EXAMPLE 1 **Solve a radical equation**

Solve $\sqrt[3]{2x+7} = 3$**.**

$\sqrt[3]{2x+7} = 3$ **Write original equation.**

$(\sqrt[3]{2x+7})^3 = 3^3$ **Cube each side to eliminate the radical.**

$2x+7 = 27$ **Simplify.**

$2x = 20$ **Subtract 7 from each side.**

$x = 10$ **Divide each side by 2.**

EXAMPLE 1 **Solve a radical equation**

CHECK

Check $x = 10$ in the original equation.

$\sqrt[3]{2(10)+7} \overset{?}{=} 3$ **Substitute 10 for x.**

$\sqrt[3]{27} \overset{?}{=} 3$ **Simplify.**

$3 = 3$ ✔   **Solution checks.**

GUIDED PRACTICE **for Example 1**

Solve equation. Check your solution.

1. $\sqrt[3]{x} - 9 = -1$

$\sqrt[3]{x} - 9 = -1$ **Write original equation.**

$\sqrt[3]{x} = 8$ **Add 9 to each side.**

$(\sqrt[3]{x})^3 = (8)^3$ **Use each side to eliminate the radical.**

$x = 512$

GUIDED PRACTICE for Example 1

Solve equation. Check your solution.

2. $(\sqrt{x+25}) = 4$

$(\sqrt{x+25}) = 4$ **Write original equation.**

$(\sqrt{x+25})^2 = 4^2$ **Square each side to eliminate the radical.**

$x + 25 = 16$ **Simplify.**

$x = -9$ **Subtract** 25 **from each side.**

GUIDED PRACTICE **for Example 1**

Solve equation. Check your solution.

3. $(2\sqrt[3]{x-3}) = 4$

$(2\sqrt[3]{x-3}) = 4$ **Write original equation.**

$\sqrt[3]{x-3} = 2$ **Divided 2 from each side.**

$(\sqrt[3]{x-3})^3 = 2^3$ **Cube each side to eliminate the radical.**

$x - 3 = 8$ **Simplify.**

$x = 11$ **Add 3 to each side.**

EXAMPLE 2 **Solve a radical equation given a function**

Wind Velocity

In a hurricane, the mean sustained wind velocity v (in meters per second) is given by

$$v(p) = 6.3\sqrt{1013 - p}$$

where p is the air pressure (in millibars) at the center of the hurricane. Estimate the air pressure at the center of a hurricane when the mean sustained wind velocity is 54.5 meters per second.

EXAMPLE 2 — Solve a radical equation given a function

SOLUTION

$$v(p) = 6.3\sqrt{1013 - p}$$ **Write given function.**

$$54.5 = 6.3\sqrt{1013 - p}$$ **Substitute** 54.5 **for** $v(p)$**.**

$$8.65 \simeq \sqrt{1013 - p}$$ **Divide each side by** 6.3**.**

$$(8.65)^2 \simeq \left(\sqrt{1013 - p}\right)^2$$ **Square each side.**

$$74.8 \simeq 1013 - p$$ **Simplify.**

$$938.2 \simeq -p$$ **Subtract** 1013 **from each side.**

$$-938.2 \simeq -p$$ **Divide each side by** -1**.**

http://www.classzone.com/cz/books/algebra_2_2011_na/book_home.htm

EXAMPLE 2 **Solve a radical equation given a function**

ANSWER

The air pressure at the center of the hurricane is about 938 millibars**.**

GUIDED PRACTICE *for Example 2*

4. **What If? Use the function in Example** 2 **to estimate the air pressure at the center of a hurricane when the mean sustained wind velocity is** 48.3 meters per second**.**

SOLUTION

$v(p) = 6.3\sqrt{1013 - p}$ **Write given function.**

$48.3 = 6.3\sqrt{1013 - p}$ **Substitute** 48.3 **for** $v(p)$**.**

$7.67 \simeq \sqrt{1013 - p}$ **Divide each side by** 6.3**.**

$(7.67)^2 \simeq \left(\sqrt{1013 - p}\right)^2$ **Square each side.**

GUIDED PRACTICE *for Example 2*

$59 \simeq 1013 - p$ **Simplify.**

$-954 \simeq -p$ **Subtract** 1013 **from each side.**

$954 \simeq p$ **Divide each side by** -1**.**

ANSWER

The air pressure at the center of the hurricane is about 954 **mille bars.**

EXAMPLE 3 **Standardized Test Practice**

What are the solutions of the equation $4x^{2/3} = 36$?

(A) ± 3 (B) ± 6 (C) ± 21 (D) ± 27

SOLUTION

$4x^{2/3} = 36$ **Write original equation.**

$x^{2/3} = 9$ **Divide each side by 4.**

$(x^{2/3})^{3/2} = 9^{3/2}$ **Raise each side to the power $\dfrac{3}{2}$.**

$x = \pm 27$ **Simplify.**

ANSWER

The correct answer is D. (A) (B) (C) (D)

EXAMPLE 4   **Solve an equation with a rational exponent**

Solve $(x + 2)^{3/4} - 1 = 7$.

$(x + 2)^{3/4} - 1 = 7$ **Write original equation.**

$(x + 2)^{3/4} = 8$ **Add 1 to each side.**

$\left[(x + 2)^{3/4}\right]^{4/3} = 8^{4/3}$ **Raise each side to the power** $\dfrac{4}{3}$**.**

$x + 2 = (8^{1/3})^4$ **Apply properties of exponents.**

$x + 2 = 2^4$ **Simplify.**

$x + 2 = 16$ **Simplify.**

$x = 14$ **Subtract 2 from each side.**

EXAMPLE 4 Solve an equation with a rational exponent

ANSWER

The solution is 14. Check this in the original equation.

GUIDED PRACTICE   **for Examples 3 and 4**

Solve the equation. Check your solution.

5. $3x^{3/2} = 375$

$3x^{3/2} = 375$ **Write original equation.**

$x^{3/2} = 125$ **Divide each side by 3.**

$(x^{3/2})^{2/3} = (125)^{2/3}$ **Raise each side to the power $\dfrac{2}{3}$.**

$x = 25$ **Simplify.**

GUIDED PRACTICE for Examples 3 and 4

Solve the equation. Check your solution.

6. $-2x^{3/4} = -16$

$-2x^{3/4} = -16$ — **Write original equation.**

$x^{3/4} = 8$ — **Divide each side by -2.**

$(x^{3/4})^{4/3} = 8^{4/3}$ — **Raise each side to the power $\dfrac{4}{3}$.**

$x = (8^{1/3})^4$ — **Apply properties of exponent.**

$x = 16$ — **Simplify.**

Solve the equation. Check your solution.

7. $\quad -\dfrac{2}{3}x^{1/5} = -2$

$-\dfrac{2}{3}x^{1/5} = -2$ **Write original equation.**

$x^{1/5} = 3$ **Divide each side by $-2/3$.**

$(x^{1/5})^5 = 3^5$ **Raise each side to the power 5.**

$x = 243$ **Simplify.**

GUIDED PRACTICE   **for Examples 3 and 4**

Solve the equation. Check your solution.

8. $(x + 3)^{5/2} = 32$

$(x + 3)^{5/2} = 32$ **Write original equation.**

$[(x + 3)^{5/2}]^{2/5} = 32^{2/5}$ **Raise each side to the power** 2/5**.**

$x + 3 = (32 \cdot \frac{1}{5})^2$ **Apply properties of exponent.**

$x + 3 = 4$ **Simplify.**

$x = 1$ **Simplify.**

GUIDED PRACTICE for Examples 3 and 4

Solve the equation. Check your solution.

9. $(x-5)^{4/3} = 81$

$(x-5)^{4/3} = 81$	**Write original equation.**
$[(x-5)^{4/3}]^{3/4} = (81)^{3/4}$	**Raise each side to the power** 3/4**.**
$x-5 = (81^{1/4})^3$	**Apply properties of exponent.**
$x-5 = \pm 3^3$	**Simplify.**
$x-5 = \pm 27$	**Simplify.**
$x-5 = 27$ or $x-5 = 27$	**Let (x – 5) equal 27 and –27.**
$x = 32$ or $x = -22$	**Subtract 5 from both sides of each equation.**

GUIDED PRACTICE **for Examples 3 and 4**

Solve the equation. Check your solution.

10. $(x + 2)^{2/3} + 3 = 7$

$(x + 2)^{2/3} + 3 = 7$ **Write original equation.**

$(x + 2)^{2/3} = 4$ **Subtract each side by 3.**

$[(x + 2)^{2/3}]^{3/2} = 4^{3/2}$ **Raise each side to the power 3/2.**

$x + 2 = (4^{1/2})^3$ **Apply properties of exponent.**

$x + 2 = 8 \text{ or } x + 2 = -8$ **Simplify.**

$x = -10 \text{ or } 6$ **Simplify.**

EXAMPLE 5 **Solve an equation with an extraneous solution**

Solve $x + 1 = \sqrt{7x + 15}$.

$$x + 1 = \sqrt{7x + 15}$$ **Write original equation.**

$$(x + 1)^2 = (\sqrt{7x + 15})^2$$ **Square each side.**

$$x^2 + 2x + 1 = 7x + 15$$ **Expand left side and simplify right side.**

$$x^2 - 5x - 14 = 0$$ **Write in standard form.**

$$(x - 7)(x + 2) = 0$$ **Factor.**

$$x - 7 = 0 \quad \textbf{or} \quad x + 2 = 0$$ **Zero-product property**

$$x = 7 \quad \textbf{or} \quad x = -2$$ **Solve for x.**

EXAMPLE 5 **Solve an equation with an extraneous solution**

CHECK 1

Check $x = 7$ in the original equation.

$$x + 1 = \sqrt{7x + 15}$$

$$7 + 1 \overset{?}{=} \sqrt{7(7) + 15}$$

$$8 \overset{?}{=} \sqrt{64}$$

$$8 = 8 \checkmark$$

Check $x = -2$ in the original equation.

$$x + 1 = \sqrt{7x + 15}$$

$$-2 + 1 \overset{?}{=} \sqrt{7(-2) + 15}$$

$$-1 \overset{?}{=} \sqrt{1}$$

$$-1 \neq 1$$

ANSWER **The only solution is 7. (The apparent solution 22 is extraneous.)**

EXAMPLE 6   **Solve an equation with two radicals**

Solve $\sqrt{x+2} + 1 = \sqrt{3-x}$.

SOLUTION

METHOD 1   **Solve using algebra.**

$$\sqrt{x+2} + 1 = \sqrt{3-x}$$ **Write original equation.**

$$\left(\sqrt{x+2} + 1\right)^2 = \left(\sqrt{3-x}\right)^2$$ **Square each side.**

$$x + 2 + 2\sqrt{x+2} + 1 = 3 - x$$ **Expand left side and simplify right side.**

$$2\sqrt{x+2} = -2x$$ **Isolate radical expression.**

EXAMPLE 6 — **Solve an equation with two radicals**

$$\sqrt{x + 2} = -x$$
Divide each side by 2.

$$\left(\sqrt{x + 2}\right)^2 = (-x)^2$$
Square each side again.

$$x + 2 = x^2$$
Simplify.

$$0 = x^2 - x - 2$$
Write in standard form.

$$0 = (x - 2)(x + 1)$$
Factor.

$$x - 2 = 0 \quad \textbf{or} \quad x + 1 = 0$$
Zero-product property.

$$x = 2 \quad \textbf{or} \quad x = -1$$
Solve for x.

EXAMPLE 6   **Solve an equation with two radicals**

Check $x = 2$ **in the original equation.**

$$\sqrt{x+2} + 1 = \sqrt{3-x}$$

$$\sqrt{2+2} + 1 \overset{?}{=} \sqrt{3-2}$$

$$\sqrt{4} + 1 \overset{?}{=} \sqrt{1}$$

$$3 \neq -1$$

Check $x = -1$ **in the original equation.**

$$\sqrt{x+2} + 1 = \sqrt{3-x}$$

$$\sqrt{-1+2} + 1 \overset{?}{=} \sqrt{3-(-1)}$$

$$\sqrt{1} + 1 \overset{?}{=} \sqrt{4}$$

$$2 = 2 \; \checkmark$$

ANSWER

**The only solution is -1.
(The apparent solution 2 is extraneous.)**

EXAMPLE 6 **Solve an equation with two radicals**

METHOD 2

Use: a graph to solve the equation. Use a graphing calculator to graph $y_1 = \sqrt{x+2} + 1$ and $y_2 = \sqrt{3-x}$. Then find the intersection points of the two graphs by using the *intersect* feature. You will find that the only point of intersection is $(21, 2)$. Therefore, 21 is the only solution of the equation

$$\sqrt{x+2} + 1 = \sqrt{3-x}$$


```
Intersection
X=-1             Y=2
```
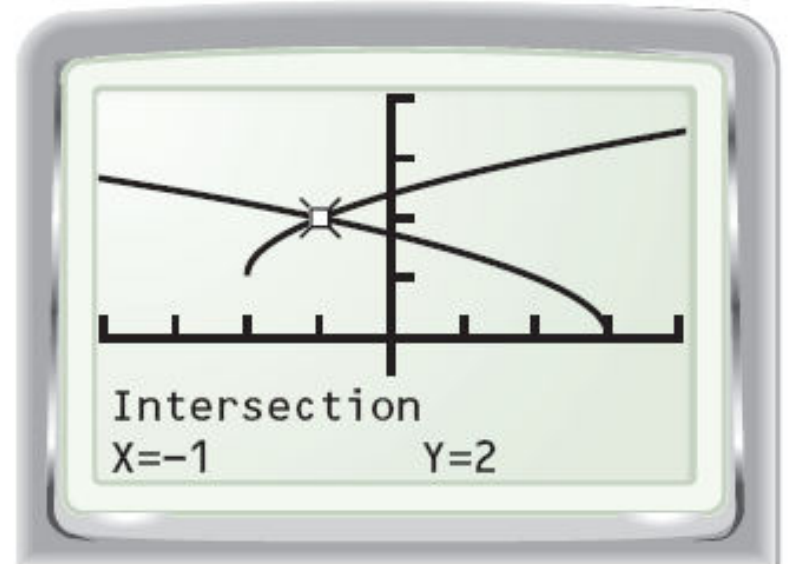
GUIDED PRACTICE for Examples 5 and 6

Solve the equation. Check for extraneous solutions

11. $x - \dfrac{1}{2} = \sqrt{\dfrac{1}{4}x}$

$x - \dfrac{1}{2} = \sqrt{\dfrac{1}{4}} \ x$ **Write original equation.**

$(x + \dfrac{1}{2})^2 = \left(\sqrt{\dfrac{1}{4}} \ x \right)^2$ **Square each side.**

$x^2 - x + \dfrac{1}{4} = \dfrac{1}{4} \ x$ **Expand left side and simplify right side.**

$x^2 - \dfrac{5}{4}x + \dfrac{1}{4} = 0$

$4x^2 - 5x + 1 = 0$ **Write in standard form.**

$(4x - 1)(x - 1) = 0$ **Factor.**

GUIDED PRACTICE   **for Examples 5 and 6**

$4x - 1 = 0$ **or** $\quad x - 1 = 0$ **Zero-product property.**

$x = \dfrac{1}{4}$ **or** $x = 1$ **Solve for x.**

Check $x = 1$ in the original equation.

$$x - \frac{1}{2} = \sqrt{\frac{1}{4}x}$$

$$1 - \frac{1}{2} \overset{?}{=} \sqrt{\frac{1}{4}x\,1}$$

$$\frac{1}{2} \overset{?}{=} \sqrt{\frac{1}{4}}$$

$$\frac{1}{2} = \frac{1}{2}$$

Check $x = \dfrac{1}{4}$ in the original equation.

$$x - \frac{1}{2} = \sqrt{\frac{1}{4}x}$$

$$\frac{1}{4} - \frac{1}{2} \overset{?}{=} \sqrt{\frac{1}{4}x\frac{1}{4}}$$

$$-\frac{1}{4} \overset{?}{=} \sqrt{\frac{1}{16}}$$

$$-\frac{1}{4} \neq \frac{1}{4}$$

The only solution is 1 (the apparent solution $1/4$ is extraneous)

GUIDED PRACTICE **for Examples 5 and 6**

Solve the equation. Check for extraneous solutions

12. $\sqrt{10x + 9} = x + 3$

$\sqrt{10x + 9} = x + 3$ **Write original equation.**

$\left(\sqrt{10x + 9}\right)^2 \overset{?}{=} (x + 3)^2$ **Square each side.**

$10x + 9 = x^2 + 6x + 9$ **Expand right side and simplify left side.**

$x^2 - 4x = 0$ **Write in standard form.**

$x(x - 4) = 0$ **Factor.**

$(x - 4) = 0$ **or** $x = 0$ **Zero-product property.**

$x = 4$ **or** $x = 0$ **Solve for x.**

Check $x = 4$ **in the original equation.**

$$\sqrt{10x + 9} = x + 3$$

$$\sqrt{10x\ 4 + 9} \overset{?}{=} 4 + 3$$

$$\sqrt{40 + 9} \overset{?}{=} 7$$

$$\sqrt{49} \overset{?}{=} 7$$

$$7 = 7$$

Check $x = 0$ **in the original equation.**

$$\sqrt{10x + 9} = x + 3$$

$$\sqrt{10x\ 0 + 9} \overset{?}{=} 0 + 3$$

$$\sqrt{9} \overset{?}{=} 3$$

$$3 = 3$$

The solution are 4 **and** 0**.**

GUIDED PRACTICE **for Examples 5 and 6**

Solve the equation. Check for extraneous solutions

13. $\sqrt{2x+5} = \sqrt{x+7}$

$\sqrt{2x+5} = \sqrt{x+7}$ **Write original equation.**

$\left(\sqrt{2x+5}\right)^2 = \left(\sqrt{x+7}\right)^2$ **Square each side.**

$2x + 5 = x + 7$ **Simplify both the sides.**

$x - 2 = 0$ **Simplify.**

$x = 2$ **Simplify.**

GUIDED PRACTICE   **for Examples 5 and 6**

Check $x = 2$ **in the original equation**

$$\sqrt{2x + 5} = \sqrt{x + 7}$$

$$\sqrt{2 \cdot 2 + 5} \overset{?}{=} \sqrt{2 + 7}$$

$$\sqrt{9} \overset{?}{=} \sqrt{9}$$

$$3 = 3$$

The solution is 2**.**

GUIDED PRACTICE **for Examples 5 and 6**

Solve the equation. Check for extraneous solutions

14. $\sqrt{x + 6} - 2 = \sqrt{x - 2}$

Solve

$$\sqrt{x + 6} - 2 = \sqrt{x - 2}$$ **Write original equation.**

$$\left(\sqrt{x + 6} - 2\right)^2 = \left(\sqrt{x - 2}\right)^2$$ **Square each side.**

$$x + 6 - 4\sqrt{x + 6} + 4 = x - 2$$ **Simplify each side.**

$$-4\sqrt{x + 6} = -12$$

$$\sqrt{x + 6} = 3$$

GUIDED PRACTICE   for Examples 5 and 6

$$\sqrt{x + 6} = 3$$ **Divide each side by -4.**

$$\left(\sqrt{x + 6}\right)^2 = 3^2$$ **Square each side.**

$$x + 6 = 9$$ **Simplify.**

$$x = 3$$ **Simplify.**

GUIDED PRACTICE **for Examples 5 and 6**

Check $x = 3$ **in the original equation**

$$\sqrt{x + 6} - 2 = \sqrt{x - 2}$$

$$\sqrt{3 + 6} - 2 \overset{?}{=} \sqrt{3 - 2}$$

$$\sqrt{9} - 2 \overset{?}{=} \sqrt{1}$$

$$1 = 1$$