

EXAMPLE 1**Simplify a rational expression**

$$\text{Simplify : } \frac{x^2 - 2x - 15}{x^2 - 9}$$

SOLUTION

$$\frac{x^2 - 2x - 15}{x^2 - 9} = \frac{(x + 3)(x - 5)}{(x + 3)(x - 3)}$$

Factor numerator and denominator.

$$= \frac{\cancel{(x + 3)}(x - 5)}{\cancel{(x + 3)}(x - 3)}$$

Divide out common factor.

$$= \frac{x - 5}{x - 3}$$

Simplified form

ANSWER

$$\frac{x - 5}{x - 3}$$

EXAMPLE 2

Solve a multi-step problem

Packaging

A company makes a tin to hold flavored popcorn. The tin is a rectangular prism with a square base. The company is designing a new tin with the same base and twice the height of the old tin.



- Find the surface area and volume of each tin.
- Calculate the ratio of surface area to volume for each tin.
- What do the ratios tell you about the efficiencies of the two tins?

EXAMPLE 2**Solve a multi-step problem****SOLUTION****Old tin****New tin****STEP 1**

$$S = 2s^2 + 4sh$$

$$S = 2s^2 + 4s(2h) \quad \text{Find surface area, } S.$$

$$= 2s^2 + 8sh$$

$$V = s^2h$$

$$V = s^2(2h) \quad \text{Find volume, } V.$$

$$= 2s^2h$$

STEP 2

$$\frac{S}{V} = \frac{2s^2 + 4sh}{s^2h}$$

$$\frac{S}{V} = \frac{2s^2 + 8sh}{2s^2h}$$

Write ratio of S to V .

$$= \frac{\cancel{s}(2s + 4h)}{\cancel{s}(sh)}$$

$$= \frac{\cancel{2s}(s + 4h)}{\cancel{2s}(sh)}$$

Divide out common factor.

$$= \frac{2s + 4h}{sh}$$

$$= \frac{s + 4h}{sh}$$

Simplified form

EXAMPLE 2**Solve a multi-step problem**

STEP 3
$$\frac{2s + 4h}{sh} > \frac{s + 4h}{sh}$$

Because the left side of the inequality has a greater numerator than the right side and both have the same (positive) denominator. The ratio of surface area to volume is *greater* for the old tin than for the new tin. So, the old tin is *less* efficient than the new tin.

Simplify the expression, if possible.

1. $\frac{2(x + 1)}{(x + 1)(x + 3)}$

SOLUTION

$$\begin{aligned}\frac{2(x + 1)}{(x + 1)(x + 3)} &= \frac{2\cancel{(x + 1)}}{\cancel{(x + 1)}(x + 3)} \\ &= \frac{2}{x + 3}\end{aligned}$$

Divide out common factor.

Simplified form

ANSWER

$$\frac{2}{x + 3}$$

GUIDED PRACTICE

for Examples 1 and 2

Multiply and Divide Rational Expressions

$$2. \frac{40x + 20}{10x + 30}$$

SOLUTION

$$\frac{40x + 20}{10x + 30} = \frac{20(2x + 1)}{10(x + 3)}$$

Factor numerator and denominator.

$$= \frac{\cancel{20}(2x + 1)}{\cancel{10}(x + 3)}$$

Divide out common factor.

$$= \frac{2(2x + 1)}{x + 3}$$

Simplified form

ANSWER

$$\frac{2(2x + 1)}{x + 3}$$

GUIDED PRACTICE

for Examples 1 and 2

Multiply and Divide Rational Expressions

3. $\frac{4}{x(x+2)}$

SOLUTION

$$\frac{4}{x(x+2)}$$

Simplified form

ANSWER

$$\frac{4}{x(x+2)}$$

4. $\frac{x + 4}{x^2 - 16}$

SOLUTION

$$\frac{x + 4}{x^2 - 16} = \frac{(x + 4)}{(x + 4)(x - 4)}$$

Factor numerator and denominator.

$$= \frac{\cancel{(x + 4)}}{\cancel{(x + 4)}(x - 4)}$$

Divide out common factor.

$$= \frac{1}{x - 4}$$

Simplified form

ANSWER

$$\frac{1}{x - 4}$$

GUIDED PRACTICE

for Examples 1 and 2

Multiply and Divide Rational Expressions

5. $\frac{x^2 - 2x - 3}{x^2 - x - 6}$

SOLUTION

$$\frac{x^2 - 2x - 3}{x^2 - x - 6} = \frac{(x - 3)(x + 1)}{(x - 3)(x + 2)}$$

Factor numerator and denominator.

$$= \frac{\cancel{(x - 3)}(x + 1)}{\cancel{(x - 3)}(x + 2)}$$

Divide out common factor.

$$= \frac{x + 1}{x + 2}$$

Simplified form

ANSWER

$$\frac{x + 1}{x + 2}$$

GUIDED PRACTICE

for Examples 1 and 2

Multiply and Divide Rational Expressions

6.
$$\frac{2x^2 + 10x}{3x^2 + 16x + 5}$$

SOLUTION

$$\begin{aligned}\frac{2x^2 + 10x}{3x^2 + 16x + 5} &= \frac{2x(x + 5)}{(3x + 1)(x + 5)} \\ &= \frac{2x\cancel{(x + 5)}}{(3x + 1)\cancel{(x + 5)}} \\ &= \frac{2x}{3x + 1}\end{aligned}$$

Factor numerator and denominator.

Divide out common factor.

Simplified form

ANSWER

$$\frac{2x}{3x + 1}$$

7. **What If?** In Example 2, suppose the new popcorn tin is the same height as the old tin but has a base with sides twice as long. What is the ratio of surface area to volume for this tin?

SOLUTION
Old tin
New tin

STEP 1 $S = 2s^2 + 4sh$ $S = 2(2s)^2 + 4(2s)h$ **Find surface area, S .**
 $= 8s^2 + 8sh$

$$V = s^2h$$

$$V = (2s)^2h$$

$$= 4s^2h$$

Find volume, V .

GUIDED PRACTICE**for Examples 1 and 2**

STEP 2 $\frac{S}{V} = \frac{2s^2 + 4sh}{s^2h}$ $\frac{S}{V} = \frac{8s^2 + 8sh}{2s^2h}$ **Write ratio of S to V.**

$= \frac{\cancel{s}(2s + 4h)}{\cancel{s}(sh)}$ $= \frac{\cancel{4s}(2s + 2h)}{\cancel{4s}(sh)}$ **Divide out common factor.**

$= \frac{2s + 4h}{sh}$ $= \frac{2s + 4h}{sh}$ **Simplified form**

ANSWER

$$\frac{2s + 4h}{sh}$$

EXAMPLE 3**Standardized Test Practice**

What is a simplified form of $\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y}$?

(A) $\frac{5}{2}x^6y$

(B) $7x^6y$

(C) $7x^{11}y$

(D) $7x^7y^{4/3}$

SOLUTION

$$\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y} = \frac{56x^7y^4}{8xy^3}$$

$$= \frac{\cancel{8} \cdot 7 \cdot \cancel{x} \cdot x^6 \cdot \cancel{y^3} \cdot y}{\cancel{8} \cdot \cancel{x} \cdot \cancel{y^3}}$$

$$= 7x^6y$$

Multiply numerators and denominators.

Factor and divide out common factors.

Simplified form

ANSWER

The correct answer is **B**. (A) (B) (C) (D)

EXAMPLE 4**Multiply rational expressions**

Multiply: $\frac{3x - 3x^2}{x^2 + 4x - 5} \cdot \frac{x^2 + x - 20}{3x}$

SOLUTION

$$\begin{aligned} & \frac{3x - 3x^2}{x^2 + 4x - 5} \cdot \frac{x^2 + x - 20}{3x} \\ = & \frac{3x(1-x)}{(x-1)(x+5)} \cdot \frac{(x+5)(x-4)}{3x} \\ = & \frac{3x(1-x)(x+5)(x-4)}{(x-1)(x+5)(3x)} \\ = & \frac{3x(-1)(x-1)(x+5)(x-4)}{(x-1)(x+5)(3x)} \\ = & \frac{\cancel{3x}(-1)(\cancel{x-1})(\cancel{x+5})(x-4)}{(\cancel{x-1})(\cancel{x+5})(\cancel{3x})} \end{aligned}$$

Factor numerators and denominators.

Multiply numerators and denominators.

Rewrite $1-x$ as $(-1)(x-1)$.

Divide out common factors.

EXAMPLE 4**Multiply rational expressions**

$$= (-1)(x - 4)$$

Simplify.

$$= -x + 4$$

Multiply.**ANSWER**

$$-x + 4$$

EXAMPLE 5**Multiply a rational expression by a polynomial**

Multiply: $\frac{x+2}{x^3-27} \cdot (x^2+3x+9)$

SOLUTION

$$\begin{aligned} & \frac{x+2}{x^3-27} \cdot (x^2+3x+9) \\ = & \frac{x+2}{x^3-27} \cdot \frac{x^2+3x+9}{1} \\ = & \frac{(x+2)(x^2+3x+9)}{(x-3)(x^2+3x+9)} \\ = & \frac{(x+2)\cancel{(x^2+3x+9)}}{(x-3)\cancel{(x^2+3x+9)}} \\ = & \frac{x+2}{x-3} \end{aligned}$$

Write polynomial as a rational expression.

Factor denominator.

Divide out common factors.

Simplified form

ANSWER

$$\frac{x+2}{x-3}$$

GUIDED PRACTICE**for Examples 3, 4 and 5**

Multiply the expressions. Simplify the result.

$$8. \quad \frac{3x^5 y^2}{8xy} \cdot \frac{6xy^2}{9x^3 y}$$

SOLUTION

$$\frac{3x^5 y^2}{2xy} \cdot \frac{6xy^2}{9x^3 y} = \frac{18x^6 y^4}{72x^4 y^2}$$

$$= \frac{\cancel{18} \cdot \cancel{x^4} \cdot \cancel{y^2} \cdot x^2 \cdot y^2}{\cancel{18} \cdot 4 \cdot \cancel{x^4} \cdot \cancel{y^2}}$$

$$= \frac{x^2 y^2}{4}$$

**Multiply numerators
and denominators.**

**Factor and divide out
common factors.**

Simplified form

GUIDED PRACTICE

for Examples 3, 4 and 5

$$9. \frac{2x^2 - 10x}{x^2 - 25} \cdot \frac{x + 3}{2x^2}$$

SOLUTION

$$\begin{aligned} & \frac{2x^2 - 10x}{x^2 - 25} \cdot \frac{x + 3}{2x^2} \\ = & \frac{2x(x - 5)}{(x - 5)(x + 5)} \cdot \frac{x + 3}{2x(x)} \\ = & \frac{2x(x - 5)(x + 3)}{(x - 5)(x + 5)2x(x)} \\ = & \frac{\cancel{2x}(x - \cancel{5})(x + 3)}{(x - \cancel{5})(x + 5)\cancel{2x}(x)} \\ = & \frac{x + 3}{x(x + 5)} \end{aligned}$$

Factor numerators and denominators.

Multiply numerators and denominators.

Divide out common factors.

Simplified form

GUIDED PRACTICE**for Examples 3, 4 and 5**

$$10. \frac{x+5}{x^3-1} \cdot x^2+x+1$$

SOLUTION

$$= \frac{x+5}{(x-1)(x^2+x+1)} \cdot \frac{x^2+x+1}{1}$$

Factor denominators.

$$= \frac{(x+5)(x^2+x+1)}{(x-1)(x^2+x+1)}$$

Multiply numerators and denominators.

$$= \frac{(x+5)\cancel{(x^2+x+1)}}{(x-1)\cancel{(x^2+x+1)}}$$

Divide out common factors.

$$= \frac{x+5}{x-1}$$

Simplified form

EXAMPLE 6**Divide rational expressions**

Divide : $\frac{7x}{2x-10} \div \frac{x^2-6x}{x^2-11x+30}$

SOLUTION

$$\begin{aligned} & \frac{7x}{2x-10} \div \frac{x^2-6x}{x^2-11x+30} \\ = & \frac{7x}{2x-10} \cdot \frac{x^2-11x+30}{x^2-6x} \\ = & \frac{7x}{2(x-5)} \cdot \frac{(x-5)(x-6)}{x(x-6)} \\ = & \frac{\cancel{7x}(x-5)\cancel{(x-6)}}{2\cancel{(x-5)}\cancel{(x)}\cancel{(x-6)}} \\ = & \frac{7}{2} \end{aligned}$$

Multiply by reciprocal.

Factor.

Divide out common factors.

Simplified form

ANSWER

$$\frac{7}{2}$$

EXAMPLE 7**Divide a rational expression by a polynomial**

Divide : $\frac{6x^2 + x - 15}{4x^2} \div (3x^2 + 5x)$

SOLUTION

$$\begin{aligned} & \frac{6x^2 + x - 15}{4x^2} \div (3x^2 + 5x) \\ = & \frac{6x^2 + x - 15}{4x^2} \cdot \frac{1}{3x^2 + 5x} \\ = & \frac{(3x + 5)(2x - 3)}{4x^2} \cdot \frac{1}{x(3x + 5)} \\ = & \frac{\cancel{(3x + 5)}(2x - 3)}{4x^2(x)\cancel{(3x + 5)}} \\ = & \frac{2x - 3}{4x^3} \end{aligned}$$

Multiply by reciprocal.

Factor.

Divide out common factors.

Simplified form

ANSWER

$$\frac{2x - 3}{4x^3}$$

Divide the expressions. Simplify the result.

11.
$$\frac{4x}{5x - 20} \div \frac{x^2 - 2x}{x^2 - 6x + 8}$$

SOLUTION

$$\begin{aligned} & \frac{4x}{5x - 20} \div \frac{x^2 - 2x}{x^2 - 6x + 8} \\ = & \frac{4x}{5x - 20} \cdot \frac{x^2 - 6x + 8}{x^2 - 2x} \\ = & \frac{4(x)(x - 4)(x - 2)}{5(x - 4)(x)(x - 2)} \\ = & \frac{4\cancel{(x)}\cancel{(x - 4)}\cancel{(x - 2)}}{5\cancel{(x - 4)}\cancel{(x)}\cancel{(x - 2)}} \\ = & \frac{4}{5} \end{aligned}$$

Multiply by reciprocal.

Factor.

Divide out common factors.

Simplified form

$$12. \frac{2x^2 + 3x - 5}{6x} \div (2x^2 + 5x)$$

SOLUTION

$$\frac{2x^2 + 3x - 5}{6x} \div (2x^2 + 5x)$$

$$= \frac{2x^2 + 3x - 5}{6x} \cdot \frac{1}{(2x^2 + 5x)}$$

$$= \frac{(2x + 5)(x - 1)}{6x(x)(2x + 5)}$$

$$= \frac{\cancel{(2x + 5)}(x - 1)}{6x(x)\cancel{(2x + 5)}}$$

$$= \frac{x - 1}{6x^2}$$

Multiply by reciprocal.

Factor.

Divide out common factors.

Simplified form