EXAMPLE 1 Simplify natural base expressions

Simplify the expression.

a. $e^2 \cdot e^5 = e^{2+5}$

$= e^7$

b. $\dfrac{12e^4}{3e^3} = 4e^{4-3}$

$= 4e$

c. $(5e^{-3x})^2 = 5^2(e^{-3x})^2$

$= 25e^{-6x}$

$= \dfrac{25}{e^{6x}}$

EXAMPLE 2 Evaluate natural base expressions

Use a calculator to evaluate the expression.

	Expression	Keystrokes	Display
a.	e^4	2nd $[e^x]$ 4) ENTER	54.59815003
b.	$e^{-0.09}$	2nd $[e^x]$ (−) 0.09) ENTER	0.9139311853

GUIDED PRACTICE for Examples 1 and 2

Simplify the expression.

1. $e^7 \cdot e^4 = e^{7+4}$

$= e^{11}$

2. $2e^{-3} \cdot 6e^5 = (2 \cdot 6)\, e^{-3+5}$

$= 12e^2$

3. $\dfrac{24e^8}{4e^5} = 6 \cdot \dfrac{e^8}{e^5}$

$= 6\, e^{8-5}$

$= 6e^3$

GUIDED PRACTICE for Examples 1 and 2

Simplify the expression.

4. $(10e^{-4x})^3 = 10^3(e^{-4x})^3$

$= 1000\,e^{-12x}$

$= \dfrac{1000}{e^{12x}}$

5. Use a calculator to evaluate $e^{3/4}$**.**

SOLUTION $e^{3/4} = 2.117$

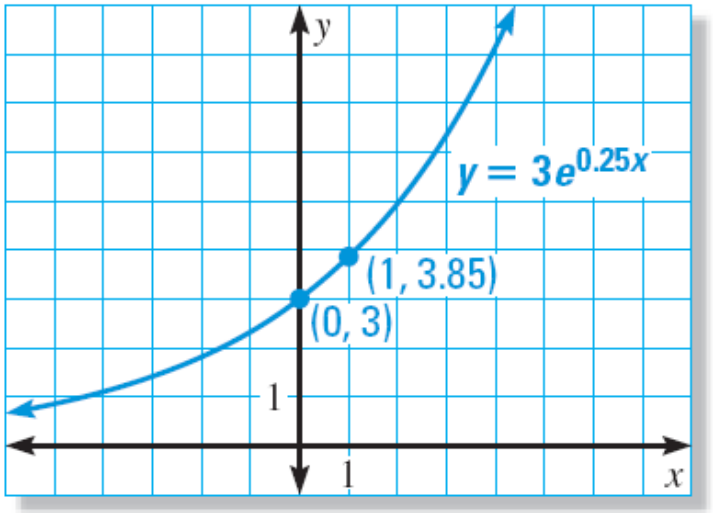
EXAMPLE 3 Graph natural base functions

Graph the function. State the domain and range.

a. $y = 3e^{0.25x}$

SOLUTION

Because $a = 3$ is positive and $r = 0.25$ is positive, the function is an exponential growth function. Plot the points $(0, 3)$ and $(1, 3.85)$ and draw the curve.



The domain is all real numbers, and the range is $y > 0$.

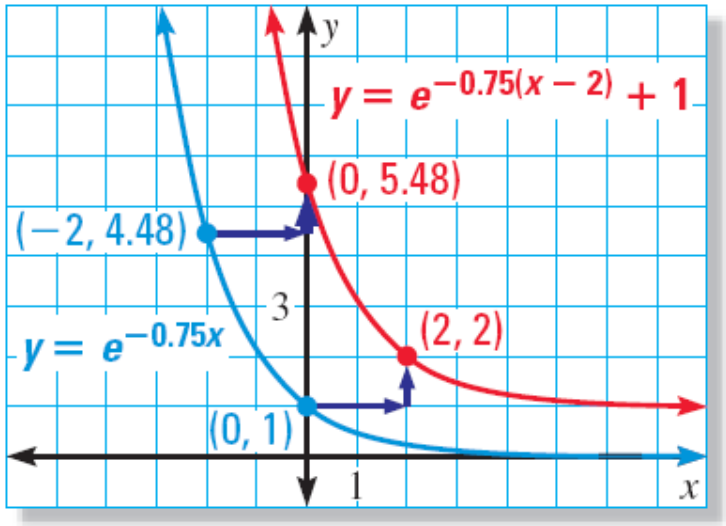
EXAMPLE 3 Graph natural base functions

Graph the function. State the domain and range.

b. $y = e^{-0.75(x-2)} + 1$

SOLUTION

$a = 1$ **is positive and** $r = -0.75$ **is negative, so the function is an exponential decay function. Translate the graph of** $y = e^{-0.75x}$ **right** 2 **units and up** 1 **unit.**



The domain is all real numbers, and the range is $y > 1$.

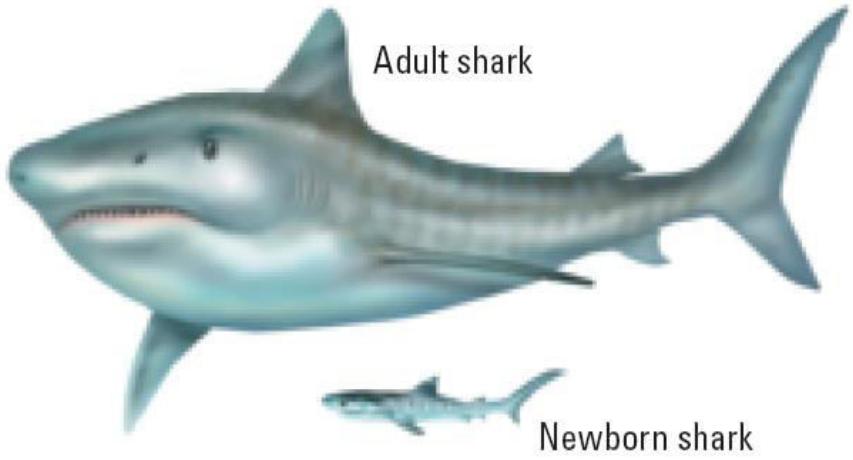
EXAMPLE 4 Solve a multi-step problem

Biology

The length l (in centimeters) **of a tiger shark can be modeled by the function**

$$l = 337 - 276\,e^{-0.178t}$$

where t **is the shark's age** (in years)**.**



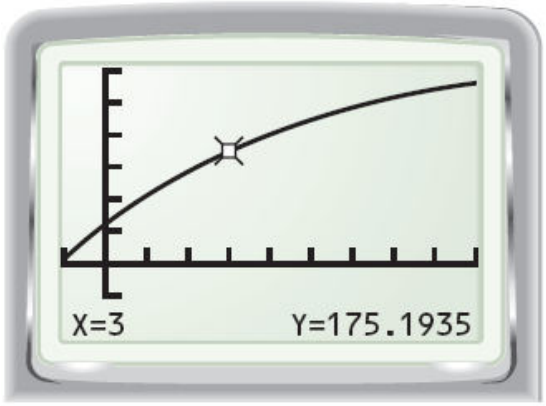
- **Graph the model.**
- **Use the graph to estimate the length of a tiger shark that is** 3 years **old.**

EXAMPLE 4 Solve a multi-step problem

SOLUTION

STEP 1 Graph the model, as shown.

STEP 2 Use the *trace* feature to determine that $l \approx 175$ when $t = 3$.



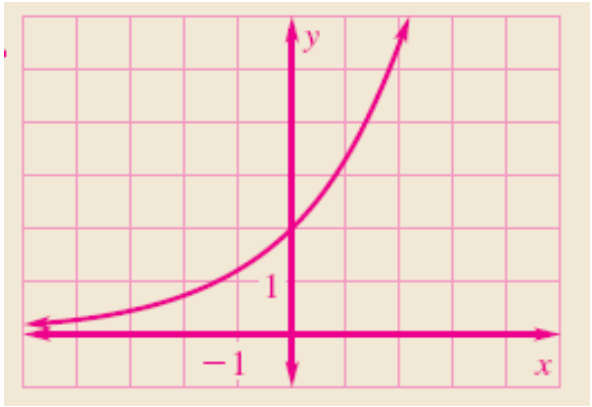
ANSWER

The length of a 3year**-old tiger shark is about** 175 centimeters**.**

GUIDED PRACTICE for Examples 3 and 4

Graph the function. State the domain and range.

6. $y = 2e^{0.5x}$

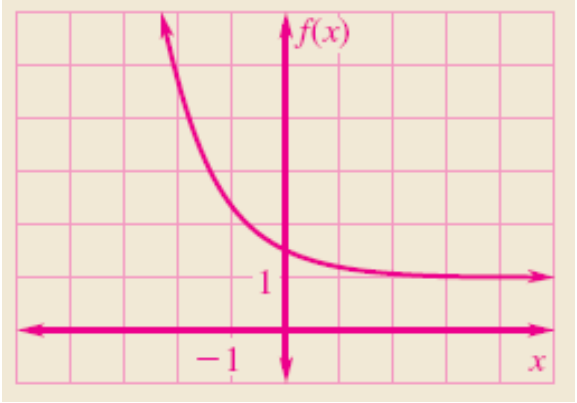


domain: all real numbers,
range: $y>0$

GUIDED PRACTICE for Examples 3 and 4

Graph the function. State the domain and range.

7. $f(x) = \frac{1}{2} e^{-x} + 1$

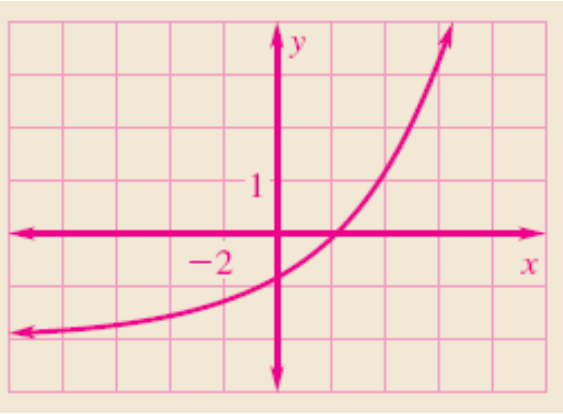


domain: all real numbers,
range: $y>1$

GUIDED PRACTICE for Examples 3 and 4

Graph the function. State the domain and range.

8. $y = 1.5e^{0.25(x-1)} - 2$



domain: all real numbers,
range: $y > -2$

GUIDED PRACTICE for Examples 3 and 4

9. WHAT IF? In Example 4, use the given function to estimate the length of a tiger shark that is 5 years old.

SOLUTION

$l = 337 - 276\,e^{-0.178t}$

where t is the shark's age (in years).

$t = 5$

$l = 337 - 276\,e^{-0.178 \times 5}$

$= 337 - 113.4$

$= 224\text{cm}$

ANSWER **The length of the tiger shark is 224 cm.**

EXAMPLE 5 Model continuously compounded interest

Finance

You deposit $4000 in an account that pays 6% annual interest compounded continuously. What is the balance after 1 year?

SOLUTION

Use the formula for continuously compounded interest.

$$A = Pe^{rt}$$ **Write formula.**

$$= 4000\, e^{0.06(1)}$$ **Substitute 4000 for P, 0.06 for r, and 1 for t.**

$$\approx 4247.35$$ **Use a calculator.**

ANSWER **The balance at the end of 1 year is $4247.35.**

GUIDED PRACTICE for Example 5

FINANCE: You deposit $2500 in an account that pays 5% annual interest compounded continuously. Find the balance after each amount of time?

a. 2 **years**

SOLUTION

Use the formula for continuously compounded interest.

$$A = Pe^{rt}$$ **Write formula.**

$$= 2500\, e^{0.05 \times 2}$$ **Substitute 2500 for P, 0.05 for r, and 2 for t.**

$$= 2500\, e^{0.10}$$

$$= 2500 \times 1.105$$

$$\approx 2762.9$$

ANSWER **The balance at the end of 2 years is $2762.9.**

GUIDED PRACTICE for Example 5

FINANCE: You deposit $2500 in an account that pays 5% annual interest compounded continuously. Find the balance after each amount of time?

b. 5 years

SOLUTION

Use the formula for continuously compounded interest.

$$A = Pe^{rt}$$ **Write formula.**

$$= 2500 \times e^{0.05 \times 5}$$ **Substitute 2500 for P, 0.05 for r and 5 for t.**

$$= 2500 \times e^{0.25}$$

$$= 2500 \times 1.2840$$

$$\approx 3210.06$$

ANSWER **The balance at the end of 5 years is $3210.06.**

GUIDED PRACTICE for Example 5

FINANCE: You deposit $2500 in an account that pays 5% annual interest compounded continuously. Find the balance after each amount of time?

c. 7.5 **years**

SOLUTION

Use the formula for continuously compounded interest.

$$A = Pe^{rt}$$ **Write formula.**

$$= 2500 \times e^{0.05 \times 7.5}$$ **Substitute 2500 for P, 0.05 for r and 7.5 for t.**

$$= 2500 \times e^{0.375}$$

$$= 2500 \times 1.459$$

$$\approx 3637.48$$

ANSWER **The balance at the end of 7.5 year is $3637.48.**

GUIDED PRACTICE for Example 5

11. FINANCE: Find the amount of interest earned in parts (a) – (c) of Exercise 10.

SOLUTION

a. $A = 2500$

The balance at the end of 2 years is $2762.93.

The amount of interest = balance − deposit

$$= \$2762.93 - \$2500$$

$$= \$262.93$$

GUIDED PRACTICE for Example 5

11. FINANCE: Find the amount of interest earned in parts (a) – (c) of Exercise 10.

SOLUTION

b. $A = 2500$

The balance at the end of 5 years is $3210.06.

The amount of interest = balance − deposit

$$= \$3210.06 - \$2500$$

$$= \$710.06$$

GUIDED PRACTICE for Example 5

11. FINANCE: Find the amount of interest earned in parts (a) – (c) of Exercise 10.

SOLUTION

c. $A = 2500$

The balance at the end of 7.5 years is $3637.48.

The amount of interest = balance − deposit

$$= \$3637.48 - \$2500$$

$$= \$1137.48$$