Complex Numbers

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Complex Numbers

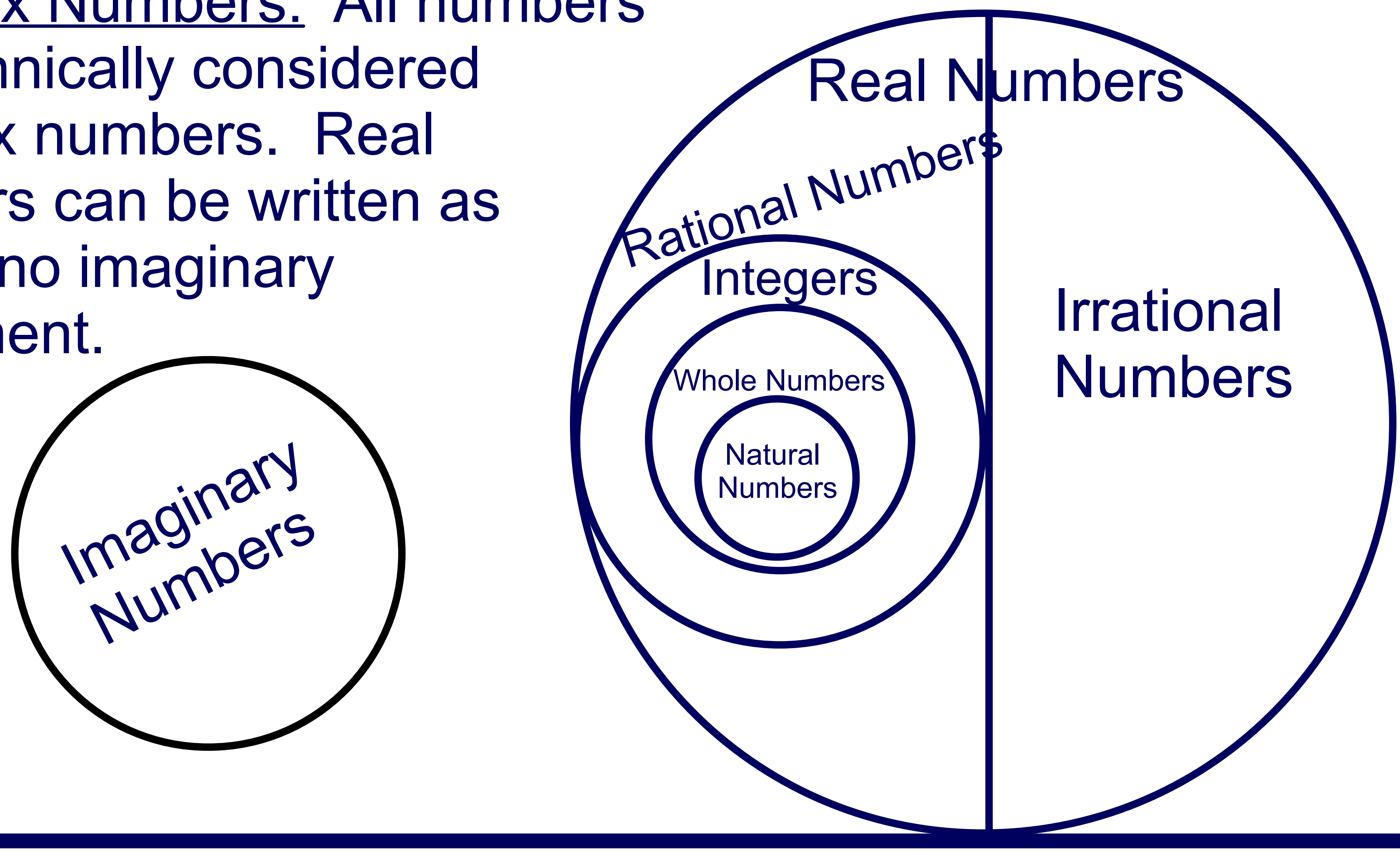
The square root of a negative number has no real solution,
but it does have an imaginary one:

$$\sqrt{-1} = i$$

An expression is **complex** (also called imaginary) if it has an i in it.

Complex Numbers

Complex Numbers: All numbers are technically considered complex numbers. Real Numbers can be written as a + 0i - no imaginary component.



Complex Numbers

$$i^2 = -1$$

$$i^3 = -i$$

$$i^4 = 1$$

Why does this work?

Teacher Notes

Complex Numbers

Higher order *i*'s can be simplified into *i*, -1, -*i*, or 1.

If the power of *i* is even:

...and the exponent is a multiple of 4 , then it simplifies to 1.

...and the exponent is a multiple of 2,but not 4 , then it simplifies to -1.

If the power of *i* is odd:

...factor out one *i* to create an even exponent. Use the rules for even exponents and leave the factored *i.*

Complex Numbers

Remember

$i^2 = -1$

$i^4 = 1$

Simplify the following:

i^{87} i^{64} $-i^{18}$ $-i^{27}$

Teacher Notes

Complex Numbers

Remember

$i^2 = -1$

$i^4 = 1$

More examples:

i^{72} i^{105} i^{102} i^{89} $i^{4000000}$

Teacher Notes

142 Simplify: i^{19}

A i

B -1

C $-i$

D 1

Answer

143 Simplify: i^{91}

A i

B -1

C $-i$

D 1

Answer

144 Simplify: i^{100}

A i

B -1

C $-i$

D 1

Answer

145 Simplify: i^{77}

A i

B -1

C $-i$

D 1

Answer

Complex Numbers

Simplify radical expressions that have a negative by taking out *i* first. Then, perform the indicated operation(s). Simplify any expression that has a power of *i* greater than one.

$\sqrt{-16} \cdot \sqrt{-9}$

$\sqrt{-15} \cdot \sqrt{-30}$

$\sqrt{-16x^2}$

Teacher Notes

Complex Numbers

Examples:

$-\sqrt{-45a^2}$

$\sqrt{-4m^2n^4} \cdot \sqrt{-24m^4n}$

$\sqrt{-3p^3} \cdot \sqrt{-27p}$

Teacher Notes

146 Simplify: $\sqrt{-25}$

A $5i$

B $-5i$

C $5\sqrt{i}$

D -5

Answer

147 Simplify: $\sqrt{-49a^2b^4}$

A $7ab^2i$

B $-7abi$

C $7|a|b^2i$

D $7|ab^2i|$

Answer

148 Simplify: $-\sqrt{-64}$

A 8

B -8

C $8i$

D $-8i$

Answer

149 Simplify: $\sqrt{-9}\sqrt{-25}$

A 15

B -15

C $15i$

D $-15i$

Answer

150 Simplify: $\sqrt{-10a^2}\sqrt{-30a^4}$

A $10|a^3|i\sqrt{3}$

B $-10a^3i\sqrt{3}$

C $10|a^3i|\sqrt{3}$

D $-10|a^3|\sqrt{3}$

Answer

Working with Complex Numbers

Operations, such as addition, subtraction, multiplication and division, can be done with i.

Treat i like any other variable, except at the end make sure i is at most to the first power.

Working with Complex Numbers

Answers of complex numbers are left in standard form.
The standard form of a complex number is a + bi.

Examples of standard form of a complex number:

3 - 2i   0 + 3i   8 + 0i $\frac{4}{13} - \frac{3i}{13}$ $\frac{1}{2} + \frac{3i}{4}$

Adding or Subtracting Complex Numbers

When adding or subtracting complex numbers, collect like terms. Leave answers in standard form.

$$(7+4i)+(3-2i)$$

$$(4+3i)-(5-6i)$$

$$(5-6i)+(4+8i)$$

$$(12+3i)-(12-3i)$$

Teacher Notes

Multiplying Complex Numbers

When multiplying, multiply numbers, multiply *i's* and simplify any *i* with a power greater than one.

$$(7i)(3i)(2i)$$

$$(3i)^2(2i)$$

Teacher Notes

Multiplying Complex Numbers

When multiplying, multiply numbers, multiply *i's* and simplify any *i* with a power greater than one.

$$(4i)(-5i)(3i)(-2i)$$

$$(2i)^3(4i)^2$$

Teacher Notes

Multiplying Complex Numbers

Multiply and leave answers in standard form.

$2i(3-4i)$

$(4-3i)^2$

Teacher Notes

Multiplying Complex Numbers

Multiply and leave answers in standard form.

$$(3-2i)(4+i)$$

$$(4-5i)(4+5i)$$

Teacher Notes

151 Simplify: $(3+5i)+(-2+3i)$

A $5+8i$

B $1+8i$

C $5+2i$

D $1+2i$

Answer

152 Simplify: $(3+5i)-(-2+3i)$

A $5+8i$

B $1+8i$

C $5+2i$

D $1+2i$

Answer

153 Simplify: $(3+5i)(-2+3i)$

A $-21+i$

B $-21-i$

C $-6+i$

D $6+i$

Answer

154 Simplify: $(3+5i)^2$

A $9+25i^2$

B -16

C $-16+15i$

D $-16+30i$

Answer

155 Simplify: $(3+5i)(3-5i)$

A $9-30i$

B 34

C $9+30i$

D -16

Answer

Dividing with *i*

Since *i* represents a square root, a fraction is not insimplified form if there is an *i* in the denominator. And, similar to roots, if the denominator is a monomial just multiply top and bottom of the fraction by *i* to rationalize.

$$\frac{3}{2i}$$

Teacher Notes

Dividing with *i*

Simplify:

$$\frac{4}{3i}$$

$$\frac{-5}{10i}$$

Teacher Notes

Dividing with *i*

Simplify:

$$\frac{7i}{6}$$

$$\frac{5+4i}{i}$$

Teacher Notes

156 Simplify: $\frac{3}{7i}$

A $\frac{3i}{7}$

B $\frac{-3i}{7}$

C $\frac{21i}{7}$

D $\frac{7i}{3}$

Answer

157 Simplify: $\frac{-5}{10i}$

A $\frac{i}{2}$

B $\frac{-i}{2}$

C $\frac{5i}{10}$

D $\frac{-5i}{10}$

Answer

158 Simplify: $\dfrac{5-2i}{4i}$

A $\dfrac{2+5i}{4}$

B $\dfrac{-2+5i}{4}$

C $\dfrac{1-5i}{2}$

D $-\dfrac{1}{2}-\dfrac{5i}{4}$

Answer

Rationalizing Complex Numbers

If the denominator is a binomial including *i*, rationalize it by multiplying top and bottom by its conjugate. Remember using conjugates earlier in this unit: the conjugate of 4 - 3*i* is 4 + 3*i*.

Example: $$\frac{5}{4-3i}$$

Teacher Notes

Rationalizing Complex Numbers

Simplify:

$$\frac{2}{3-2i} \qquad \frac{4+i}{5+3i} \qquad \frac{1-6i}{1+6i}$$

Teacher Notes

159 Simplify: $\dfrac{-1}{1-i}$

A $1+\frac{1}{2}i$

B $\frac{1}{2}-\frac{1}{2}i$

C $-\frac{1}{2}-\frac{1}{2}i$

D $-1-i$

Answer

160 Simplify: $\dfrac{3-i}{5+4i}$

A $\dfrac{19}{41}+\dfrac{17}{41}i$

B $\dfrac{19}{41}-\dfrac{17}{41}i$

C $\dfrac{11}{41}+\dfrac{17}{41}i$

D $\dfrac{11}{41}-\dfrac{17}{41}i$

Answer

161 Simplify: $\frac{4+i}{6-2i}$

A $\frac{22}{40}+\frac{14}{40}i$

B $\frac{22}{40}-\frac{14}{40}i$

C $\frac{11}{20}+\frac{7}{20}i$

D $\frac{11}{20}-\frac{7}{20}i$

Answer

162 Simplify: $\dfrac{1+2i}{3+6i}$

A $\dfrac{15}{45}-\dfrac{12}{45}i$

B $\dfrac{1}{3}-\dfrac{4}{15}i$

C $\dfrac{4}{15}i$

D $\dfrac{1}{3}$

Answer