

EXAMPLE 1**Solve a rational equation by cross multiplying**

Solve: $\frac{3}{x+1} = \frac{9}{4x+1}$

$$\frac{3}{x+1} = \frac{9}{4x+1}$$

$$3(4x + 5) = 9(x + 1)$$

$$12x + 15 = 9x + 9$$

$$3x + 15 = 9$$

$$3x = -6$$

$$x = -2$$

Write original equation.

Cross multiply.

Distributive property

Subtract $9x$ from each side.

Subtract 15 from each side.

Divide each side by 3.

ANSWER

The solution is -2 . Check this in the original equation.

EXAMPLE 2**Write and use a rational model****ALLOYS**

An *alloy* is formed by mixing two or more metals. Sterling silver is an alloy composed of 92.5% silver and 7.5% copper by weight. Jewelry silver is composed of 80% silver and 20% copper by weight. How much pure silver should you mix with 15 ounces of jewelry silver to make sterling silver?

SOLUTION

$$\text{Percent of copper in mixture} = \frac{\text{Weight of copper in mixture}}{\text{Total weight of mixture}}$$

EXAMPLE 2**Write and use a rational model**

$$\frac{7.5}{100} = \frac{0.2(15)}{15 + x}$$

$$7.5(15 + x) = 100(0.2)(15)$$

$$112.5 + 7.5x = 300$$

$$7.5x = 187.5$$

$$x = 25$$

x is the amount of silver added.

Cross multiply.

Simplify.

Subtract 112.5 from each side.

Divide each side by 7.5.

ANSWER

You should mix 25 ounces of pure silver with the jewelry silver.

GUIDED PRACTICE**for Examples 1 and 2**

$$1. \frac{3}{5x} = \frac{2}{x-7}$$

SOLUTION

$$\frac{3}{5x} = \frac{2}{x-7}$$

Write original equation.

$$3(x-7) = 2(5x)$$

Cross multiply.

$$3x - 21 = 10x$$

Distributive property

$$-7x - 21 = 0$$

Subtract $10x$ from each side.

$$-7x = 21$$

Subtract 21 from each side.

$$x = -3$$

Divide each side by 7.

ANSWER

The solution is -3 . Check this in the original equation

GUIDED PRACTICE**for Examples 1 and 2**

$$2. \frac{-4}{x+3} = \frac{5}{x-3}$$

SOLUTION

$$\frac{-4}{x+3} = \frac{5}{x-3}$$

$$-4(x-3) = 5(x+3)$$

$$-4x + 12 = 5x + 15$$

$$-9x + 12 = 15$$

$$-9x = 3$$

$$x = \frac{-1}{3}$$

ANSWER

The solution is $\frac{-1}{3}$. Check this in the original equation

Write original equation.

Cross multiply.

Distributive property

Subtract $5x$ from each side.

Subtract 12 from each side.

Divide each side by 9.

$$3. \frac{1}{2x+5} = \frac{x}{11x+8}$$

SOLUTION

$$\frac{1}{2x+5} = \frac{x}{11x+8}$$

Write original equation.

$$1(11x+8) = x(2x+5)$$

Cross multiply.

$$11x+8 = 2x^2+5x$$

Distributive property

$$2x^2 - 6x^2 + 8 = 0$$

Subtract $2x^2$, and 5 from each side.

$$x^2 - 3x^2 + 4 = 0$$

$$(x-4)(x+1) = 0$$

GUIDED PRACTICE

for Examples 1 and 2

$$x = 4, x = -1$$

ANSWER

$$x = 4, x = -1$$

4. **What If?** In Example 2, suppose you have 10 ounces of jewelry silver. How much pure silver must be mixed with the jewelry silver to make sterling silver?

SOLUTION

$$\text{Percent of copper in mixture} = \frac{\text{Weight of copper in mixture}}{\text{Total weight of mixture}}$$

GUIDED PRACTICE**for Examples 1 and 2**

$$\frac{7.5}{100} = \frac{0.2(10)}{10 + x}$$

x is the amount silver added .

$$7.5 + (10 + x) = 100 (0.2)(10) \quad \text{Cross multiply.}$$

$$75 + 7.5x = 200$$

Simplify.

$$7.5x = 125$$

Subtract 7.5 from each side.

$$x = 16 \frac{2}{3}$$

Divide each side by 7.5.

You should mix $16 \frac{2}{3}$ oz of pure silver with the jeweler silver

EXAMPLE 3**Standardized Test Practice**

What is the solution of $\frac{5}{x} + \frac{7}{4} = -\frac{9}{x}$?

(A) -10

(B) -8

(C) -4

(D) 6

SOLUTION

$$\frac{5}{x} + \frac{7}{4} = -\frac{9}{x}$$

$$4x\left(\frac{5}{x} + \frac{7}{4}\right) = 4x\left(-\frac{9}{x}\right)$$

$$20 + 7x = -36$$

$$7x = -56$$

$$x = -8$$

Write original equation.

Multiply each side by the *LCD*, $4x$.

Simplify.

Subtract 20 from each side.

Divide each side by 7.

EXAMPLE 3

Standardized Test Practice

ANSWER

The correct answer is *B*. A B C D

EXAMPLE 4**Solve a rational equation with two solutions**

Solve: $1 - \frac{8}{x-5} = \frac{3}{x}$

$$1 - \frac{8}{x-5} = \frac{3}{x}$$

Write original equation.

$$x(x-5) \left(1 - \frac{8}{x-5}\right) = x(x-5) \cdot \frac{3}{x}$$

Multiply each side by the LCD, $x(x-5)$.

$$x(x-5) - 8x = 3(x-5)$$

Simplify.

$$x^2 - 5x - 8x = 3x - 15$$

Simplify.

$$x^2 - 16x + 15 = 0$$

Write in standard form.

$$(x-1)(x-15) = 0$$

Factor.

$$x = 1 \text{ or } x = 15$$

Zero product property

EXAMPLE 4**Solve a rational equation with two solutions****ANSWER**

The solutions are 1 and 15. Check these in the original equation.

EXAMPLE 5**Check for extraneous solutions**

Solve: $\frac{6}{x-3} = \frac{8x^2}{x^2-9} - \frac{4x}{x+3}$

SOLUTION

Write each denominator in factored form.

The LCD is $(x+3)(x-3)$.

$$\frac{6}{x-3} = \frac{8x^2}{(x+3)(x-3)} - \frac{4x}{x+3}$$

$$(x+3)(x-3) \cdot \frac{6}{x-3} = (x+3)(x-3) \cdot \frac{8x^2}{(x+3)(x-3)} - (x+3)(x-3) \cdot \frac{4x}{x+3}$$

$$6(x+3) = 8x^2 - 4x(x-3)$$

$$6x + 18 = 8x^2 - 4x^2 + 12x$$

EXAMPLE 5**Check for extraneous solutions**

$$0 = 4x^2 + 6x - 18$$

$$0 = 2x^2 + 3x - 9$$

$$0 = (2x - 3)(x + 3)$$

$$2x - 3 = 0 \text{ or } x + 3 = 0$$

$$x = \frac{3}{2} \text{ or } x = -3$$

You can use algebra or a graph to check whether either of the two solutions is extraneous.

Algebra

The solution checks, $\frac{3}{2}$ but the apparent solution -3 is extraneous, because substituting it in the equation results in division by zero, which is undefined.

EXAMPLE 5**Check for extraneous solutions****Graph**Graph $y = \frac{6}{x-3}$ and

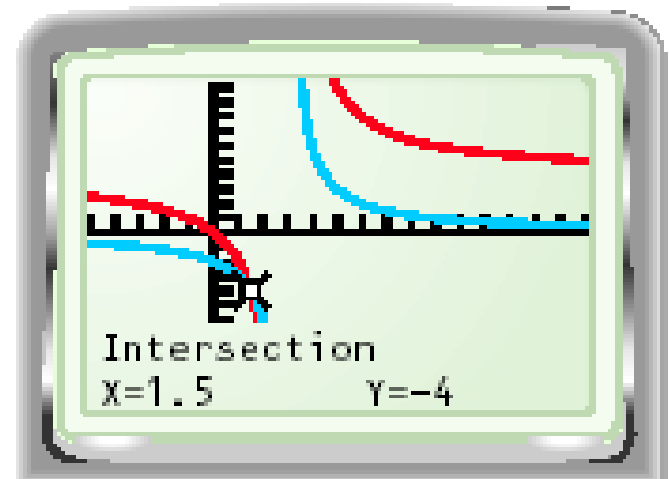
$$y = \frac{8x^2}{x^2-9} - \frac{4x}{x+3}$$

The graphs intersect when $x = \frac{3}{2}$
but not when $x = -3$.

ANSWER

The solution is $\frac{3}{2}$.

$$\frac{6}{-3-3} \neq \frac{8(-3)^2}{(-3)^2-9} - \frac{4(-3)}{-3+3}$$



GUIDED PRACTICE**for Examples 3, 4 and 5**

Solve the equation by using the *LCD*. Check for extraneous solutions.

$$5. \frac{7}{2} + \frac{3}{x} = 3$$

SOLUTION

Write each denominator in factored form. The *LCD* is $2x$

$$\frac{7}{2} + \frac{3}{x} = 3$$

$$2x \cdot \frac{7}{2} + 2x \cdot \frac{3}{x} = 2x \cdot 3$$

$$7x + 6 = 6x$$

$$x = -6$$

GUIDED PRACTICE**for Examples 3, 4 and 5**

$$6. \frac{2}{x} + \frac{4}{3} = 2$$

SOLUTION

Write each denominator in factored form. The *LCD* is $3x$

$$\frac{2}{x} + \frac{4}{3} = 2$$

$$3x \cdot \frac{2}{x} + 3x \cdot \frac{4}{3} = 3x \cdot 2$$

$$6 + 4x = 6x$$

$$6 = 2x$$

$$x = 3$$

GUIDED PRACTICE**for Examples 3, 4 and 5**

$$7. \frac{3}{7} + \frac{8}{x} = 1$$

SOLUTION

Write each denominator in factored form. The *LCD* is $7x$

$$\frac{3}{7} + \frac{8}{x} = 1$$

$$7x \cdot \frac{3}{7} + 7x \cdot \frac{8}{x} = 7x \cdot 1$$

$$3x + 56 = 7x$$

$$56 = 4x$$

$$x = 14$$

GUIDED PRACTICE**for Examples 3, 4 and 5**

$$8. \frac{3}{2} + \frac{4}{x-1} = \frac{x+1}{x-1}$$

SOLUTION

Write each denominator in factored form. The LCD is $2(x-1)$

$$\frac{3}{2} + \frac{4}{x-1} = \frac{x+1}{x-1}$$

$$(x-1)(2) \cdot \frac{3}{2} + (x-1)(2) \cdot \frac{4}{x-1} = (x-1)(2) \cdot \frac{x+1}{x-1}$$

$$3x - 3 + 8 = 2x + 2$$

$$x = -3$$

GUIDED PRACTICE**for Examples 3, 4 and 5**

$$9. \frac{3x}{x+1} - \frac{5}{2x} = \frac{3}{2x}$$

SOLUTION

Write each denominator in factored form. The *LCD* is $(x + 1)(2x)$

$$\frac{3x}{x+1} - \frac{5}{2x} = \frac{3}{2x}$$

$$2x(x+1) \cdot \frac{3x}{x+1} - 2x(x+1) \cdot \frac{5}{2x} = 2x(x+1) \frac{3}{2x}$$

$$6x^2 - 5x - 5 = 3x + 3$$

$$0 = 3x + 3 - 6x^2 + 5x + 5$$

$$0 = -6x^2 + 8x + 8$$

$$0 = (3x + 2)(x - 2)$$

$$3x + 2 = 0 \quad \text{or} \quad x - 2 = 0$$

$$x = -\frac{2}{3} \quad \text{or} \quad x = 2$$

GUIDED PRACTICE**for Examples 3, 4 and 5**

$$10. \frac{5x}{x-2} = 7 + \frac{10}{x-2}$$

SOLUTION

Write each denominator in factored form. The LCD is $x - 2$

$$\frac{5x}{x-2} = 7 + \frac{10}{x-2}$$

$$x - 2 \cdot \frac{5x}{x-2} = (x - 2) \cdot 7 + (x - 2) \cdot \frac{10}{x-2}$$

$$5x = 7x - 14 + 10$$

$$4 = 2x$$

$$x = 2$$

$x=2$ results in no solution.

EXAMPLE 6**Solve a rational equation given a function****Video Game Sales**

From 1995 through 2003, the annual sales S (in billions of dollars) of entertainment software can be modeled by

$$S(t) = \frac{848t^2 + 3220}{115t^2 + 1000} \quad 0 \leq t \leq 8$$

where t is the number of years since 1995. For which year were the total sales of entertainment software about \$5.3 billion?

SOLUTION

$$S(t) = \frac{848t^2 + 3220}{115t^2 + 1000} \quad \text{Write given function.}$$

$$5.3 = \frac{848t^2 + 3220}{115t^2 + 1000} \quad \text{Substitute 5.3 for } S(t).$$

EXAMPLE 6**Solve a rational equation given a function**

$$5.3(115t^2 + 1000) = 848t^2 + 3220$$

Multiply each side by
 $115t^2 + 1000$.

$$609.5t^2 + 5300 = 848t^2 + 3220$$

Simplify.

$$5300 = 238.5t^2 + 3220$$

Subtract $609.5t^2$ from
each side.

$$2080 = 238.5t^2$$

Subtract 3220 from
each side.

$$8.72 \approx t^2$$

Divide each side
by 238.5.

$$\pm 2.95 \approx t$$

Take square roots of
each side.

**Because -2.95 is not in the domain ($0 \leq t \leq 8$),
the only solution is 2.95 .**

EXAMPLE 6**Solve a rational equation given a function****ANSWER**

So, the total sales of entertainment software were about \$5.3 billion about 3 years after 1995, or in 1998.

GUIDED PRACTICE**for Example 6**

11. **What If? Use the information in Example 6 to determine in which year the total sales of entertainment software were about \$4.5 billion.**

SOLUTION

$$S(t) = \frac{848t^2 + 3220}{115t^2 + 1000}$$

Write given function.

$$4.5 = \frac{848t^2 + 3220}{115t^2 + 1000}$$

Substitute 5.3 for $S(t)$.

$$4.5(115t^2 + 1000) = 848t^2 + 3220$$

Multiply each side by $115t^2 + 1000$.

$$517.5t^2 + 4500 = 848t^2 + 3220$$

Simplify.

$$1280 = 330.5t^2$$

Subtract.

GUIDED PRACTICE**for Example 6**

$$3.88 = + 2$$

Divide each side
by 330.5.

$$\pm 1.95 \approx t$$

Take square roots of
each side.

Because -1.95 is not the domain ($0 \leq t \leq 8$), the only solution is 1.95

So , the total sales of entertainment software were about \$ 4.5 million about two your after 1995, or in 1997