + g(x)$

SOLUTION

$$f(x) + g(x) = 4x^{1/2} + (-9x^{1/2}) = [4 + (-9)]x^{1/2} = -5x^{1/2}$$

b. $f(x) - g(x)$

SOLUTION

$$f(x) - g(x) = 4x^{1/2} - (-9x^{1/2}) = [4 - (-9)]x^{1/2} = 13x^{1/2}$$

EXAMPLE 1 Add and subtract functions

c. the domains of $f + g$ and $f - g$

SOLUTION

The functions f and g each have the same domain: all nonnegative real numbers. So, the domains of $f + g$ and $f - g$ also consist of all nonnegative real numbers.

EXAMPLE 2 Multiply and divide functions

Let $f(x) = 6x$ and $g(x) = x^{3/4}$. Find the following.

a. $f(x) \cdot g(x)$

SOLUTION

$$f(x) \cdot g(x) = (6x)(x^{3/4}) = 6x^{(1 + 3/4)} = 6x^{7/4}$$

b. $\dfrac{f(x)}{g(x)}$

SOLUTION

$$\frac{f(x)}{g(x)} = \frac{6x}{x^{3/4}} = 6x^{(1 - 3/4)} = 6x^{1/4}$$

EXAMPLE 2 Multiply and divide functions

c. the domains of $f \cdot g$ and $\frac{f}{g}$

SOLUTION

The domain of *f* consists of all real numbers, and the domain of g consists of all nonnegative real numbers. So, the domain of $f \cdot g$ consists of all nonnegative real numbers. Because $g(0) = 0$, the domain of $\frac{f}{g}$ is restricted to all ***positive*** real numbers.

EXAMPLE 3 Solve a multi-step problem

Rhinos

For a white rhino, heart rate r (in beats per minute) and life span s (in minutes) are related to body mass m (in kilograms) by these functions:

$$r(m) = 241m^{-0.25} \qquad s(m) = (6 \times 10^6)m^{0.2}$$

- **Find $r(m) \cdot s(m)$.**
- **Explain what this product represents.**

EXAMPLE 3 Solve a multi-step problem

SOLUTION

STEP 1

Find and simplify $r(m) \cdot s(m)$.

$$r(m) \cdot s(m) = 241m^{-0.25}\,[\,(6 \times 10^6)m^{0.2}\,]$$ **Write product of $r(m)$ and $s(m)$.**

$$= 241(6 \times 10^6)m^{(-0.25 + 0.2)}$$ **Product of powers property**

$$= (1446 \times 10^6)m^{-0.05}$$ **Simplify.**

$$= (1.446 \times 10^9)m^{-0.05}$$ **Use scientific notation.**

EXAMPLE 3 Solve a multi-step problem

STEP 2

Interpret $r(m) \cdot s(m)$.

Multiplying heart rate by life span gives the total number of heartbeats for a white rhino over its entire lifetime.

GUIDED PRACTICE for Examples 1, 2, and 3

Let $f(x) = -2x^{2/3}$ and $g(x) = 7x^{2/3}$. Find the following.

1. $f(x) + g(x)$

SOLUTION

$$f(x) + g(x) = -2x^{2/3} + 7x^{2/3} = (-2 + 7)x^{2/3} = 5x^{2/3}$$

2. $f(x) - g(x)$

SOLUTION

$$f(x) - g(x) = -2x^{2/3} - 7x^{2/3} = [-2 + (-7)]x^{2/3} = -9x^{2/3}$$

GUIDED PRACTICE for Examples 1, 2, and 3

3. the domains of $f + g$ and $f - g$

SOLUTION

The domains of f and g have the same domain: all non-negative real numbers. So , the domain of $f + g$ and $f - g$ also consist of all non-negative real numbers.

GUIDED PRACTICE for Examples 1, 2, and 3

Let $f(x) = 3x$ and $g(x) = x^{1/5}$. Find the following.

4. $f(x) \cdot g(x)$

SOLUTION

$$f(x) \cdot g(x) = 3x \cdot x^{1/5} = 3(x)^{1 + 1/5} = 3x^{6/5}$$

5. $\dfrac{f(x)}{g(x)}$

SOLUTION

$$\frac{f(x)}{g(x)} = \frac{3x}{x^{1/5}} = 3(x)^{1 - 1/5} = 3x^{4/5}$$

GUIDED PRACTICE for Examples 1, 2, and 3

6. the domains of $f \cdot g$ and $\frac{f}{g}$

SOLUTION

The domain of $f \cdot g$ consists of all real numbers.

The domain of $\frac{f}{g}$ consists of all real numbers except $x = 0$.

GUIDED PRACTICE for Examples 1, 2, and 3

Rhinos

7. Use the result of Example 3 to find a white rhino's number of heartbeats over its lifetime if its body mass is 1.7×10^5 kilograms.

ANSWER **about 7.92×10^8 heartbeats**

EXAMPLE 4 Standardized Test Practice

Let $f(x) = 2x - 7$ and $g(x) = x^2 + 4$. What is the value of $g(f(3))$?

(A) -5 (B) -3 (C) 3 (D) 5

SOLUTION

To evaluate $g(f(3))$, you first must find $f(3)$.

$$f(3) = 2(3) - 7 = -1$$

Then $g(f(3)) = g(-1) = (-1)^2 + 4 = 1 + 4 = 5$.

So, the value of $g(f(3))$ is 5.

EXAMPLE 4 Standardized Test Practice

ANSWER

The correct answer is D. (A) (B) (C) (D)

EXAMPLE 5 Find compositions of functions

Let $f(x) = 4x^{-1}$ and $g(x) = 5x - 2$. Find the following.

a. $f(g(x))$

SOLUTION

$$f(g(x)) = f(5x - 2) = 4(5x - 2)^{-1} = \frac{4}{5x - 2}$$

b. $g(f(x))$

SOLUTION

$$g(f(x)) = g(4x^{-1}) = 5(4x^{-}1) - 2 = -20x^{-1} - 2 = \frac{20}{x} - 2$$

EXAMPLE 5 Find compositions of functions

c. $f(f(x))$

SOLUTION

$$f(f(x)) = f(4x^{-1})$$

$$= 4(4x^{-1})^{-1}$$

$$= 4(4^{-1}\ x)$$

$$= 4^{0}x$$

$$= x$$

EXAMPLE 5 Find compositions of functions

d. The domain of $f(g(x))$ consists of all real numbers except $x \frac{1}{2}$ because $g\left(\frac{2}{5}\right) = 0$ is not in the domain of f. (Note that $f(0) = \left(\frac{4}{0}\right)$, which is undefined.) The domains of $g(f(x))$ and $f(f(x))$ consist of all real numbers except $x = 0$, again because 0 is not in the domain of f.

EXAMPLE 6 Solve a multi-step problem

Paint Store

You have a $10 gift certificate to a paint store. The store is offering 15% off your entire purchase of any paints and painting supplies. You decide to purchase a $30 can of paint and $25 worth of painting supplies.

Use composition of functions to do the following:

- **Find the sale price of your purchase when the $10 gift certificate is applied before the 15% discount.**

http://www.classzone.com/cz/books/algebra_2_2011_na/book_home.htm

EXAMPLE 6 Solve a multi-step problem

- **Find the sale price of your purchase when the 15% discount is applied before the $\$10$ gift certificate.**

SOLUTION

STEP 1

Find: the total amount of your purchase. The total amount for the paint and painting supplies is $\$30 + \$25 = \$55$.

STEP 2

Write: functions for the discounts. Let x be the regular price, $f(x)$ be the price after the $\$10$ gift certificate is applied, and $g(x)$ be the price after the 15% discount is applied.

EXAMPLE 6 Solve a multi-step problem

Function for $10 gift certificate: $f(x) = x - 10$

Function for 15% discount: $g(x) = x - 0.15x = 0.85x$

STEP 3

Compose: the functions.

The composition $g(f(x))$ represents the sale price when the $10 gift certificate is applied before the 15% discount.

$$g(f(x)) = g(x - 10) = 0.85(x - 10)$$

EXAMPLE 6 Solve a multi-step problem

The composition $f(g(x))$ represents the sale price when the 15% discount is applied before the $\$10$ gift certificate.

$$f(g(x)) = f(0.85x) = 0.85x - 10$$

EXAMPLE 6 Solve a multi-step problem

STEP 4

Evaluate: the functions $g(f(x))$ and $f(g(x))$ when $x = 55$.

$$g(f(55)) = 0.85(55 - 10) = 0.85(45) = \$38.25$$

$$f(g(55)) = 0.85(55) - 10 = 46.75 - 10$$

ANSWER

The sale price is $\$38.25$ when the $\$10$ gift certificate is applied before the 15% discount. The sale price is $\$36.75$ when the 15% discount is applied before the $\$10$ gift certificate.

GUIDED PRACTICE for Examples 4, 5, and 6

Let $f(x) = 3x - 8$ and $g(x) = 2x^2$. Find the following.

8. $g(f(5))$

SOLUTION

To evaluate $g(f(5))$, you first must find $f(5)$.

$$f(5) = 3(5) - 8 = 7$$

Then $g(f(3)) = g(7) = 2(7)^2 = 2(49) = 98$.

ANSWER **So, the value of $g(f(5))$ is 98.**

GUIDED PRACTICE for Examples 4, 5, and 6

Let $f(x) = 3x - 8$ and $g(x) = 2x^2$. Find the following.

9. $f(g(5))$

SOLUTION

To evaluate $f(g(5))$, you first must find $g(5)$.

$$g(5) = 2(5)^2 = 2(25) = 50$$

Then $f(g(5)) = f(50) = 3(50) - 8 = 150 - 8 = 142$.

ANSWER **So, the value of $f(g(5))$ is 142.**

GUIDED PRACTICE for Examples 4, 5, and 6

Let $f(x) = 3x - 8$ and $g(x) = 2x^2$. Find the following.

10. $f(f(5))$

SOLUTION

To evaluate $f(f(5))$, you first must find $f(5)$.

$$f(5) = 3(5) - 8 = 7$$

Then $f(f(5)) = f(7) = 3(7) - 8 = 21 - 8 = 13$.

ANSWER **So, the value of $f(g(5))$ is 13.**

GUIDED PRACTICE for Examples 4, 5, and 6

Let $f(x) = 3x - 8$ and $g(x) = 2x^2$. Find the following.

11. $g(g(5))$

SOLUTION

To evaluate $g(g(5))$, you first must find $g(5)$.

$$g(5) = 2(5)^2 = 2(25) = 50$$

Then $g(g(5)) = g(50) = 2(50)^2 = 2(2500) = 5000$.

ANSWER **So, the value of $g(g(5))$ is 5000.**

GUIDED PRACTICE for Examples 4, 5, and 6

12. **Let $f(x) = 2x^{-1}$ and $g(x) = 2x + 7$. Find $f(g(x))$, $g(f(x))$, and $f(f(x))$. Then state the domain of each composition.**

SOLUTION

$$f(g(x)) = f(2x + 7) = 2(2x + 7)^{-1} = \frac{2}{2x + 7}$$

$$g(f(x)) = f(2x^{-1}) = 2(2x^{-1}) + 7 = 4x^{-1} + 7 = \frac{4}{x} + 7$$

$$f(f(x)) = f(2x^{-1}) = 2(2x^{-1})^{-1} = x$$

GUIDED PRACTICE for Examples 4, 5, and 6

ANSWER

The domain of $f(g(x))$ consists of all real numbers except $x = -3.5$. The domain of $g(f(x))$ consists of all real numbers except $x = 0$.

GUIDED PRACTICE for Examples 4, 5, and 6

13. **What If? In Example 6, how do your answers change if the gift certificate to the paint store is $15 and the store discount is 20%?**

- **Find the sale price of your purchase when the 15% discount is applied before the $10 gift certificate.**

EXAMPLE 6 Solve a multi-step problem

SOLUTION

STEP 1

Find: the total amount of your purchase. The total amount for the paint and painting supplies is $\$30 + \$25 = \$55$**.**

STEP 2

Write: functions for the discounts. Let x **be the regular price,** $f(x)$ **be the price after the** $\$15$ **gift certificate is applied, and** $g(x)$ **be the price after the** 20% **discount is applied.**

EXAMPLE 6 Solve a multi-step problem

Function for $\$15$ gift certificate: $f(x) = x - 15$

Function for 20% discount: $g(x) = x - 0.2x = 0.8x$

STEP 3

Compose: the functions.

The composition $g(f(x))$ represents the sale price when the $\$15$ gift certificate is applied before the 20% discount.

$$g(f(x)) = g(x - 15) = 0.8(x - 15)$$

EXAMPLE 6 **Solve a multi-step problem**

The composition $f(g(x))$ represents the sale price when the 20% discount is applied before the $\$15$ gift certificate.

$$f(g(x)) = f(0.8x) \quad = 0.8x - 15$$

EXAMPLE 6 Solve a multi-step problem

STEP 4

Evaluate: the functions $g(f(x))$ and $f(g(x))$ when $x = 55$.

$$g(f(55)) = 0.8(55 - 15) \quad = 0.8(40) \quad = \$32$$

$$f(g(55)) = 0.8(55) - 15 \quad = \$29$$

ANSWER

The sale price is $\$32$ when the $\$15$ gift certificate is applied before the 20% discount. The sale price is $\$29$ when the 20% discount is applied before the $\$15$ gift certificate.