

**SIMPLIFYING RADICAL EXPRESSIONS**

$\sqrt{25}$ , read “the square root of 25,” means one of the two positive equal factors of 25.

$$\sqrt{25} = \sqrt{5 \cdot 5} = 5; \quad \sqrt{36} = \sqrt{6 \cdot 6} = 6; \quad \sqrt{100} = \sqrt{10^2} = 10; \quad \sqrt{49} = \sqrt{7^2} = 7$$

$$\sqrt{a^2} = \sqrt{a \cdot a} = a; \quad \sqrt{a^6} = \sqrt{a^3 \cdot a^3} = a^3; \quad \sqrt{m^{16}} = \sqrt{m^8 \cdot m^8} = m^8; \quad \sqrt{y^{10}} = y^5$$

$\sqrt{a}$  is called a radical, **a** is called the radicand.

**General Rules for Radicals:**

$$\sqrt{x} \cdot \sqrt{y} = \sqrt{xy}$$

$$\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$$

$$\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$$

$$\sqrt{x^2} = (\sqrt{x})^2 = x$$

$$\sqrt{2} \cdot \sqrt{5} = \sqrt{10}$$

$$\sqrt{18} = \sqrt{9} \cdot \sqrt{2}$$

$$\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}}$$

$$\sqrt{159^2} = (\sqrt{159})^2 = 159$$

$$a\sqrt{x} + b\sqrt{x} = (a + b)\sqrt{x}$$

$$2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$$

**Simplifying Square Roots:**

Sometimes it is convenient to leave square roots in radical form instead of using a calculator to find approximations (decimal values). Look for perfect squares (i.e., 4, 9, 16, 25, 36, 49, ...) as factors of the number that is inside the radical sign (radicand) and take the square root of any perfect square factor. Multiply the root of the perfect square times the reduced radical. When there is an existing value that multiplies the radical, multiply any root(s) times that value.

**Examples:**

$$\begin{aligned} 1. \quad \sqrt{20} &= \sqrt{4 \cdot 5} \\ &= \sqrt{4} \cdot \sqrt{5} \\ &= 2\sqrt{5} \end{aligned}$$

$$\begin{aligned} 2. \quad \sqrt{48} &= \sqrt{16 \cdot 3} \\ &= \sqrt{16} \cdot \sqrt{3} \\ &= 4\sqrt{3} \end{aligned}$$

$$\begin{aligned} 3. \quad \sqrt{a^5} &= \sqrt{a \cdot a \cdot a \cdot a \cdot a} \\ &= a \cdot a\sqrt{a} \\ &= a^2\sqrt{a} \end{aligned}$$

*(note that one a comes “out” for each pair of a’s in the radical)*

**Simplify Completely:**

|                       |                    |                       |                          |                           |
|-----------------------|--------------------|-----------------------|--------------------------|---------------------------|
| 1. $\sqrt{4}$         | 2. $\sqrt{16}$     | 3. $-\sqrt{100}$      | 4. $\sqrt{a^8}$          | 5. $\sqrt{w^{12}}$        |
| 6. $\sqrt{a^6b^{10}}$ | 7. $\sqrt{9a^2}$   | 8. $-\sqrt{81m^{64}}$ | 9. $\sqrt{49a^4b^{12}}$  | 10. $\sqrt{121x^{14}y^6}$ |
| 11. $\sqrt{18}$       | 12. $\sqrt{125}$   | 13. $\sqrt{72}$       | 14. $\sqrt{180}$         | 15. $\sqrt{a^3}$          |
| 16. $\sqrt{b^7}$      | 17. $\sqrt{m^9}$   | 18. $\sqrt{75x^7y^5}$ | 19. $\sqrt{27a^{11}b^7}$ | 20. $\sqrt{32a^7b^4}$     |
| 21. $\sqrt{9a^8}$     | 22. $\sqrt{45a^7}$ | 23. $\sqrt{36x^2y^6}$ | 24. $\sqrt{12x^{20}y}$   | 25. $-\sqrt{200}$         |

Bonus:  $2\sqrt{45} - 2\sqrt{5} =$

**Rationalizing the Denominator** means eliminating all irrational numbers from the denominator. When eliminating a radical from a denominator, it is important that you always multiply by one – otherwise you will change the value of the fraction. Multiplying by 1 (the multiplicative identity element) does not change the value of what is being multiplied.

**Single Radical Term Denominators:** If your fraction has a denominator with only one term that contains a radical, build a fraction with the radical in both its numerator and denominator. This fraction will always have a value of 1, since its numerator and denominator are identical. Then multiply the original fraction by the new fraction that you build. The radical in the denominator will be gone in the product, and the product will have the same value as the original fraction.

|  |   |
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| <p>Example</p> <p>Given: <math>\frac{3}{\sqrt{5}}</math></p> <p>Build a 1 = <math>\frac{\sqrt{5}}{\sqrt{5}}</math></p> <p>Multiply: <math>\frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{3\sqrt{5}}{5}}</math></p> | <p>Example</p> <p>Given: <math>\frac{9}{\sqrt{6}}</math></p> <p>Build a 1 = <math>\frac{\sqrt{6}}{\sqrt{6}}</math></p> <p>Multiply and Simplify: <math>\frac{9}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{9\sqrt{6}}{6} = \boxed{\frac{3\sqrt{6}}{2}}</math></p> |
|--|---|

Rewrite the following expressions with rational denominators.  
Be sure to completely simplify your answers.

|   |                             |
|---|-----------------------------|
| $\frac{4}{\sqrt{10}}$                                     | $\frac{6}{\sqrt{2}}$        |
| $\frac{3}{\sqrt{5}}$                                      | $\frac{9}{\sqrt{48}}$       |
| $\frac{4}{\sqrt{8}}$                                      | $\frac{5}{\sqrt{125}}$      |
| $\frac{-4}{3\sqrt{2}}$                                    | $\frac{5}{\sqrt{3}}$        |
| $\frac{\sqrt{3}}{\sqrt{8}}$                               | $\frac{\sqrt{2}}{\sqrt{8}}$ |
| <p>Bonus: <math>\frac{\sqrt{5} + 3}{\sqrt{10}}</math></p> |                             |