# Solving Quadratic Equations by Completing the Square

Completing the Square

In algebra, "Completing the Square" is a technique for changing a quadratic expression from standard form: $ax^2 + bx + c$
to the vertex/graphing form: $a(x + h)^2 + k$.

It can also be used as a method for solving quadratic equations.

Completing the Square

Consider the following square. If its side length is (x + 3) then the area of the square would be _____?

$x+3$

$x+3$
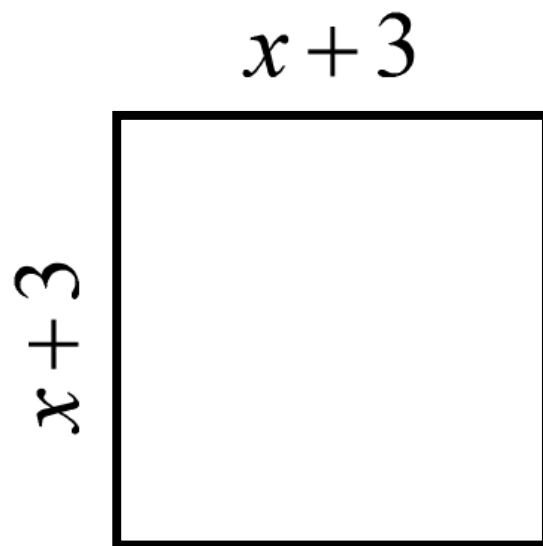
$$Area = (x+3)(x+3)$$
$$= (x+3)^2$$
$$= x^2 + 6x + 9$$

Notice all 3 expressions are equivalent.

In the standard form, b = 6 and c = 9.

That is, $(b/2)^2 = c$.

Completing the Square

Form a perfect square trinomial with lead coefficient of 1

$x^2 + bx + c$ where $c = (b/2)^2$

Find the value that completes the square.

$$x^2 + 8x + \underline{\quad} = (x \underline{\quad})^2$$

$$x^2 + 20x + \underline{\quad} = (x \underline{\quad})^2$$

$$x^2 - 16x + \underline{\quad} = (x \underline{\quad})^2$$

$$x^2 - 2x + \underline{\quad} = (x \underline{\quad})^2$$

35 Find $\left(\frac{b}{2}\right)^2$ if b = 14.

36 Find $\left(\frac{b}{2}\right)^2$ if b = 10.

37 Find $\left(\frac{b}{2}\right)^2$ if b = -12.

38 **Complete the square to form a perfect square trinomial.**

$x^2 + 18x + ?$

39 **Complete the square to form a perfect square trinomial.**

$x^2 - 6x + ?$

Completing the Square

Step 1 - Write the equation in the form $x^2 + bx = c$.

Step 2 - Find $(b \div 2)^2$.

Step 3 - Complete the square by adding $(b \div 2)^2$ to both sides of the equation.

Step 4 - Factor the perfect square trinomial.

Step 5 - Take the square root of both sides.

Step 6 - Write two equations, using both the positive and negative square root and solve each equation.

Completing the Square

Let's look at an example to solve:

$$x^2 + 14x - 15 = 0$$

Step 1 - Rewrite Equation

Step 2 - Find $(b/2)^2$

Step 3 - Add the result to both sides

Step 4 - Factor & Simplify

Step 5 - Take Square Root of both sides

Step 6 - Write 2 Equations & Solve

How can you check your solutions?

Answer

119

Completing the Square

Let's look at an example to solve:

$$x^2 - 2x - 2 = 0$$

Step 1 - Rewrite Equation

Step 2 - Find $(b/2)^2$

Step 3 - Add the result to both sides

Step 4 - Factor & Simplify

Step 5 - Take Square Root of both sides

Step 6 - Write 2 Equations & Solve

How can you check your solutions?

Answer

40 **Solve the following by completing the square :**

$$x^2 + 6x = -5$$

A   -5

B   -2

C   -1

D   5

E   2

41 Solve the following by completing the square :

$$x^2 - 8x = 20$$

A   -10

B   -2

C   -1

D   10

E   2

42 Solve the following by completing the square :

$-36x = 3x^2 + 108$

Hint: Look for G(

A -6

B $\sqrt{6}$

C 0

D 6

E $-\sqrt{6}$

Completing the Square

Challenge:

$$3x^2 - 10x = -3$$

Step 1 - Rewrite Equation

Step 2 - Find $(b/2)^2$

Step 3 - Add the result to both sides

Step 4 - Factor & Simplify

Step 5 - Take Square Root of both sides

Step 6 - Write 2 Equations & Solve

Answer

Completing the Square

Challenge:

$4x^2 - 17x + 4 = 0$

Note: There is no GCF to factor out.

Step 1 - Rewrite Equation

Step 2 - Find $(b/2)^2$

Step 3 - Add the result to both sides

Step 4 - Factor & Simplify

Step 5 - Take Square Root of both sides

Step 6 - Write 2 Equations & Solve

Answer

Completing the Square

Challenge:

$-6x^2 - 25x - 25 = 0$

*Note: There is no GCF
to factor out.*

Answer

43 Solve the following by completing the square :

$$4x^2 - 10x - \frac{7}{4} = 0$$

A $\quad \dfrac{5}{4} + \sqrt{2}$

B $\quad \dfrac{5}{4} - \sqrt{2}$

C $\quad -\dfrac{5}{4} + \sqrt{2}$

D $\quad -\dfrac{5}{4} - \sqrt{2}$

E $\quad \dfrac{5 \pm \sqrt{2}}{2}$