EXAMPLE 1 Rewrite logarithmic equations

	Logarithmic Form	Exponential Form
a.	$\log_2 8 = 3$	$2^3 = 8$
b.	$\log_4 1 = 0$	$4^0 = 1$
c.	$\log_{12} 12 = 1$	$12^1 = 12$
d.	$\log_{1/4} 4 = -1$	$\left(\frac{1}{4}\right)^{-1} = 4$

GUIDED PRACTICE for Example 1

Rewrite the equation in exponential form.

	Logarithmic Form	Exponential Form
1.	$\log_3 81 = 4$	$3^4 = 81$
2.	$\log_7 7 = 1$	$7^1 = 7$
3.	$\log_{14} 1 = 0$	$14^0 = 1$
4.	$\log_{1/2} 32 = -5$	$\left(\frac{1}{2}\right)^{-5} = 32$

EXAMPLE 2 Evaluate logarithms

Evaluate the logarithm.

a. $\log_4 64$

SOLUTION

To help you find the value of $\log_b y$**, ask yourself what power of** b **gives you** y**.**

a. 4 **to what power gives** 64**?** $4^3 = 64$**, so** $\log_4 64 = 3$**.**

b. $\log_5 0.2$

b. 5 **to what power gives** 0.2**?** $5^{-1} = -0.2$**, so** $\log_5 0.2 = -1$**.**

EXAMPLE 2 Evaluate logarithms

Evaluate the logarithm.

c. $\log_{1/5} 125$

SOLUTION

To help you find the value of $\log_b y$, ask yourself what power of b gives you y.

c. $\frac{1}{5}$ **to what power gives 125?** $\left(\frac{1}{5}\right)^{-3} = 125$, **so** $\log_{1/5} 125 = -3$.

d. $\log_{36} 6$

d. 36 **to what power gives 6?** $36^{1/2} = 6$, **so** $\log_{36} 6 = \frac{1}{2}$.

EXAMPLE 3 Evaluate common and natural logarithms

Expression	Keystrokes	Display	Check
a. $\log 8$	LOG 8) ENTER	0.903089987	$10^{0.903} \approx 8$ ✓
b. $\ln 0.3$	LN .3) ENTER	–1.203972804	$e^{-1.204} \approx 0.3$ ✓

EXAMPLE 4 Evaluate a logarithmic model

Tornadoes

The wind speed s (in miles per hour) near the center of a tornado can be modeled by

$$s = 93 \log d + 65$$

where d is the distance (in miles) that the tornado travels. In 1925, a tornado traveled 220 miles through three states. Estimate the wind speed near the tornado's center.

Not drawn to scale

EXAMPLE 4 Evaluate a logarithmic model

SOLUTION

$s = 93 \log d + 65$	Write function.
$= 93 \log 220 + 65$	Substitute 220 for d.
$\approx 93(2.342) + 65$	Use a calculator.
$= 282.806$	Simplify.

ANSWER

The wind speed near the tornado's center was about 283 miles per hour.

GUIDED PRACTICE for Examples 2, 3 and 4

Evaluate the logarithm. Use a calculator if necessary.

5. $\log_2 32$

SOLUTION

2 **to what power gives** 32**?**

$2^5 = 32$, **so** $\log_2 32 = 5$.

6. $\log_{27} 3$

SOLUTION

27 **to what power gives** 3**?**

$27^{1/3} = 3$, **so** $\log_{27} 3 = \frac{1}{3}$.

GUIDED PRACTICE for Examples 2, 3 and 4

Evaluate the logarithm. Use a calculator if necessary.

Expression	Keystrokes	Display	Check
7. log 12	LOG 12) ENTER	1.079	$10^{1.079} \approx 12$ ✓
8. ln 0.75	LN .75) ENTER	–0.288	$e^{-0.288} \approx 0.75$ ✓

GUIDED PRACTICE for Examples 2, 3 and 4

9. **WHAT IF?** **Use the function in Example 4 to estimate the wind speed near a tornado's center if its path is 150 miles long.**

SOLUTION

$s = 93 \log d + 65$	Write function.
$= 93 \log 150 + 65$	Substitute 150 for d.
$\approx 93(2.1760) + 65$	Use a calculator.
$= 267$	Simplify.

ANSWER

The wind speed near the tornado's center is about 267 miles per hour.

EXAMPLE 5 Use inverse properties

Simplify the expression.

a. $10^{\log 4}$

b. $\log_5 25^x$

SOLUTION

a. $10^{\log 4} = 4$

$b^{\log_b x} = x$

b. $\log_5 25^x = \log_5 (5^2)^x$ — Express 25 as a power with base 5.

$= \log_5 5^{2x}$ — Power of a power property

$= 2x$ — $\log_b b^x = x$

EXAMPLE 6 Find inverse functions

Find the inverse of the function.

a. $y = 6^x$

b. $y = \ln(x + 3)$

SOLUTION

a. From the definition of logarithm, the inverse of $y = 6^x$ is $y = \log_6 x$.

b.

$y = \ln(x + 3)$ — Write original function.

$x = \ln(y + 3)$ — Switch x and y.

$e^x = (y + 3)$ — Write in exponential form.

$e^x - 3 = y$ — Solve for y.

ANSWER The inverse of $y = \ln(x + 3)$ is $y = e^x - 3$.

GUIDED PRACTICE for Examples 5 and 6

Simplify the expression.

10. $8^{\log_8 x}$

SOLUTION

$8^{\log_8 x} = x$ $\quad b^{\log_b b} = x$

11. $\log_7 7^{-3x}$

SOLUTION

$\log_7 7^{-3x} = -3x$ $\quad \log_a a^x = x$

GUIDED PRACTICE for Examples 5 and 6

Simplify the expression.

12. $\log_2 64^x$

SOLUTION

$\log_2 64^x = \log_2 (2^6)^x$ — Express 64 as a power with base 2.

$= \log_2 2^{6x}$ — Power of a power property

$= 6x$ — $\log_b b^x = x$

13. $e^{ln 20}$

SOLUTION

$e^{ln 20} = e^{\log_e 20} = 20$ — $e^{\log_e x} = x$

GUIDED PRACTICE for Examples 5 and 6

14. Find the inverse of $y = 4^x$

SOLUTION

From the definition of logarithm, the inverse of $y = 6$ is $y = \log_4 x$.

15. Find the inverse of $y = \ln(x - 5)$.

SOLUTION

$y = \ln(x - 5)$ Write original function.

$x = \ln(y - 5)$ Switch x and y.

$e^x = (y - 5)$ Write in exponential form.

$e^x + 5 = y$ Solve for y.

ANSWER The inverse of $y = \ln(x - 5)$ is $y = e^x + 5$.

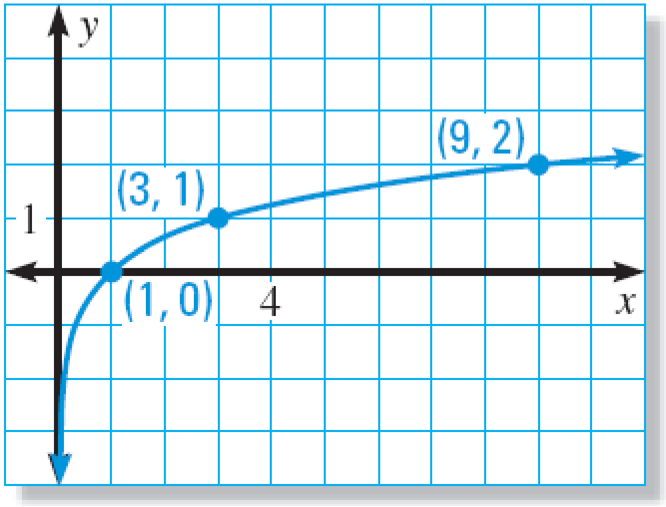
EXAMPLE 7 Graph logarithmic functions

Graph the function.

a. $y = \log_3 x$

SOLUTION

Plot several convenient points, such as (1, 0), (3, 1), and (9, 2). The y-axis is a vertical asymptote.



From *left* to *right*, draw a curve that starts just to the right of the y-axis and moves up through the plotted points, as shown below.

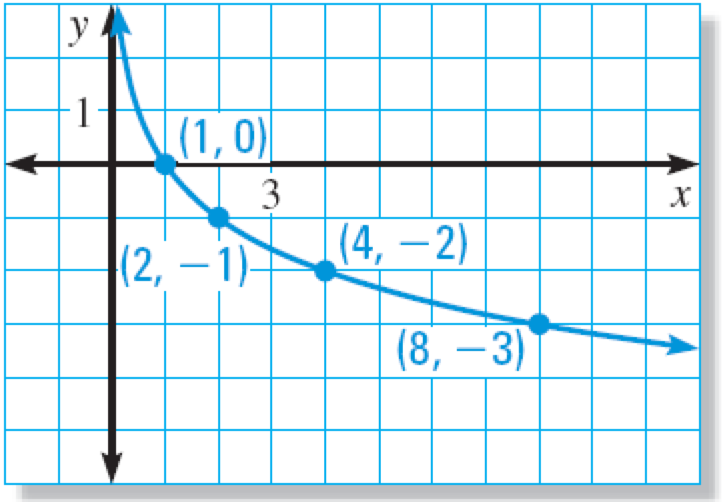
EXAMPLE 7 Graph logarithmic functions

Graph the function.

b. $y = \log_{1/2} x$

SOLUTION

Plot several convenient points, such as (1, 0), (2, –1), (4, –2), and (8, –3). The y-axis is a vertical asymptote.

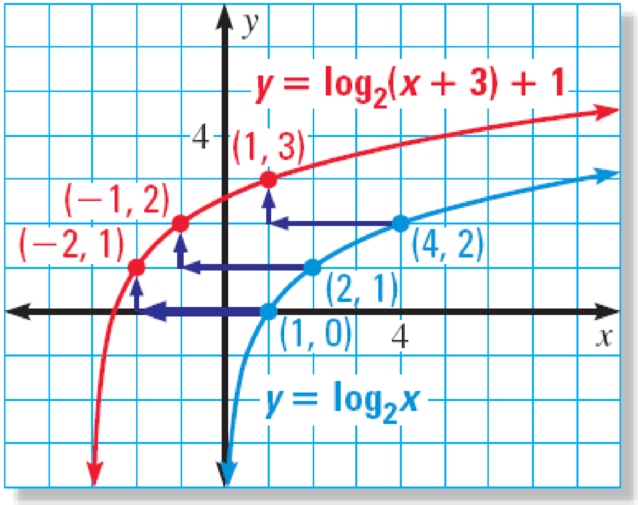


From *left* to *right*, draw a curve that starts just to the right of the y-axis and moves down through the plotted points, as shown below.

EXAMPLE 8 Translate a logarithmic graph

Graph $y = \log_2(x + 3) + 1$. State the domain and range.

SOLUTION



STEP 1 **Sketch the graph of the parent function $y = \log_2 x$, which passes through $(1, 0)$, $(2, 1)$, and $(4, 2)$.**

STEP 2 **Translate the parent graph left 3 units and up 1 unit. The translated graph passes through $(–2, 1)$, $(–1, 2)$, and $(1, 3)$. The graph's asymptote is $x = –3$. The domain is $x > –3$, and the range is all real numbers.**

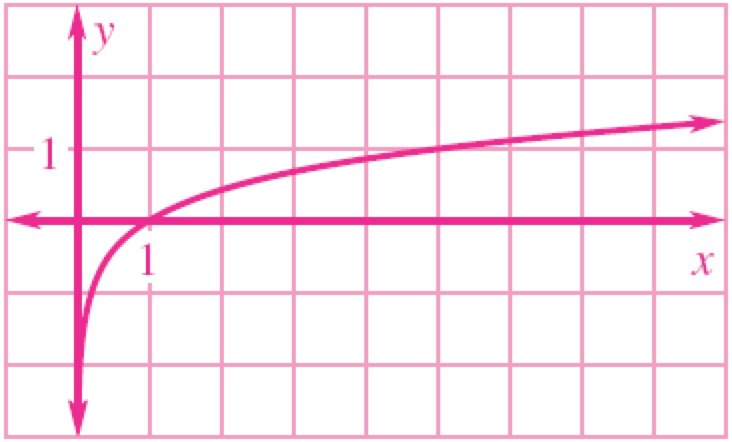
GUIDED PRACTICE for Examples 7 and 8

Graph the function. State the domain and range.

16. $y = \log_5 x$

SOLUTION

If $x = 1$ $y = 0$,
$x = 5$ $y = 1$,
$x = 10$ $y = 2$



Plot several convenient points, such as (1, 0), (5, 1), **and** (10, 2). **The** y**-axis is a vertical asymptote.**

GUIDED PRACTICE for Examples 7 and 8

From *left* to *right*, draw a curve that starts just to the right of the y-axis and moves up through the plotted points.

The domain is $x > 0$, and the range is all real numbers.

GUIDED PRACTICE for Examples 7 and 8

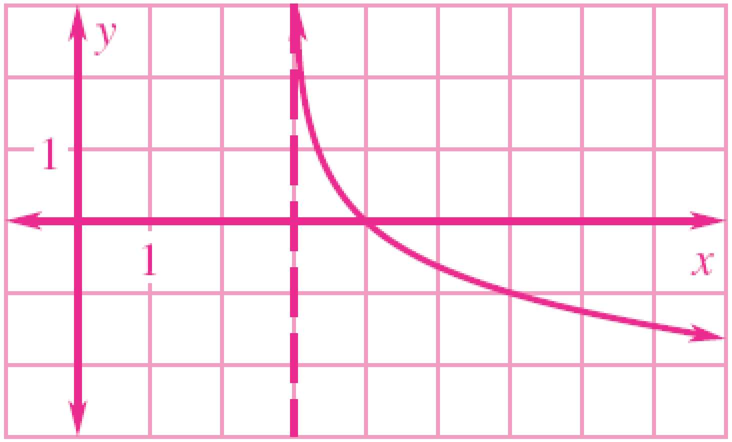
Graph the function. State the domain and range.

17. $y = \log_{1/3}(x - 3)$

SOLUTION

domain: $x > 3$,

range: all real numbers



GUIDED PRACTICE for Examples 7 and 8

Graph the function. State the domain and range.

18. $y = \log_4 (x + 1) - 2$

SOLUTION

domain: $x > 21$**,**

range: all real numbers