EXAMPLE 1 Use properties of logarithms

Use $\log_4 3 \approx 0.792$ and $\log_4 7 \approx 1.404$ to evaluate the logarithm.

a. $\log_4 \frac{3}{7} = \log_4 3 - \log_4 7$ Quotient property

$\approx 0.792 - 1.404$ Use the given values of $\log_4 3$ and $\log_4 7$.

$= -0.612$ Simplify.

b. $\log_4 21 = \log_4 (3 \cdot 7)$ Write 21 as $3 \cdot 7$.

$= \log_4 3 + \log_4 7$ Product property

$\approx 0.792 + 1.404$ Use the given values of $\log_4 3$ and $\log_4 7$.

$= 2.196$ Simplify.

EXAMPLE 1 Use properties of logarithms

Use $\log_4 3 \approx 0.792$ **and** $\log_4 7 \approx 1.404$ **to evaluate the logarithm.**

c. $\log_4 49 = \log_4 7^2$ — **Write** 49 **as** 7^2

$= 2\log_4 7$ — **Power property**

$\approx 2(1.404)$ — **Use the given value of** $\log_4 7$.

$= 2.808$ — **Simplify.**

GUIDED PRACTICE for Example 1

Use $\log_6 5 \approx 0.898$ and $\log_6 8 \approx 1.161$ to evaluate the logarithm.

1. $\log_6 \frac{5}{8} = \log_6 5 - \log_6 8$ **Quotient property**

$\approx 0.898 - 1.161$ **Use the given values of $\log_6 5$ and $\log_6 8$.**

$= -0.263$ **Simplify.**

2. $\log_6 40 = \log_6(8 \cdot 5)$ **Write 40 as $8 \cdot 5$.**

$= \log_6 8 + \log_6 5$ **Product property**

$\approx 1.161 + 0.898$ **Use the given values of $\log_6 5$ and $\log_6 8$.**

$= 2.059$ **Simplify.**

GUIDED PRACTICE for Example 1

Use $\log_6 5 \approx 0.898$ and $\log_6 8 \approx 1.161$ to evaluate the logarithm.

3. $\log_6 64 = \log_6 8^2$ — Write 64 as 8^2

$= 2\log_6 8$ — Power property

$\approx 2(1.161)$ — Use the given value of $\log_6 8$.

$= 2.322$ — Simplify.

4. $\log_6 125 = \log_6 5^3$ — Write 125 as 5^3

$= 3\log_6 5$ — Power property

$\approx 3(0.898)$ — Use the given value of $\log_6 5$.

$= 2.694$ — Simplify.

EXAMPLE 2 Expand a logarithmic expression

Expand $\log_6 \frac{5x^3}{y}$

SOLUTION

$\log_6 \frac{5x^3}{y} = \log_6 5x^3 - \log_6 y$ **Quotient property**

$= \log_6 5 + \log_6 x^3 - \log_6 y$ **Product property**

$= \log_6 5 + 3\log_6 x - \log_6 y$ **Power property**

EXAMPLE 3 Standardized Test Practice

Which of the following is equivalent to $\log 9 + 3\log 2 - \log 3$?

Ⓐ $\log 8$ Ⓑ $\log 14$ Ⓒ $\log 18$ Ⓓ $\log 24$

SOLUTION

$$\log 9 + 3\log 2 - \log 3 = \log 9 + \log 2^3 - \log 3$$ Power property

$$= \log(9 \cdot 2^3) - \log 3$$ Product property

$$= \log \frac{9 \cdot 2^3}{3}$$ Quotient property

$$= \log 24$$ Simplify.

ANSWER The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

GUIDED PRACTICE for Examples 2 and 3

5. Expand $\log 3x^4$.

SOLUTION

$\log 3x^4 = \log 3 + \log x^4$ **Product property**

$= \log 3 + 4 \log x$ **Power property**

GUIDED PRACTICE for Examples 2 and 3

6. Condense $\ln 4 + 3 \ln 3 - \ln 12$.

SOLUTION

$\ln 4 + 3 \ln 3 - \ln 12 = \ln 4 + \ln 3^3 - \ln 12$ — Power property

$= \ln (4 \cdot 3^3) - \ln 12$ — Product property

$= \ln \dfrac{4 \cdot 3^3}{12}$ — Quotient property

$= \ln 9$ — Simplify.

EXAMPLE 4 Use the change-of-base formula

Evaluate $\log_3 8$ using common logarithms and natural logarithms.

SOLUTION

Using common logarithms:

$$\log_3 8 = \frac{\log 8}{\log 3} \approx \frac{0.9031}{0.4771} \approx 1.893$$

Using natural logarithms:

$$\log_3 8 = \frac{\ln 8}{\ln 3} \approx \frac{2.0794}{1.0986} \approx 1.893$$

EXAMPLE 5 Use properties of logarithms in real life

Sound Intensity

For a sound with intensity I (in watts per square meter), the loudness $L(I)$ of the sound (in decibels) is given by the function

$$L(I) = 10 \log \frac{I}{I_0}$$

where I_0 is the intensity of a barely audible sound (about 10^{-12} watts per square meter). An artist in a recording studio turns up the volume of a track so that the sound's intensity doubles. By how many decibels does the loudness increase?

EXAMPLE 5 Use properties of logarithms in real life

SOLUTION

Let I be the original intensity, so that $2I$ is the doubled intensity.

Increase in loudness	$= L(2I) - L(I)$	**Write an expression.**
	$= 10 \log \frac{2I}{I_0} - 10 \log \frac{I}{I_0}$	**Substitute.**
	$= 10\left(\log \frac{2I}{I_0} - \log \frac{I}{I_0}\right)$	**Distributive property**
	$= 10\left(\log 2 + \log \frac{I}{I_0} - \log \frac{I}{I_0}\right)$	**Product property**
	$= 10 \log 2$	**Simplify.**
	≈ 3.01	**Use a calculator.**

ANSWER **The loudness increases by about** 3 **decibels.**

GUIDED PRACTICE for Examples 4 and 5

Use the change-of-base formula to evaluate the logarithm.

7. $\log_5 8$

SOLUTION

$$\log_5 8 = \frac{\log 8}{\log 5} \approx \frac{0.9031}{0.6989} \approx 1.292$$

8. $\log_8 14$

SOLUTION

$$\log_8 14 = \frac{\log 14}{\log 8} \approx \frac{1.146}{0.9031} \approx 1.269$$

GUIDED PRACTICE for Examples 4 and 5

Use the change-of-base formula to evaluate the logarithm.

9. $\log_{26} 9$

SOLUTION

$$\log_{26} 9 = \frac{\log 9}{\log 26} \approx \frac{0.9542}{1.4149} \approx 0.674$$

10. $\log_{12} 30$

SOLUTION

$$\log_{12} 30 = \frac{\log 30}{\log 12} \approx \frac{1.4777}{1.076} \approx 1.369$$

GUIDED PRACTICE for Examples 4 and 5

11. **WHAT IF? In Example 5, suppose the artist turns up the volume so that the sound's intensity triples. By how many decibels does the loudness increase?**

SOLUTION

$$L(I) = 10 \log \frac{I}{I_0}$$

Let I be the original intensity, so that $3I$ is the tripled intensity.

GUIDED PRACTICE for Examples 4 and 5

Increase in loudness $= L(3I) - L(I)$	Write an expression.
$= 10 \log \frac{3I}{I_0} - 10 \log \frac{I}{I_0}$	Substitute.
$= \left(10 \log \frac{3I}{I_0} - \log \frac{I}{I_0}\right)$	Distributive property
$= 10\left(\log 3 + \log \frac{I}{I_0} - \log \frac{I}{I_0}\right)$	Product property
$= 10 \log 3$	Simplify.
≈ 4.771	Use a calculator.

ANSWER

The loudness increases by about 4.771 decibels.