EXAMPLE 1 Solve by equating exponents

Solve $4^{x} = \left(\frac{1}{2}\right)^{x-3}$

SOLUTION

$4^{x} = \left(\frac{1}{2}\right)^{x-3}$	Write original equation.
$(2^{2})^{x} = (2^{-1})^{x-3}$	Rewrite 4 and $\frac{1}{2}$ as powers with base 2.
$2^{2x} = 2^{-x+3}$	Power of a power property
$2x = -x + 3$	Property of equality for exponential equations
$x = 1$	Solve for x.

ANSWER The solution is 1.

EXAMPLE 1

Solve by equating exponents

Check: Check the solution by substituting it into the original equation.

$$4^{1} \stackrel{?}{=} \left(\frac{1}{2}\right)^{1-3}$$ Substitute 1 for x.

$$4 \stackrel{?}{=} \left(\frac{1}{2}\right)^{-2}$$ Simplify.

$$4 = 4$$ Solution checks.

GUIDED PRACTICE for Example 1

Solve the equation.

1. $9^{2x} = 27^{x-1}$

$9^{2x} = 27$

SOLUTION

$9^{2x} = 27^{x-1}$ **Write original equation.**

$9^{2x} = (9 \cdot 3)^{x-1}$

$(3^2)^{2x} = (3^3)^{x-1}$ **Rewrite 9 and 27 as powers with base 3.**

$3^{4x} = 3^{3x-3}$ **Power of a power property**

$4x = 3x - 3$ **Property of equality for exponential equations**

GUIDED PRACTICE for Example 1

$4x - 3x = -3$

Property of equality for exponential equations

$= -3$

Solve for x.

ANSWER **The solution is -3.**

GUIDED PRACTICE for Example 1

Solve the equation.

2. $100^{7x+1} = 1000^{3x-2}$

SOLUTION

$100^{7x+1} = 1000^{3x-2}$ — **Write original equation.**

$(10^2)^{7x+1} = (10^3)^{3x-2}$ — **Rewrite 100 and 1000 as powers with base 10.**

$(10)^{14x+2} = (10)^{9x-6}$ — **Power of a power property**

$14x + 2 = 9x - 6$ — **Property of equality for exponential equations**

GUIDED PRACTICE for Example 1

$14x - 9x = -6 - 2$ Property of equality for exponential equations

$5x = -8$ Property of equality for exponential equations

$x = \frac{-8}{5}$ Solve for x.

ANSWER The solution is $\frac{-8}{5}$.

Solve the equation.

3. $81^{3-x} = \left(\frac{1}{3}\right)^{5x-6}$

SOLUTION

$81^{3-x} = \left(\frac{1}{3}\right)^{5x-6}$ Write original equation.

$(3^4)^{3-x} = (3^{-1})^{5x-6}$ Rewrite 81 and 1/3 as powers with base 3.

$3^{12-4x} = 3^{-5x+6}$ Power of a power property

$12 - 4x = -5x+6$ Property of equality for exponential equations

GUIDED PRACTICE for Example 1

$-4x + 5x = 6 - 12$ **Property of equality for exponential equations**

$x = -6$ **Solve for x.**

ANSWER **The solution is -6 .**

EXAMPLE 2 Take a logarithm of each side

Solve $4^x = 11$.

SOLUTION

$4^x = 11$ **Write original equation.**

$\log_4 4^x = \log_4 11$ **Take $\log_4$ of each side.**

$x = \log_4 11$ $\log_b b^x = x$

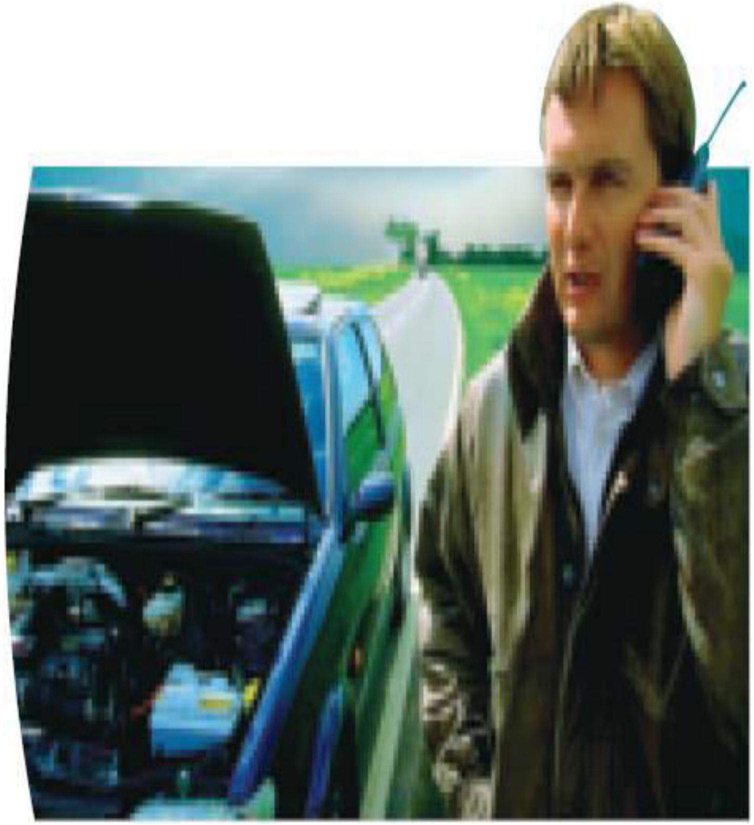
$x = \dfrac{\log 11}{\log 4}$ **Change-of-base formula**

$x \approx 1.73$ **Use a calculator.**

ANSWER **The solution is about 1.73. Check this in the original equation.**

EXAMPLE 3 Use an exponential model

Cars You are driving on a hot day when your car overheats and stops running. It overheats at 280°F and can be driven again at 230°F. If $r = 0.0048$ and it is 80°F outside, how long (in minutes) do you have to wait until you can continue driving?



EXAMPLE 3 Use an exponential model

SOLUTION

$T = (T_o - T_R)e^{-rt} + T_R$	**Newton's law of cooling**
$230 = (280 - 80)e^{-0.0048t} + 80$	**Substitute for** T, T_o, T_R **and** r.
$150 = 200e^{-0.0048t}$	**Subtract** 80 **from each side.**
$0.75 = e^{-0.0048t}$	**Divide each side by** 200.
$\ln 0.75 = \ln e^{-0.0048t}$	**Take natural log of each side.**
$-0.2877 \approx -0.0048t$	**ln** $e^x = \log_e e^x = x$
$60 \approx t$	**Divide each side by** -0.0048.

EXAMPLE 3 Use an exponential model

ANSWER

You have to wait about 60 minutes until you can continue driving.

GUIDED PRACTICE for Examples 2 and 3

Solve the equation.

4. $2^x = 5$

SOLUTION

$2^x = 5$ — Write original equation.

$\text{Log}_2 2^x = \log_2 5$ — Take $\log_2$ of each side.

$x = \log_2 5$ — $\log_b b^x = x$

$x = \dfrac{\log 5}{\log 2}$ — Change-of-base formula

$x \approx 2.32$ — Use a calculator.

GUIDED PRACTICE for Examples 2 and 3

ANSWER **The solution is about 2.32. Check this in the original equation.**

GUIDED PRACTICE for Examples 2 and 3

Solve the equation.

5. $7^{9x} = 15$

SOLUTION

$7^{9x} = 15$ Write original equation.

$\text{Log}_7 7^{9x} = \log_7 15$ Take $\log_7$ of each side.

$9x = \log_7 15$ $\log_b b^x = x$

$9x = \dfrac{\log 15}{\log 7}$ Change-of-base formula

$x \approx 0.155$ Use a calculator.

GUIDED PRACTICE for Examples 2 and 3

ANSWER **The solution is about** 0.155**. Check this in the original equation.**

GUIDED PRACTICE **for Examples 2 and 3**

Solve the equation.

6. $4e^{-0.3x} - 7 = 13$

SOLUTION

$$4e^{-0.3x} - 7 = 13$$

$$4e^{-0.3x} = 13 + 7$$

$$4e^{-0.3x} = 20$$

$$e^{-0.3x} = \frac{20}{4} = 5$$

$$\log_e e^{-0.3x} = \log_e 5$$

GUIDED PRACTICE for Examples 2 and 3

$-0.3x = \text{In } 5$

In $e^x = \log_e e^x = x$

$-0.3x = 1.6094$

Divide each side by $-0.3x$**.**

$$x = \frac{-1.6094}{0.3} = -5.365$$

ANSWER **The solution is about** 5.365**. Check this in the original equation.**

EXAMPLE 4 Solve a logarithmic equation

Solve $\log_5(4x - 7) = \log_5(x + 5)$.

SOLUTION

$\log_5(4x - 7) = \log_5(x + 5)$.	Write original equation.
$4x - 7 = x + 5$	Property of equality for logarithmic equations
$3x - 7 = 5$	Subtract x from each side.
$3x = 12$	Add 7 to each side.
$x = 4$	Divide each side by 3.

ANSWER The solution is 4.

EXAMPLE 4 Solve a logarithmic equation

Check: Check the solution by substituting it into the original equation.

$\log_5(4x - 7) = \log_5(x - 5)$ Write original equation.

$\log_5(4 \cdot 4 - 7) \stackrel{?}{=} \log_5(4 + 5)$ Substitute 4 for x.

$\log_5 9 = \log_5 9$ ✓ Solution checks.

EXAMPLE 5 Exponentiate each side of an equation

Solve $\log_4(5x - 1) = 3$

SOLUTION

$\log_4(5x - 1) = 3$	**Write original equation.**
$4^{\log_4(5x-1)} = 4^3$	**Exponentiate each side using base** 4**.**
$5x - 1 = 64$	$b^{\log_b x} = x$
$5x = 65$	**Add** 1 **to each side.**
$x = 13$	**Divide each side by** 5**.**

ANSWER **The solution is** 13**.**

EXAMPLE 5 Exponentiate each side of an equation

Check: $\log_4(5x - 1) = \log_4(5 \cdot 13 - 1) = \log_4 64$

Because $4^3 = 64$, $\log_4 64 = 3$. ✓

EXAMPLE 6 Standardized Test Practice

What is (are) the solution(s) of $\log 2x + \log (x - 5) = 2$?

(A) −5, 10 (B) 5 (C) 10 (D) 5, 10

SOLUTION

$\log 2x + \log (x - 5) = 2$ — **Write original equation.**

$\log [2x(x - 5)] = 2$ — **Product property of logarithms**

$10^{\log [2x(x - 5)]} = 10^2$ — **Exponentiate each side using base 10.**

$2x(x - 5) = 100$ — **Distributive property**

EXAMPLE 6 Standardized Test Practice

$$2x^2 - 10x = 100$$ $b^{\log_b x} = x$

$$2x^2 - 10x - 100 = 0$$ **Write in standard form.**

$$x^2 - 5x - 50 = 0$$ **Divide each side by 2.**

$$(x - 10)(x + 5) = 0$$ **Factor.**

$x = 10$ **or** $x = -5$ **Zero product property**

Check: Check the apparent solutions 10 and – 5 using algebra or a graph.

Algebra: Substitute 10 and 25 for x in the original equation.

EXAMPLE 6 Standardized Test Practice

$\log 2x + \log (x - 5) = 2$

$\log (2 \cdot 10) + \log (10 - 5) = 2$

$\log 20 + \log 5 = 2$

$\log 100 = 2$

$2 = 2$ ✓

$\log 2x + \log (x - 5) = 2$

$\log [2(-5)] + \log (-5 - 5) = 2$

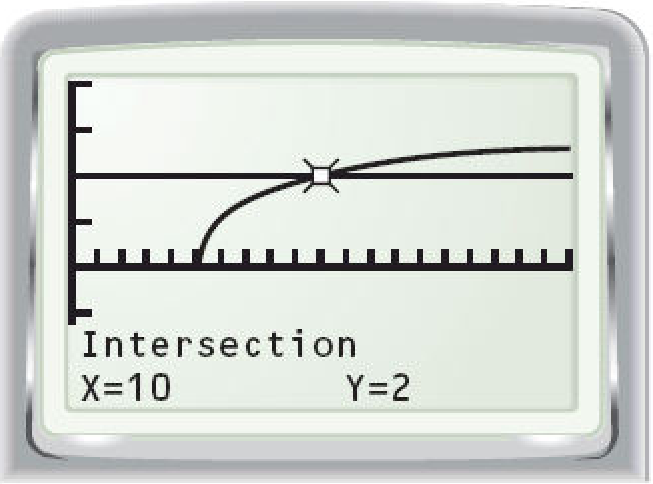
$\log (-10) + \log (-10) = 2$

Because log (-10) **is not defined,** -5 **is *not* a solution.**

So, 10 **is a solution.**

EXAMPLE 6 Standardized Test Practice

Graph: Graph $y = \log 2x + \log (x - 5)$ **and** $y = 2$ **in the same coordinate plane. The graphs intersect only once, when** $x = 10$**. So,** 10 **is the only solution.**



ANSWER

The correct answer is C.

GUIDED PRACTICE for Examples 4, 5 and 6

Solve the equation. Check for extraneous solutions.

7. $\ln(7x - 4) = \ln(2x + 11)$

SOLUTION

$\ln(7x - 4) = \ln(2x + 11)$ — Write original equation.

$7x - 4 = 2x + 11$ — Property of equality for logarithmic equations

$7x - 2x = 11 - 4$

$5x = 15$

$x = 3$ — Divide each side by 5.

ANSWER The solution is 3.

GUIDED PRACTICE for Examples 4, 5 and 6

Solve the equation. Check for extraneous solutions.

8. $\log_2(x-6)=5$

SOLUTION

$\log_2(x-6)=5$ Write original equation.

$2^{\log_2(x-6)}=2^5$ Exponentiate each side using base 2.

$x-6=32$ $b^{\log_b x}=x$

$x=32+6$ Add 6 to each side.

$x=38$

ANSWER The solution is 38.

GUIDED PRACTICE **for Examples 4, 5 and 6**

Solve the equation. Check for extraneous solutions.

9. $\log 5x + \log (x - 1) = 2$

SOLUTION

$\log 5x + \log (x - 5) = 2$ **Write original equation.**

$\log [5x(x - 1)] = 2$ **Product property of logarithms**

$10^{\log [5x(x-1)]} = 10^2$ **Exponentiate each side using base 10.**

$5x(x - 1) = 100$ **Distributive property**

GUIDED PRACTICE for Examples 4, 5 and 6

$$x^2 - x = \frac{100}{5}$$ $b^{\log_b x} = x$

$$x^2 - x = 20$$

$$x^2 - x - 20 = 0$$

$$x^2 - 5x + 4x - 20 = 0$$

$$x(x - 5) + 4(x - 5) = 0$$

$$(x - 5)(x + 4) = 0$$ **Factor.**

$x = 5$ **or** $x = -4$ **Zero product property**

Check: Check the apparent solutions 5 and -4 using algebra or a graph.

GUIDED PRACTICE for Examples 4, 5 and 6

Algebra: Substitute -4 **and** 5 **for** x **in the original equation.**

$\log 5x + \log (x - 1) = 2$	$\log 5x + \log (5x - 1) = 2$
$\log 5(-4) + \log (-4 - 1) = 2$	$\log [5(5)] + \log (5(5) - 1) = 2$
$\log -20 + \log -5 = 2$	$\log 25 + \log 24 = 2$
$\log 100 = 2$	$\log 600 = 2$
$2 = 2$ ✓	$2.778 \neq 2$

ANSWER **So,** -4 **is a solution.**

GUIDED PRACTICE for Examples 4, 5 and 6

Solve the equation. Check for extraneous solutions.

10. $\log_4(x + 12) + \log_4 x = 3$

SOLUTION

$\log_4(x + 12) + \log_4 x = 3$

$\log_4[(x + 12) \cdot x] = 3$

$\log_4(x^2 + 12x) = 3$

$4^{\log_4(x^2 + 12x)} = 4^3$

$x^2 + 12x = 4^3$

Exponentiate each side using base 4.

GUIDED PRACTICE for Examples 4, 5 and 6

$$x^2 + 12x = 64$$

$$x^2 + 12x - 64 = 0$$

$$x^2 + 16x - 4x - 64 = 0$$

$$x(x + 4) - 4(x + 4) = 0$$

$$(x - 4)(x + 4) = 0$$ **Factor.**

$x = 4$ **or** $x = -4$ **Zero product property**

Check: Check the apparent solutions 4 **and** -4 **using algebra or a graph.**

GUIDED PRACTICE for Examples 4, 5 and 6

Algebra: Substitute 4 and -4 for x in the original equation.

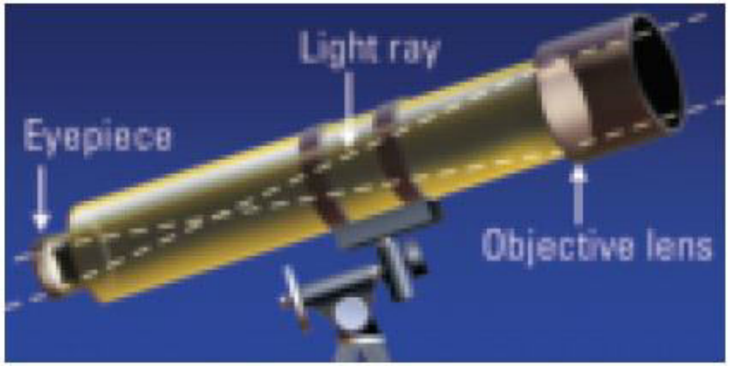
$$\log_4(4 + 12) + \log_4 4 = 3$$
$$\log_4 16 + \log_4 4 = 3$$
$$\log_4 16 + 1 = 3$$
$$\log_4 16 = 2$$
$$\frac{\log 16}{\log 4} = 2$$
$$\frac{1.204}{0.6020} = 2$$

$$\log_4(-4 + 12) + \log_{4} -4 = 3$$
$$\log_4 8 + \log_4 -4 = 3$$
$$\log_4 8 - 1 = 3$$
$$\log_4 8 = 4$$
$$\frac{\log 8}{\log 4} = 4$$
$$\frac{1.9030}{0.6020} \neq 4$$

ANSWER **So, 4 is a solution.**

EXAMPLE 7 Use a logarithmic model

Astronomy **The *apparent magnitude* of a star is a measure of the brightness of the star as it appears to observers on Earth. The apparent magnitude M of the dimmest star that can be seen with a telescope is given by the function**



$$M = 5 \log D + 2$$

where D is the diameter (in millimeters) of the telescope's objective lens. If a telescope can reveal stars with a magnitude of 12, what is the diameter of its objective lens?

EXAMPLE 7 Use a logarithmic model

SOLUTION

$M = 5 \log D + 2$	Write original equation.
$12 = 5 \log D + 2$	Substitute 12 for *M*.
$10 = 5 \log D$	Subtract 2 from each side.
$2 = \log D$	Divide each side by 5.
$10^2 = 10^{\text{Log } D}$	Exponentiate each side using base 10.
$100 = D$	Simplify.

ANSWER **The diameter is 100 millimeters.**

GUIDED PRACTICE for Example 7

11. **WHAT IF?** Use the information from Example 7 to find the diameter of the objective lens of a telescope that can reveal stars with a magnitude of 7.

SOLUTION

$M = 5 \log D + 2$ — Write original equation.

$7 = 5 \log D + 2$ — Substitute 12 for M.

$5 = 5 \log D$ — Subtract 2 from each side.

$1 = \log D$ — Divide each side by 5.

$10^1 = 10^{\text{Log } D}$

$10 = D$ — Simplify.

ANSWER The diameter is 10 millimeters.