Intro to Inverse Functions

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Goals and Objectives

Students will be able to recognize and find an inverse function:

a) using coordinates,

b) graphically and

c) algebraically.

Why Do We Need This?

Sometimes, it is important to look at a problem from the inside out.

Addition undoes subtraction.

Multiplication undoes division.

In order to look deeply into different problems,
we must try to see things from the inside out.

Inverse functions undo original functions.

Inverse Function

An **inverse function** is a function that undoes the action of another function. The inverse function has all of the same points as the original function, except the domain and range values (or x and y values) have been switched. In other words, the domain of one function will be the range of its inverse & vice-versa.

The notation for the inverse of $f(x)$ is $f^{-1}(x)$.

Read, "the inverse of f of x."

You can prove that a function is an inverse of another using the following relationship:

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

Inverse Function

Prove that $f(x)$ & $g(x)$ are inverse functions.

$$f(x) = 3x - 6 \ \ \& \ \ g(x) = \frac{1}{3}x + 2$$

Answer

Inverse Function

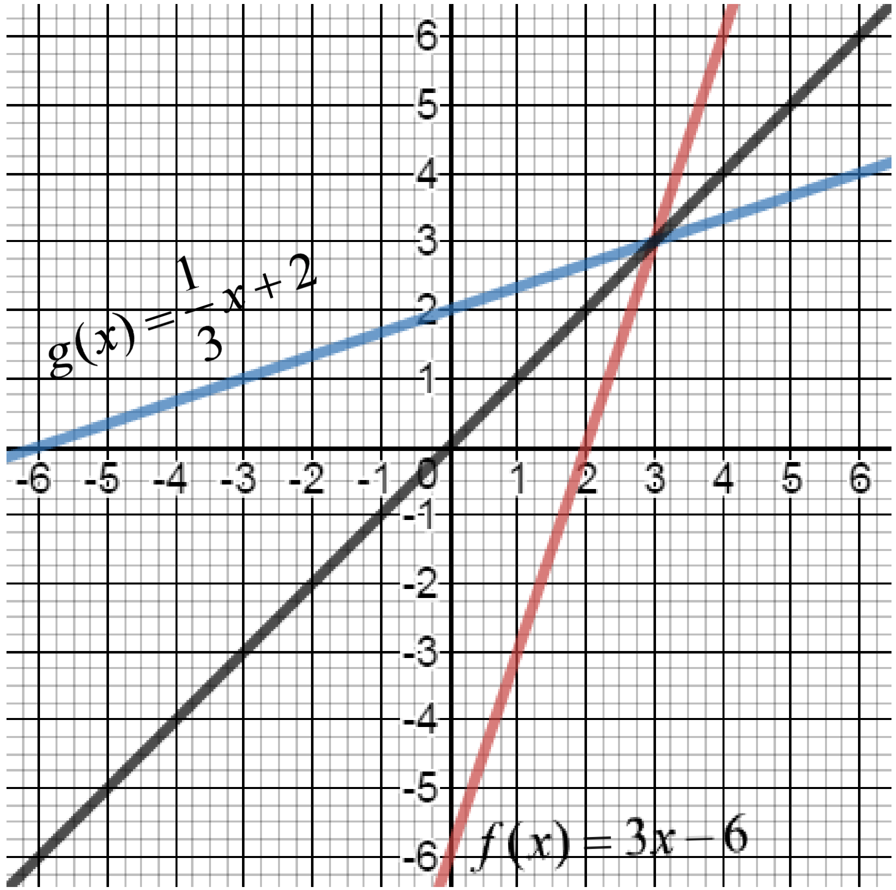
You can also prove that two functions are inverses by GRAPHING. The graph of an inverse is the reflection of the function over the line $y = x$.

The following functions are inverses of each other.

$$f(x) = 3x - 6$$

$$g(x) = \frac{1}{3}x + 2$$

What conjecture can you make about the x and y values of inverse functions? Explain your answer.



Answer

Inverse Function

The inverse of a function is the reflection over the line $y = x$. As you observed, this will result in the switching of x and y values.

Examples:

a) Find the inverse of:

$$f(x) = \{(1,\ 2),\ (3,\ 5),\ (-7,\ 6)\}$$

b) Find the inverse of:

x	y
3	2
4	4
5	–5
6	7

Answer

64 What is the inverse of $\{(1,\ 4),\ (5,\ 3),\ (2,\ -1)\}$?

A $\{(4,\ 1),\ (3,\ 5),\ (2,\ -1)\}$

B $\{(-1,\ -4),\ (-5,\ -3),\ (-2,\ 1)\}$

C $\{(4,\ 1),\ (3,\ 5),\ (-1,\ 2)\}$

D $\{(-4,\ -1),\ (-3,\ -5),\ (1,\ -2)\}$

Answer

65 If the inverse of a function is $\{(1, 0), (3, 3), (-4, -5)\}$, what was the original function?

A $\{(0, 1), (3, 3), (-5, -4)\}$

B $\{(-1, 0), (-3, -3), (4, 5)\}$

C $\{(0, -1), (-3, -3), (4, 5)\}$

D $\{(0, 1), (3, 3), (-4, -5)\}$

Answer

66 What is the inverse of:

x	y
3	4
1	0
-2	3
4	7

A

x	y
4	3
1	0
2	3
4	7

B

x	y
4	3
0	1
3	-2
7	4

C

x	y
-3	-4
-1	0
2	-3
-4	-7

D

x	y
4	3
0	0
-2	3
7	4

Answer

67 Will the inverse of the following points be a function? Why or why not?

Yes

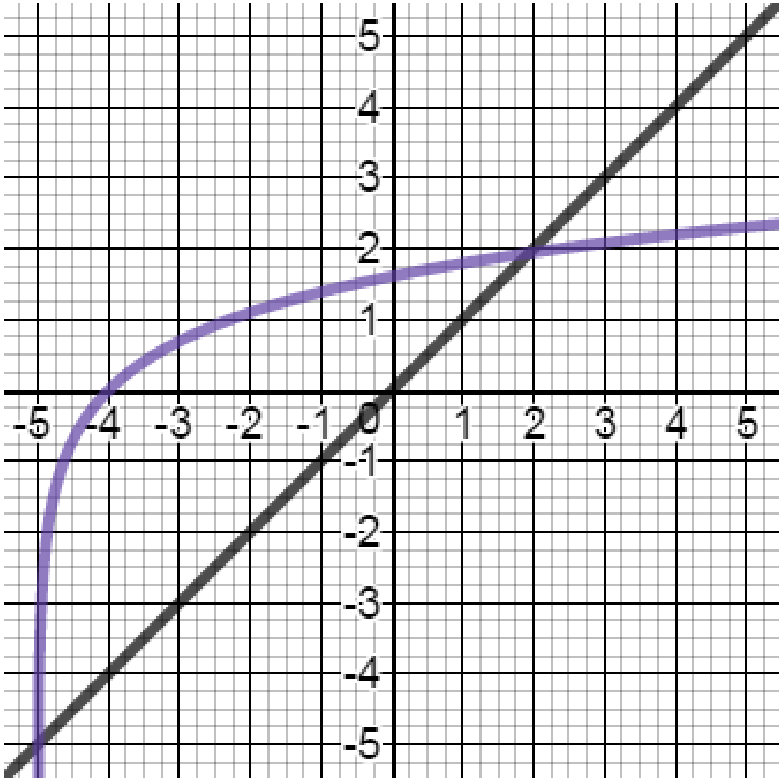
No

x	y
1	2
2	3
3	4
5	2

Answer

Inverse

Draw the inverse of the given function.

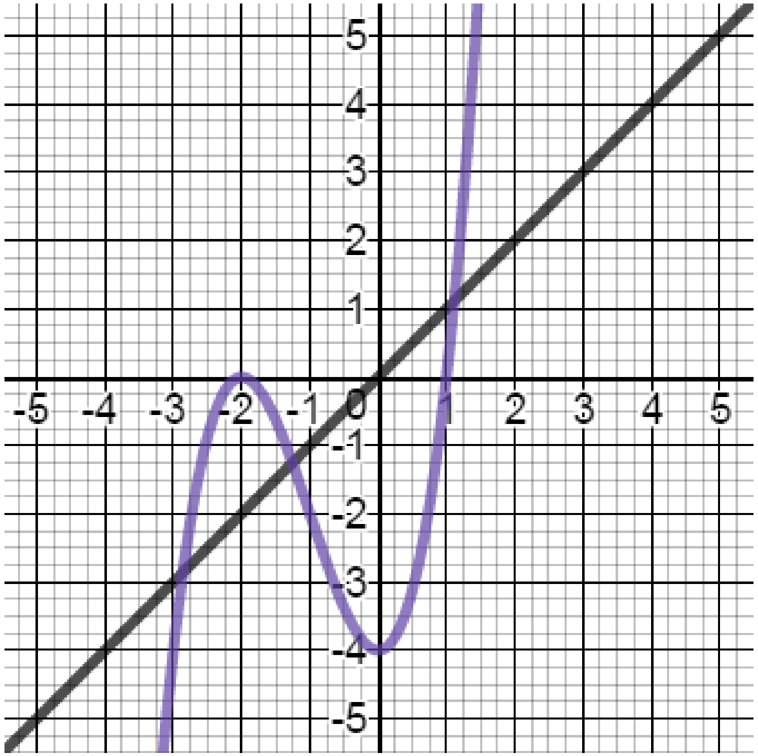


Is the inverse a function? Why or why not?

Answer

Inverse

Draw the inverse of the given function.



Is the inverse a function? Why or why not?

Answer

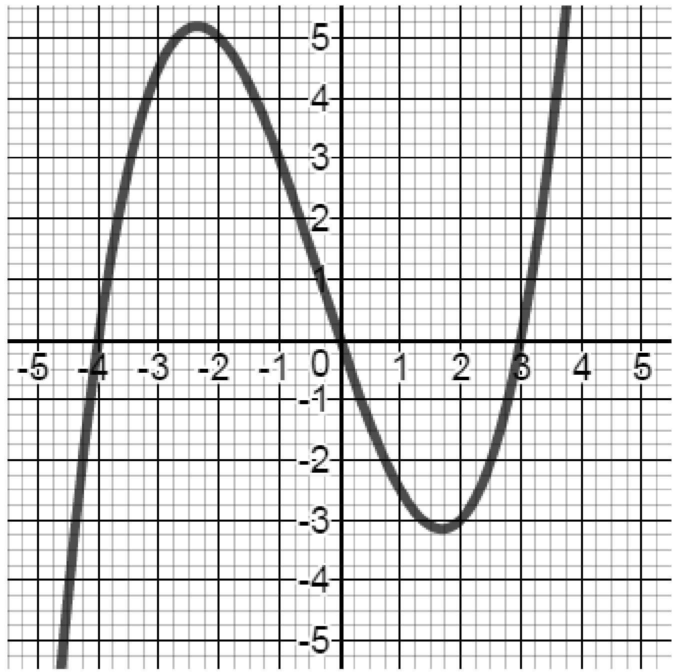
Horizontal Line Test

Just like the Vertical Line Test, there is a simple way to determine if a function's inverse is also a function, just from looking at its graph.

The **Horizontal Line Test** is used to determine if the inverse of a function is also considered to be a function. If a horizontal line crosses the function more than once, its inverse is NOT a function.

Is the inverse of the function shown to the right considered to be a function?

Move this line to check:



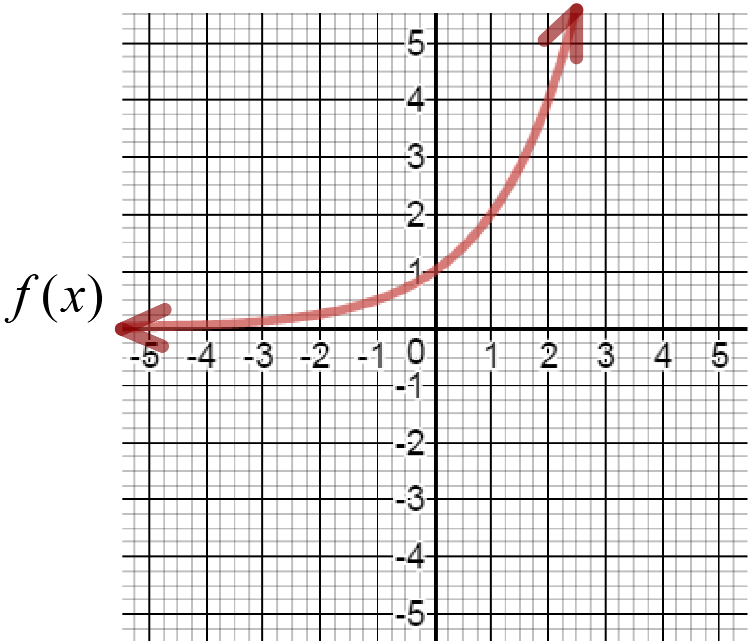
Answer

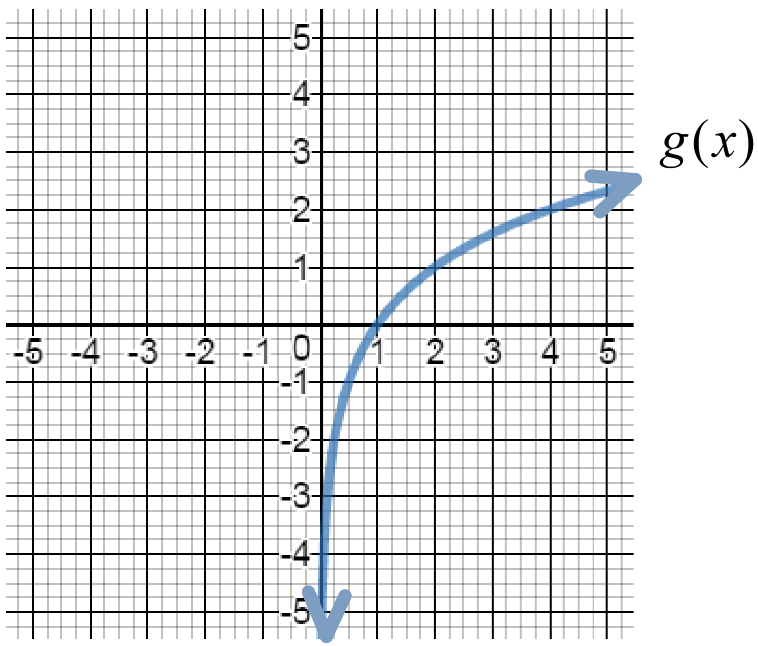
One-to-one Functions

Functions are **one-to-one** when every element in the domain is mapped, or connected to, a unique element in the range.

This happens when the original function passes both the vertical and horizontal line test.

An example of two one-to-one functions are shown in the graphs below.

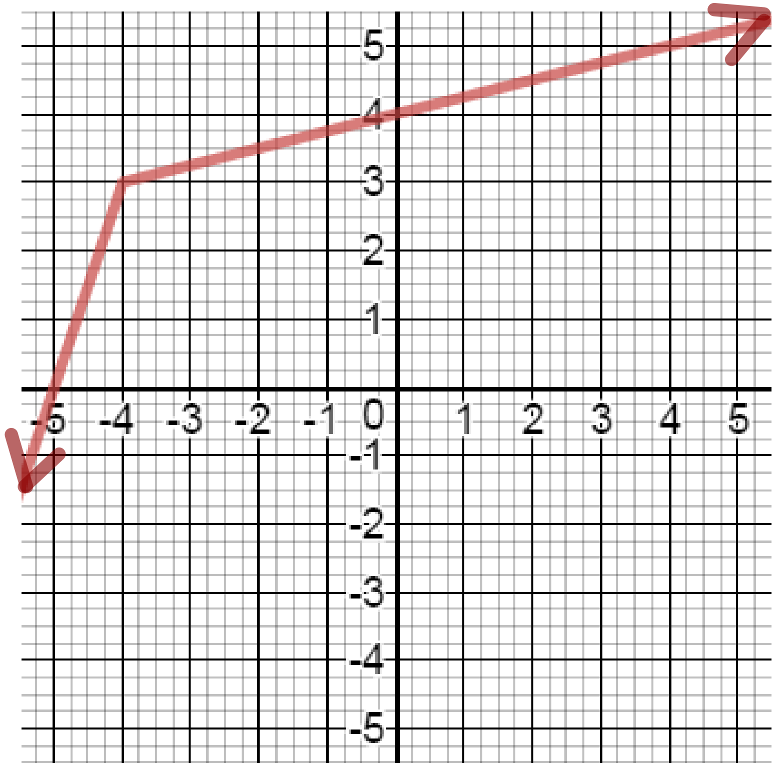




Horizontal Line Test

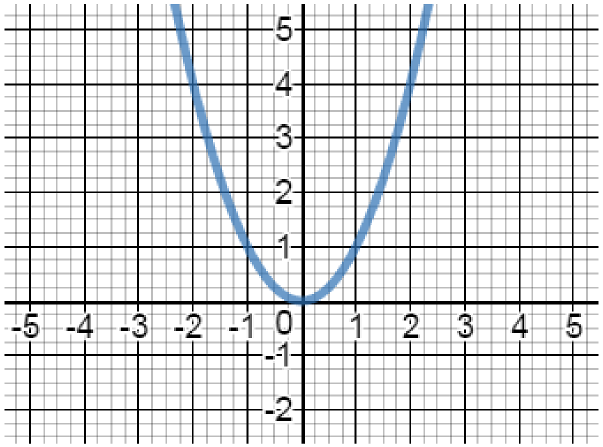
Example: Will the inverse of the given function be a function?

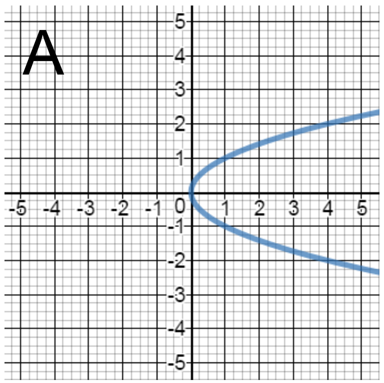
Move this line to check:

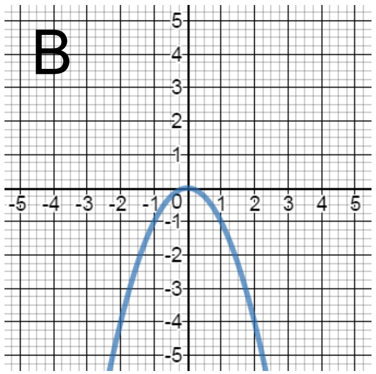


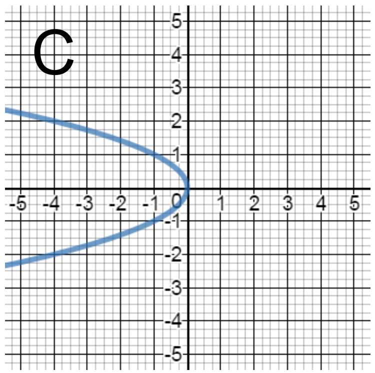
Answer

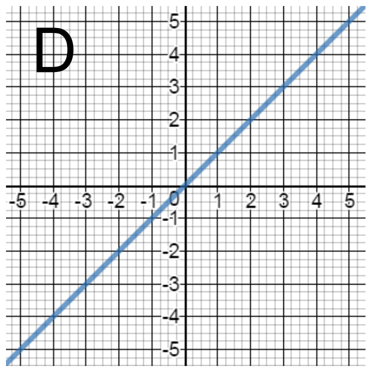
68 Which graph is the inverse of the function below?





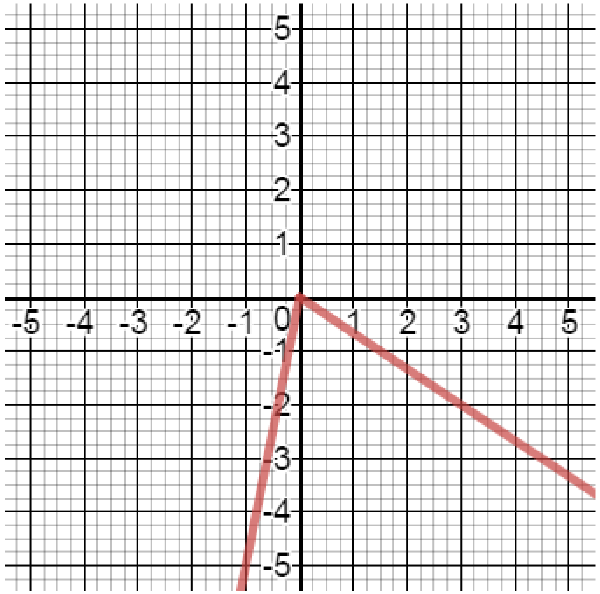


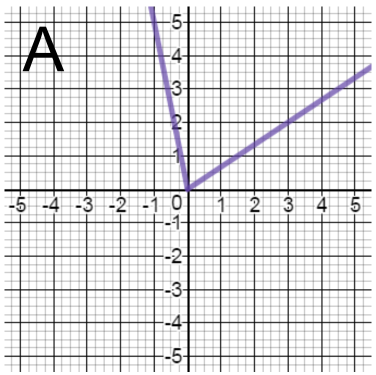


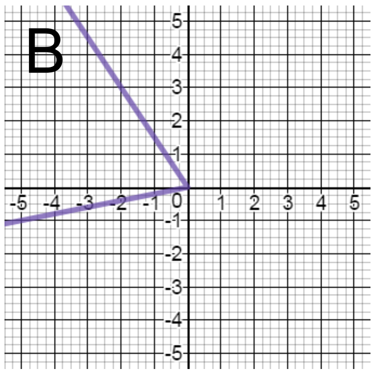


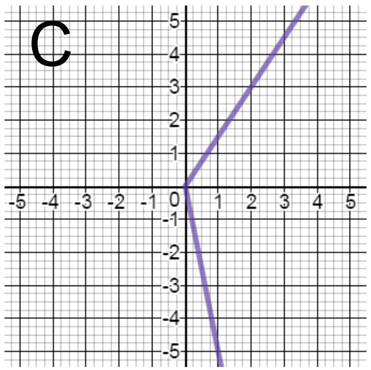
Answer

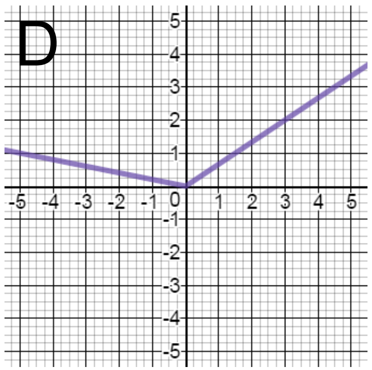
69 Which graph is the inverse of the function below?





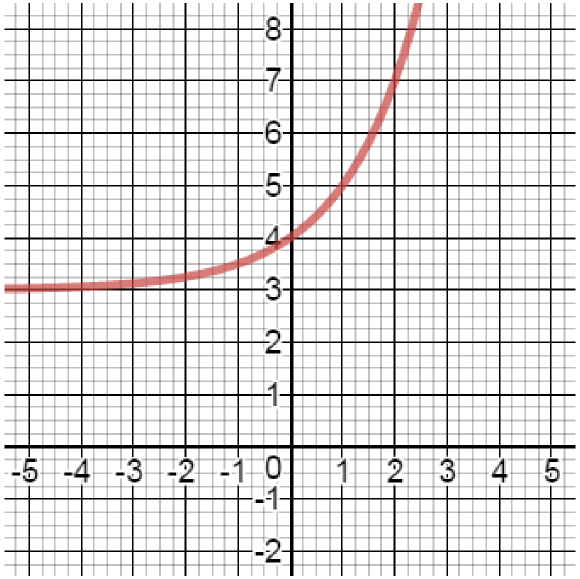


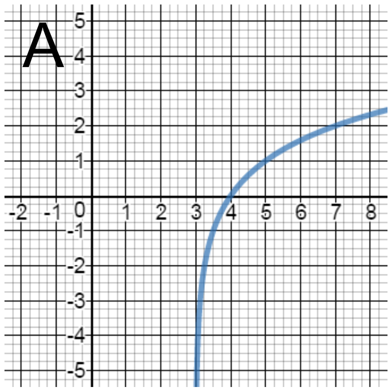


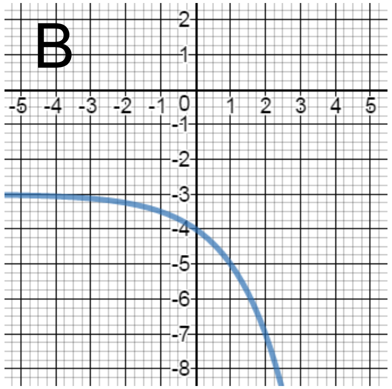


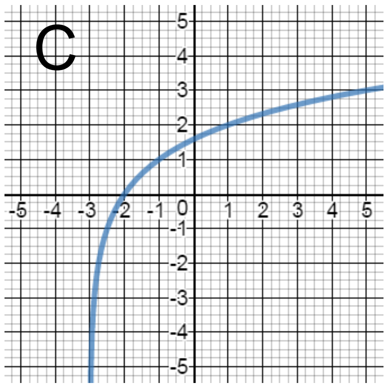
Answer

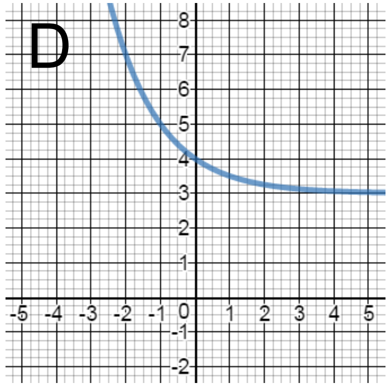
70 Which graph is the inverse of the function below?











Answer

71 Will the inverse of $f(x) = 2x^2 - 23x + 4$ be a function?

Yes

No

Answer

Finding the Inverse of a Function Algebraically

Knowing that the inverse of a function switches x and y values, we can take this concept further when given an equation.

Given: $f(x) = 3x - 1$

$y = 3x - 1$	Change $f(x)$ to y
$x = 3y - 1$	Switch x and y
$x + 1 = 3y$	Solve for y
$\frac{x+1}{3} = y$	
$f^{-1}(x) = \frac{x+1}{3}$	Change it back to function notation, using $f^{-1}(x)$, since it's the inverse

Note: If the original equation is written as "$y =$", it's inverse is y^{-1}

Inverse

Practice: Find the inverse of the following functions.

a) $y = 2x + 1$

b) $f(x) = 4x + 9$

Answer

Inverse

Practice: Find the inverse of the following function.

c) $y = 3x^2 - 1$

Answer

72 Which of the following choices is the inverse of $f(x) = 3x$?

A $f^{-1}(x) = \frac{x}{3}$

B $f^{-1}(x) = \frac{3}{x}$

C $f^{-1}(x) = -3x$

D $f^{-1}(x) = 0$

Answer

73 Which of the following is the inverse of $f(x) = 2 - 2x$?

A $f^{-1}(x) = 1 + \frac{x}{2}$

B $f^{-1}(x) = 1 - \frac{x}{2}$

C $f^{-1}(x) = -1 + \frac{x}{2}$

D $f^{-1}(x) = -1 - \frac{x}{2}$

Answer

74 Find the inverse of $y = 2x^2 - 4$

A $y^{-1} = \sqrt{\frac{x}{2} - 2}$

B $y^{-1} = \sqrt{2 - x}$

C $y^{-1} = \pm\sqrt{2 - \frac{x}{2}}$

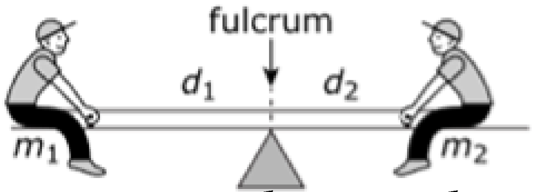
D $y^{-1} = \pm\sqrt{\frac{x}{2} + 2}$

Answer

Applications of Inverses

Part A

Two children sit on a seesaw, as illustrated. The mass, in kilograms, of the first child is m_1 and the mass, in kilograms, of the second child is m_2. In the diagram, d_1 & d_2 represent the distance, in meters, from the fulcrum (the balance point) to each child. The total distance between the children is 5 meters.



For a seesaw to be balanced, $m_1 d_1 = m_2 d_2$. Use the information in the in the table to write the function $f(x)$ that allows you to determine m_2, the mass of the second child.

m_1	d_1	d_2
45	x	5 - x

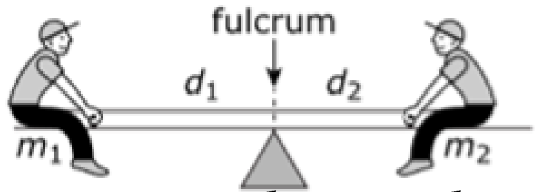
Part B

Determine the inverse function $f^{-1}(x)$ to model the distance, d_1, based on the mass of the first child. Show your work.

Applications of Inverses

Part A

Two children sit on a seesaw, as illustrated. The mass, in kilograms, of the first child is m_1 and the mass, in kilograms, of the second child is m_2. In the diagram, d_1 & d_2 represent the distance, in meters, from the fulcrum (the balance point) to each child. The total distance between the children is 5 meters.



For a seesaw to be balanced, $m_1 d_1 = m_2 d_2$. Use the information in the in the table to write the function $f(x)$ that allows you to determine m_2, the mass of the second child.

m_1	d_1	d_2
45	x	5 - x

To start, use the balance equation provided, and fill in the known values & expressions.

$$45x = m_2(5 - x)$$

Applications of Inverses

$$45x = m_2(5 - x)$$

Now, solve the equation for m_2. In the final step, write your equation as the function $f(x)$.

$$45x = m_2(5 - x)$$

$$\frac{45x}{5 - x} = m_2$$

$$f(x) = \frac{45x}{5 - x}$$

Applications of Inverses

Part B

Determine the inverse function $f^{-1}(x)$ to model the distance, d_1, based on the mass of the first child. Show your work.

$$f(x) = \frac{45x}{5-x}$$

$$y = \frac{45x}{5-x}$$

$$x = \frac{45y}{5-y}$$

$$x(5-y) = 45y$$

$$5x - xy = 45y$$

$$5x = 45y + xy$$

$$5x = y(45+x)$$

$$\frac{5x}{45+x} = y$$

$$f^{-1}(x) = \frac{5x}{45+x}$$

75 **Part A:** Every morning and evening, Audra needs to commute between her house to the office building. When traveling from her house to the office building, she leaves early enough that she can travel at a regular highway speed of 70 miles per hour, and it takes a certain length of time, t, to travel. When traveling from the office building to her home, ther is more traffic on the road, making Audra travel at a slower speed. It takes her 15 minutes more to drove from the office building to her house. Since Audra is always driving the same distance each way, $r_1 t_1 = r_2 t_2$.

Use the information provided to write the function $f(t)$ that allows you to determine r_2, Audra's speed during her trip from the office building to her house.

Answer

A $f(t) = \dfrac{70t}{t+15}$

B $f(t) = \dfrac{t+15}{70t}$

C $f(t) = \dfrac{70t+1050}{t}$

D $f(t) = \dfrac{t}{70t+1050}$

76 **Part B**

Determine the inverse function $f^{-1}(t)$ to model the time required for Audra to travel from her house to the office building. Show your work.

Click to reveal answers

Answer

77 **Part A**

A train runs its route regularly between Philadelphia, PA and Atlantic City, NJ. When traveling from Philadelphia to Atlantic City, its average speed is 60 miles per hour, and it takes a certain length of time, t, to travel. When traveling from Atlantic City to Philadelphia, it travels at a slower speed and takes 10 minutes more to arrive at Philadelphia. Since the train is always traveling the same distance each way, $r_1 t_1 = r_2 t_2$.

Use the information provided to write the function $f(t)$ that allows you to determine r_2 , the speed of the train during its trip from Atlantic City to Philadelphia.

Answer

A $f(t) = \dfrac{60t + 600}{t}$

B $f(t) = \dfrac{t}{60t + 600}$

C $f(t) = \dfrac{60t}{t + 10}$

D $f(t) = \dfrac{t + 10}{60t}$

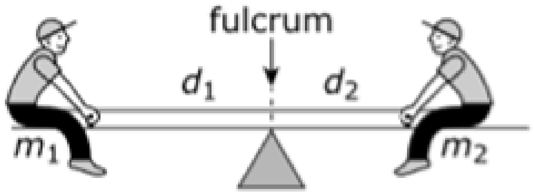
78 **Part B**

Determine the inverse function $f^{-1}(t)$ to model the time required to travel from Philadelphia to Atlantic City. Show your work.

Click to reveal answers

Answer

79 **Part A:** Two children sit on a seesaw, as illustrated. The mass, in kilograms, of the first child is m_1 and the mass, in kilograms, of the second child is m_2. In the diagram, d_1 & d_2 represent the distance, in feet from the fulcrum (the balance point) to each child. The total distance between the children is 10 feet.



For a seesaw to be balanced, $m_1 d_1 = m_2 d_2$. Use the information in the table to write the function $f(x)$ that allows you to determine m_1, the mass of the first child.

m_2	d_1	d_2
40	$10 - x$	x

A $f(x) = \dfrac{400 - 40x}{x}$

B $f(x) = \dfrac{x}{400 - 40x}$

C $f(x) = \dfrac{10 - x}{40x}$

D $f(x) = \dfrac{40x}{10 - x}$

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Answer

80 **Part B**

Determine the inverse function $f^{-1}(x)$ to model the distance, d_2 , based on the mass of the first child. Show your work.

Click to reveal answers

Answer

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