Growth and Decay

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Goals and Objectives

Students will be able to model growth and decay problems with exponential equations.

Why do we need this?

What will the population of the world be in 2050 at the current rate of growth? How long will it take the radioactive material near the nuclear reactors in Japan to dissipate to harmless levels after the destructive tsunami? These are important questions that we need to answer to plan for the future! Growth and decay can be modeled and analyzed with exponential and logarithmic functions.

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And...these functions will also model problems dealing with something we ALL need to learn to work with...

Money!

Growth and Decay

$$A = Pe^{rt}$$

This formula represents the amount (A) of money in a savings account if the interest is continuously compounded.

P = the principal (amount deposited)
r = the annual interest rate (in decimal form)
t = time in years

If $500 is invested at 4% for 2 years with continuously compounded interest, what will the account balance be?

Answer

Growth and Decay

If $500 is invested at 4%, compounded continuously, how long until the account balance is doubled?

Hint

Answer

105 If $1000 is invested at 4% for 3 years, compounded continuously, what is the account balance?

Answer

106 If $1000 is invested at 3.5%, compounded continuously, how long until the account balance is doubled?

Answer

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There are other types of interest that you may encounter...

Simple Interest: $I = Prt$

I = interest
P = principal (deposit)
r = interest rate (decimal)
t = time in years
n = number of times compounded per year
A = account balance with interest included

Account Balance with Simple Interest

$$A = P(1 + r)^t$$

Account Balance with Compound Interest:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Teacher Notes

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Calculate the simple interest if you were to buy a car from your parents for $4000 and pay for it over 4 years at 3% interest.

Answer

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Calculate the simple interest if you were to purchase a car from a used car dealer for 4000 at 10% interest over 4 years.

Answer

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Using the formula for compound interest, calculate the amount in your account if you invest $1300, over 15 years, at 4.6% interest compounded monthly.

Answer

Growth and Decay

How long would it take for an investment of $10,000 to increase to a total of $25,000 compounded quarterly at a rate of 3.5%?

Answer

107 How many years did you take to pay off a $5000 car at 7% simple interest if you paid a total of $6200 for the car?

Answer

108 What was the rate of your investment if you invested $5000 over 10 years, compounded continuously and you made $2200 in interest?

Answer

109 How much was originally invested if you have $63,710.56 in an account generating 4% interest (compounded monthly) over 15 years?

Answer

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The same formulas can be used to model growth or decay in other situations.

$$A = P(1+r)^t$$

$$A = Pe^{rt}$$

P = Population (initial amount)
r = rate of growth or decay(decimal)
t = time

When do you use each formula?

With your group, find one example of when you would use each formula.

Growth and Decay

Example: A bacteria grows constantly at a rate of 10% per hour. If there were initially 100 bacteria, how long until there are 1000?

Answer

Growth and Decay

A new car depreciates in value at a rate of 8% per year. If a 5 year old car is worth $20,000, how much was it originally worth? How will we write the rate?

Answer

Half-life Definition

In physics, half-life refers to a fixed time required for half the radioactive nuclei in a substance to decay. Half-lives of radioactive substances can range from fractions of a second to billions of years.

For example, 100 grams of Radium-229, whose half-life is four minutes, will yield only 50 grams remaining after four minutes. After 8 minutes 25 grams will be left, after 12 minutes 12.5 grams will be left, and so on.

In working with these applications, the formula $A = Pe^{rt}$ can be modified to $\frac{1}{2} = e^{rt}$.

Half-life

A certain radioactive material has a half-life of 20 years. If 100g were present to start, how much will remain in 7 years?

Hint: Use a half-life of 20 years to find *r* first.

Answer

110 If an oil spill widens continually at a rate of 15% per hour, how long will it take to grow from 2 miles wide to 3 miles wide?

Answer

111 What interest rate is needed to double your money if it's invested for 8 years compounded quarterly?

Answer

112 NASA calculates that the orbit of a communications satellite is decaying exponentially at a rate of 12% per day. If the satellite is 20,000 miles above the Earth, how long until it is visible to the naked eye at 50 miles above the earth?

Answer

113 If the half-life of an element is 50 years, at what rate does it decay?

Answer

114 Bacteria A's growth is modeled by $A = 200e^{0.02t}$ and Bacteria B's growth is modeled by $B = 50(2)^{0.12t}$ where t is measured in hours. How long until the amount of Bacteria B exceeds that of Bacteria A? Round to the nearest hour.

Answer