Solve Quadratic Equations by Factoring

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Teacher Notes

Solve by Factoring

In addition to graphing, there are additional ways to find the zeros or x-intercepts of a quadratic. This section will explore solving quadratics using the method of factoring

A complete review of factoring can be found in the Fundamental Skills of Algebra (Supplemental Review) Unit.

Fundamental Skills of Algebra (Supplemental Review) Click for Link

Solve by Factoring

Review of factoring - Factoring is simply rewriting an expression in an equivalent form which uses multiplication. To factor a quadratic, ensure that you have the quadratic in standard form: $ax^2+bx+c=0$

☆Tips for factoring quadratics:

- Check for a GCF (Greatest Common Factor).
- Check to see if the quadratic is a Difference of Squares or other special binomial product.

Solve by Factoring

Examples:

Quadratics with a GCF:

$3x^2 + 6x$ in factored form is $3x(x + 2)$

Quadratics using Difference of Squares:

$x^2 - 64$ in factored form is $(x + 8)(x - 8)$

Additional Quadratic Trinomials:

$x^2 - 12x + 27$ in factored form is $(x - 9)(x - 3)$
$2x^2 - x - 6$ in factored form is $(2x + 3)(x - 2)$

Solve by Factoring

Practice:

To factor a quadratic trinomial of the form $x^2 + bx + c$, find two factors of c whose sum is b.

Example -

To factor $x^2 + 9x + 18$, look for factors of 18 whose sum is 9.
(In other words, find 2 numbers that multiply to 18 but also add to 9.)

Factors of 18	Sum

Answer

Solve by Factoring

Practice:

Factor $x^2 + 4x - 12$, look for factors of -12 whose sum is 4.
(in other words, find 2 numbers that multiply to -12 but also add to 4.)

Factors of -12	Sum

Answer

Zero Product Property

Imagine this: If 2 numbers must be placed in the boxes and you know that when you multiply these you get ZERO, what must be true?

$$\boxed{?} \times \boxed{?} = 0$$

Teacher Notes

Zero Product Property

For all real numbers a and b, if the product of two quantities equals zero, at least one of the quantities equals zero.

If $a \cdot b = 0$ then $a = 0$ or $b = 0$

Solve by Factoring

Now... combining the 2 ideas of factoring with the Zero Product Property, we are able to solve for the x-intercepts (zeros) of the quadratic.

Example: Solve $x^2 + 4x - 12 = 0$

1. Factor the trinomial.
2. Using the Zero Product Property, set each factor equal to zero.
3. Solve each simple equation.

Answer

Solve by Factoring

Example: Solve $x^2 + 36 = 12x$

★ *Remember: The equation has to be written in standard form ($ax^2 + bx + c$).*

Answer

18 **Solve** $x^2 - 5x + 6 = 0$

A	x = -7	F	x = 3
B	x = -5	G	x = 5
C	x = -3	H	x = 6
D	x = -2	I	x = 7
E	x = 2	J	x = 15

Answer

19 Solve $m^2 + 10m + 25 = 0$

A m = -7

B m = -5

C m = -3

D m = -2

E m = 2

F m = 3

G m = 5

H m = 6

I m = 7

J m = 15

Answer

20 Solve $h^2 - h = 12$

A h = -12

B h = -4

C h = -3

D h = -2

E h = 2

F h = 3

G h = 4

H h = 6

I h = 8

J h = 12

Answer

21 **Solve** $d^2 - 35d = 2d$

A d = -7

B d = -5

C d = -3

D d = -2

E d = 0

F d = 3

G d = 5

H d = 6

I d = 7

J d = 37

Answer

Berry Method to Factor

Example: Solve $8y^2 + 2y = 3$

When a does not equal 1, check first for a GCF, then use the Berry Method.

$\boxed{\text{Berry Method to factor}}$

Step 1: Calculate ac.

Step 2: Find a pair of numbers m and n, whose product is ac, and whose sum is b.

Step 3: Create the product $(ax+m)(ax+n)$

Step 4: From each binomial in step 3, factor out and discard any common factor. The result is the factored form.

Berry Method to Factor

Solve $8y^2 + 2y = 3$

$$8y^2 + 2y = 3$$
$$8y^2 + 2y - 3 = 0$$

Use the Berry Method.
a = 8, b = 2, c = -3

Step 1 $a \cdot c = -24$

Step 2 -4 and 6 are factors of -24 that add to +2

$m = -4, n = 6$

Step 3 $(ax + m)(ax + n)$

$(8y - 4)(8y + 6)$

Step 4 Discard common factors

$(2y - 1)(4y + 3)$

Berry Method to Factor

Solve $8y^2 + 2y = 3$

Use the Zero Product Rule to solve.

$$2y - 1 = 0 \ or \ 4y + 3 = 0$$
$$2y = 1 \ or \ 4y = -3$$
$$y = \frac{1}{2} \ or \ y = -\frac{3}{4}$$

Berry Method to Factor

Solve $4x^2 - 15x - 25 = 0$

Use the Berry Method.
a = 4, b = -15, c = -25

$$a \cdot c = -100$$

$$m = 5, n = -20$$

$$(4x+5)(4x-20)$$

$$(4x+5)(x-5)$$

$$(x-5)(4x+5) = 0$$

Berry Method to Factor

Solve $4x^2 - 15x - 25 = 0$

Use the Zero Product Rule to solve.

$$x - 5 = 0 \; or \; 4x + 5 = 0$$

$$x = 5 \qquad\qquad 4x = -5$$

$$x = -\frac{5}{4}$$

Berry Method to Factor

Solve $3x^2 - 2x - 5 = 0$

Answer

Application of the Zero Product Property

In addition to finding the x-intercepts of quadratic equations, the Zero Product Property can also be used to solve real world application problems.

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Application

Example: A garden has a length of (x+7) feet and a width of (x +3) feet. The total area of the garden is 396 sq. ft. Find the width of the garden.

Answer

22 The product of two consecutive even integers is 48. Find the smaller of the two integers.

Hint: Two consecutive integers can be expressed as x and $x + 1$. Two consecutive even integers can be expressed as x and $x + 2$.

Answer

23 **The width of a rectangular swimming pool is 10 feet less than its length. The surface area of the pool is 600 square feet. What is the pool's width?**

Answer

24 A science class designed a ball launcher and tested it by shooting a tennis ball straight up from the top of a 15-story building. They determined that the motion of the ball could be described by the function $h(t) = -16t^2 + 144t + 160$, where t represents the time the ball is in the air in seconds and h(t) represents the height, in feet, of the ball above the ground at time t. What is the maximum height of the ball? At what time will the ball hit the ground? Find all key features and graph the function.

Answer

Problem is from:

engage ny

Click link for exact lesson.

25 **A ball is thrown upward from the surface of Mars with an initial velocity of 60 ft/sec. What is the ball's maximum height above the surface before it starts falling back to the surface? Graph the function. The equation for "projectile motion" on Mars is:**

$$h(t) = -6.5t^2 + 60t$$

Answer