EXAMPLE 1 Add or subtract with like denominators

Perform the indicated operation.

a. $\frac{7}{4x} + \frac{3}{4x}$

b. $\frac{2x}{x+6} - \frac{5}{x+6}$

SOLUTION

a. $\frac{7}{4x} + \frac{3}{4x} = \frac{7+3}{4x} = \frac{10}{4x} = \frac{5}{2x}$

Add numerators and simplify result.

b. $\frac{2x}{x+6} - \frac{5}{x+6} = \frac{2x-5}{x+6}$

Subtract numerators.

GUIDED PRACTICE for Example 1

Perform the indicated operation and simplify.

a. $\frac{7}{12x} + \frac{5}{12x} = \frac{7-5}{12x} = \frac{2}{12x} = \frac{1}{6x}$

Subtract numerators and simplify results .

b. $\frac{2}{3x^2} + \frac{1}{3x^2} = \frac{2+1}{3x^2} = \frac{3}{3x^2} = \frac{1}{x^2}$

Add numerators and simplify results.

c. $\frac{4x}{x-2} - \frac{x}{x-2} = \frac{4x-x}{x-2} = \frac{3x}{x-2} = \frac{3x}{3x-2}$

Subtract numerators.

d. $\frac{4x}{x^2+1} + \frac{2}{x^2+1} = \frac{2x^2+2}{x^2+1} = \frac{2(x^2+1)}{x^2+1} = 2$

Factor numerators and simplify results .

EXAMPLE 2 Find a least common multiple (LCM)

Find the least common multiple of $4x^2 - 16$ **and** $6x^2 - 24x + 24$.

SOLUTION

STEP 1

Factor each polynomial. Write numerical factors as products of primes.

$$4x^2 - 16 = 4(x^2 - 4) = (2^2)(x + 2)(x - 2)$$

$$6x^2 - 24x + 24 = 6(x^2 - 4x + 4) = (2)(3)(x - 2)^2$$

EXAMPLE 2 Find a least common multiple (LCM)

STEP 2

Form the *LCM* by writing each factor to the highest power it occurs in either polynomial.

$$LCM = (2^2)(3)(x + 2)(x – 2)^2 = 12(x + 2)(x – 2)^2$$

EXAMPLE 3 Add with unlike denominators

Add: $\frac{7}{9x^2} + \frac{x}{3x^2 + 3x}$

SOLUTION

To find the LCD**, factor each denominator and write each factor to the highest power it occurs. Note that** $9x^2 = 3^2x^2$ **and** $3x^2 + 3x = 3x(x + 1)$**, so the** LCD **is** $3^2x^2\,(x + 1)$ $= 9x^2(x\ 1\ 1)$.

$$\frac{7}{9x^2} + \frac{x}{3x^2 + 3x} = \frac{7}{9x^2} + \frac{x}{3x(x + 1)}$$ **Factor second denominator.**

$$\frac{7}{9x^2} \cdot \frac{x + 1}{x + 1} + \frac{x}{3x(x + 1)} \cdot \frac{3x}{3x}$$ LCD **is** $9x^2(x + 1)$**.**

EXAMPLE 3 Add with unlike denominators

$$= \frac{7x + 7}{9x^2(x + 1)} + \frac{3x2}{9x^2(x + 1)}$$

Multiply.

$$= \frac{3x^2 + 7x + 7}{9x^2(x + 1)}$$

Add numerators.

EXAMPLE 4 Subtract with unlike denominators

Subtract: $\dfrac{x+2}{2x-2} - \dfrac{-2x-1}{x^2-4x+3}$

SOLUTION

$$\frac{x+2}{2x-2} - \frac{-2x-1}{x^2-4x+3}$$

$$= \frac{x+2}{2(x-1)} - \frac{-2x-1}{(x-1)(x-3)}$$ **Factor denominators.**

$$= \frac{x+2}{2(x-1)} \cdot \frac{x-3}{x-3} - \frac{-2x-1}{(x-1)(x-3)} \cdot \frac{2}{2}$$ *LCD* **is** $2(x-1)(x-3)$.

$$= \frac{x^2-x-6}{2(x-1)(x-3)} - \frac{-4x-2}{2(x-1)(x-3)}$$ **Multiply.**

Subtract with unlike denominators

$$= \frac{x^2 - x - 6 - (-4x - 2)}{2(x-1)(x-3)}$$

Subtract numerators.

$$= \frac{x^2 + 3x - 4}{2(x-1)(x-3)}$$

Simplify numerator.

$$= \frac{(x-1)(x+4)}{2(x-1)(x-3)}$$

Factor numerator. Divide out common factor.

$$= \frac{x+4}{2(x-3)}$$

Simplify.

GUIDED PRACTICE for Examples 2, 3 and 4

Find the least common multiple of the polynomials.

5. $5x^3$ **and** $10x^2 - 15x$

STEP 1

Factor each polynomial. Write numerical factors as products of primes.

$5x^3 = 5(x)\,(x^2)$

$10x^2 - 15x = 5(x)\,(2x - 3)$

STEP 2

Form the *LCM* by writing each factor to the highest power it occurs in either polynomial.

$LCM = 5x^3\,(2x - 3)$

GUIDED PRACTICE for Examples 2, 3 and 4

Find the least common multiple of the polynomials.

6. $8x - 16$ **and** $12x^2 + 12x - 72$

STEP 1

Factor each polynomial. Write numerical factors as products of primes.

$8x - 16 = 8(x - 2) = 2^3(x - 2)$

$12x^2 + 12x - 72 = 12(x^2 + x - 6) = 4 \cdot 3(x - 2)(x + 3)$

STEP 2

Form the *LCM* **by writing each factor to the highest power it occurs in either polynomial.**

$LCM = 8 \cdot 3(x - 2)(x + 3)$

$= 24(x - 2)(x + 3)$

GUIDED PRACTICE for Examples 2, 3 and 4

7. $\frac{3}{4x} - \frac{1}{7}$

SOLUTION

$$\frac{3}{4x} - \frac{1}{7}$$

$$\frac{3}{4x} \cdot \frac{7}{7} - \frac{1}{7} \cdot \frac{4x}{4x}$$ *LCD* **is** $28x$

$$\frac{21}{4x(7)} - \frac{4x}{7(4x)}$$ **Multiply**

$$= \frac{21 - 4x}{28x}$$ **Simplify**

GUIDED PRACTICE for Examples 2, 3 and 4

8. $\frac{1}{3x^2} + \frac{x}{9x^2 - 12x}$

SOLUTION

$$\frac{1}{3x^2} + \frac{x}{9x^2 - 12x}$$

$$= \frac{1}{3x^2} + \frac{x}{3x(3x-4)}$$ **Factor denominators**

$$= \frac{1}{3x^2} \cdot \frac{3x-4}{3x-4} + \frac{x}{3x(3x-4)} \cdot \frac{x}{x}$$ *LCD* **is** $3x^2(3x-4)$

$$= \frac{3x-4}{3x^2(3x-4)} + \frac{x^2}{3x^2(3x-4)}$$ **Multiply**

GUIDED PRACTICE for Examples 2, 3 and 4

$$\frac{3x - 4 + x^2}{3x^2\,(3x - 4)}$$

Add numerators

$$\frac{x^2 + 3x - 4}{3x^2\,(3x - 4)}$$

Simplify

GUIDED PRACTICE for Examples 2, 3 and 4

9. $\dfrac{x}{x^2 - x - 12} + \dfrac{5}{12x - 48}$

SOLUTION

$$\frac{x}{x^2 - x - 12} + \frac{5}{12x - 48}$$

$$= \frac{x}{(x+3)(x-4)} + \frac{5}{12\,(x-4)}$$ **Factor denominators**

$$= \frac{x}{(x+3)(x-4)} \cdot \frac{12}{12} + \frac{5}{12(x-4)} \cdot \frac{x+3}{x+3}$$ ***LCD* is $12(x-4)\ (x+3)$**

$$= \frac{12x}{12(x+3)(x-4)} + \frac{5(x+3)}{12(x+3)(x-4)}$$ **Multiply**

GUIDED PRACTICE for Examples 2, 3 and 4

$$= \frac{12x + 5x + 15}{12(x + 3)(x - 4)}$$

Add numerators

$$= \frac{17x + 15}{12(x + 3)(x + 4)}$$

Simplify

GUIDED PRACTICE for Examples 2, 3 and 4

10. $\dfrac{x+1}{x^2+4x+4} - \dfrac{6}{x^2-4}$

SOLUTION

$$\frac{x+1}{x^2+4x+4} - \frac{6}{x^2-4}$$

$$= \frac{x+1}{(x+2)(x+2)} - \frac{6}{(x-2)(x+2)}$$

Factor denominators

$$= \frac{x+1}{(x+2)(x+2)} \cdot \frac{x-2}{x-2} - \frac{6}{(x-2)(x+2)} \cdot \frac{x+2}{x+2}$$

LCD **is** $(x-2)\,(x+2)^2$

$$= \frac{x^2-2x+x-2}{(x+2)(x+2)(x-2)} - \frac{6x+12}{(x-2)(x+2)(x+2)}$$

Multiply

GUIDED PRACTICE for Example 2, 3 and 4

$$= \frac{x^2 - 2x + x - 2 - (6x + 12)}{(x + 2)^2(x - 4)}$$

Subtract numerators

$$= \frac{x^2 - 7x - 14}{(x + 2)^2\,(x - 2)}$$

Simplify

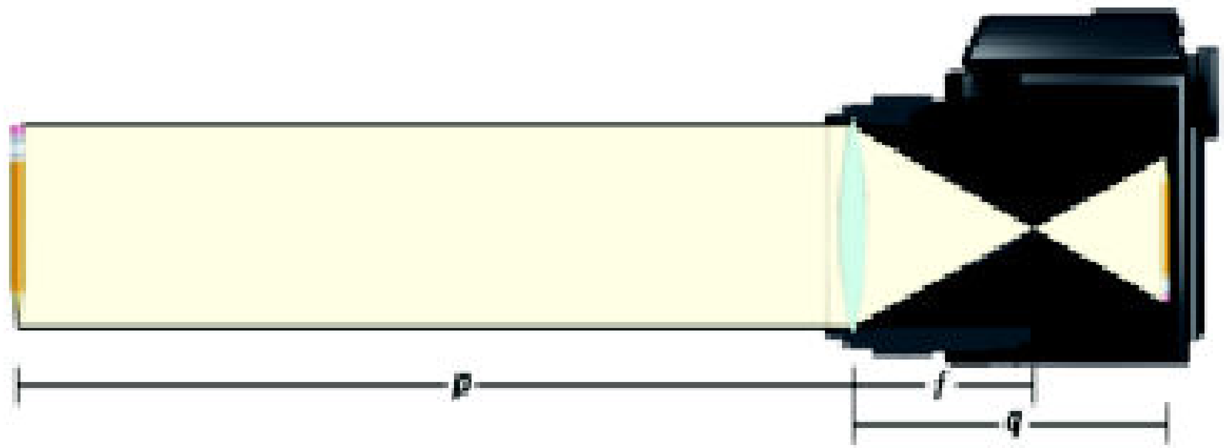
EXAMPLE 5 Simplify a complex fraction (Method 1)

Physics

Let f be the focal length of a thin camera lens, p be the distance between an object being photographed and the lens, and q be the distance between the lens and the film. For the photograph to be in focus, the variables should satisfy the *lens equation* below. Simplify the complex fraction.

Lens equation: $f = \dfrac{1}{\dfrac{1}{p} + \dfrac{1}{q}}$

EXAMPLE 5 Simplify a complex fraction (Method 1)



SOLUTION

$$f = \frac{1}{\frac{1}{p} + \frac{1}{q}} = \frac{1}{\frac{q}{pq} + \frac{p}{qp}} = \frac{1}{\frac{q+p}{pq}}$$

Write denominator as a single fraction.

$$= \frac{q+p}{pq}$$

Divide numerator by denominator.

EXAMPLE 6 Simplify a complex fraction (Method 2)

Simplify: $$\frac{\frac{5}{x+4}}{\frac{1}{x+4}+\frac{2}{x}}$$

SOLUTION

The LCD of all the fractions in the numerator and denominator is $x(x+4)$.

$$\frac{\frac{5}{x+4}}{\frac{1}{x+4}+\frac{2}{x}} = \frac{\frac{5}{x+4}}{\frac{1}{x+4}+\frac{2}{x}} \cdot \frac{x(x+4)}{x(x+4)}$$

Multiply numerator and denominator by the LCD.

$$= \frac{5x}{x+2(x+4)}$$

Simplify.

$$= \frac{5x}{3x+8}$$

Simplify.

GUIDED PRACTICE for Examples 5 and 6

11. $\dfrac{\frac{x}{6}-\frac{x}{3}}{\frac{x}{5}-\frac{7}{10}}$

$$\dfrac{\frac{x}{6}-\frac{x}{3}}{\frac{x}{5}-\frac{7}{10}} = \dfrac{\frac{x}{6}-\frac{x}{3}}{\frac{x}{5}-\frac{7}{10}} \cdot \frac{30}{30}$$

Multiply numberator and denominator by the *LCD*

$$= \frac{-5x}{3(2x-7)}$$

Simplify

GUIDED PRACTICE for Examples 5 and 6

12. $\dfrac{\frac{2}{x}-4}{\frac{2}{x}+3}$

$$\frac{\frac{2}{x}-4}{\frac{2}{x}+3} = \frac{\frac{2}{x}-4}{\frac{2}{x}+3} \cdot \frac{x}{x}$$

Multiply numberator and denominator by the *LCD*

$$= \frac{2-4x}{2+3x}$$

Simplify

$$= \frac{2(1-2x)}{2+3x}$$

Simplify

GUIDED PRACTICE for Examples 5 and 6

13. $\dfrac{\dfrac{3}{x+5}}{\dfrac{2}{x-3}+\dfrac{1}{x+5}}$

$$\frac{\dfrac{3}{x+5}}{\dfrac{2}{x-3}+\dfrac{1}{x+5}} = \frac{\dfrac{3}{x+5}}{\dfrac{2}{x-3}+\dfrac{1}{x+5}} \cdot \frac{(x+5)(x-3)}{(x+5)(x-3)}$$

Multiply numberator and denominator by the *LCD*

$$= \frac{3x-3}{3x+7}$$

Simplify

$$= \frac{3(x-3)}{3x+7}$$

Simplify