Law of Cosines

Return to Table of Contents

Law of Cosines

If you know the measures of enough sides and angles of a triangle, you can solve the triangle.

The Law of Cosines can be used to solve the triangle when the measures of all three sides (SSS) or the measures of two sides and the included angle (SAS) are known.

Law of Cosines

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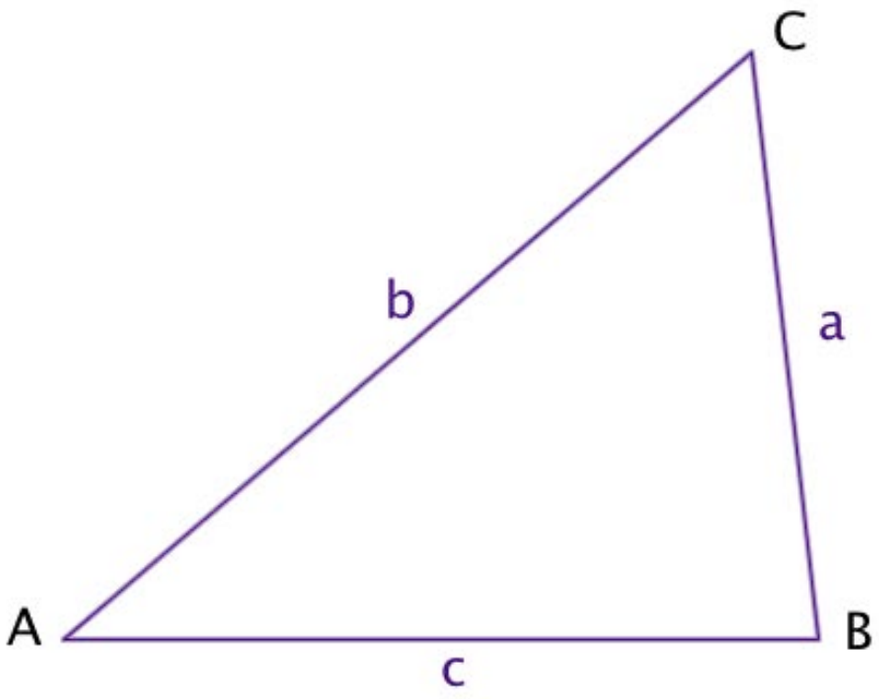
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

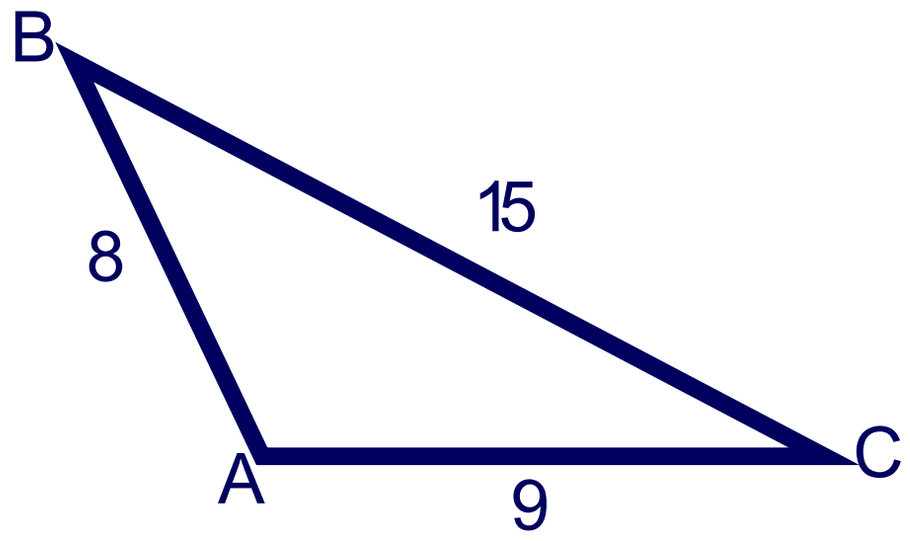


The Law of Cosines relates the length of a side of a triangle to the measure of the opposite angle.

Law of Cosines

Example 1: Solve ΔABC

Because we know all three sides, we can find any angle. Let's find A first.



$15^2=8^2 + 9^2 - 2(8)(9)\cos A$

$225 = 64 + 81 - 144\cos A$

$225 = 145 - 144\cos A$

$80 = -144\cos A$

$\cos^{-1}(-\frac{80}{144}) = A$

$m\angle A \approx 123.75^\circ$ or about 124°

(continued on next slide)

Law of Cosines

Find m∠B:

$9^2 = 15^2 + 8^2 - 2(15)(8)\cos B$

$81 = 225 + 64 - 240\cos B$

$81 = 289 - 240\cos B$

$-208 = -240\cos B$

$\cos^{-1}\left(\frac{-208}{-240}\right) = m\angle B$

$m\angle B \approx 29.9^\circ$ or 30°

Or use Law of Sines:

$$\frac{\sin 124}{15} = \frac{\sin B}{9} = \frac{\sin C}{8}$$

Once we know A and B, we subtract from 180 to find C (or we could use the Law of Cosines, but that's many more steps).

$180 - (124 + 30) = 26^\circ$

Law of Cosines

Example 2: Solve ΔABC

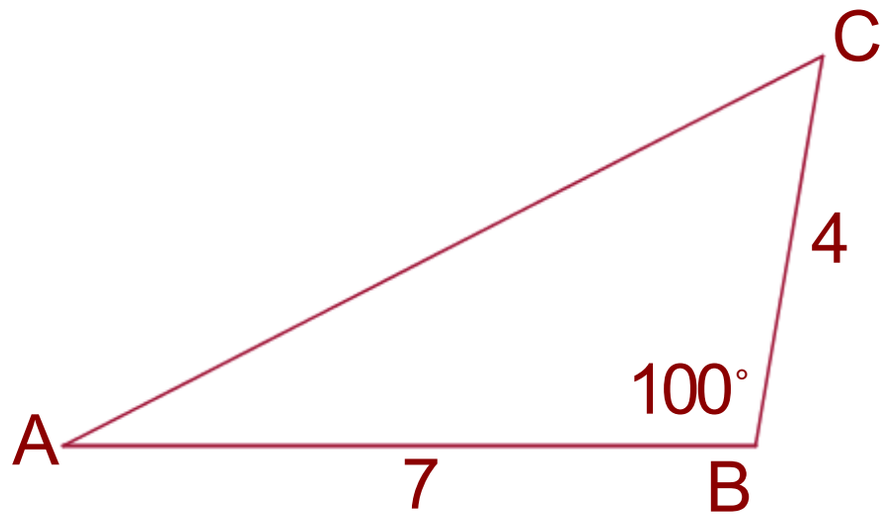
$$b^2 = 4^2 + 7^2 - 2(4)(7)\cos 100$$

$$b^2 = 16 + 49 - 28(-.1736)$$

$$b^2 \approx 65 + 4.8608$$

$$b^2 \approx 69.8608$$

$$b \approx 8.358 \text{ or about } 8.4$$

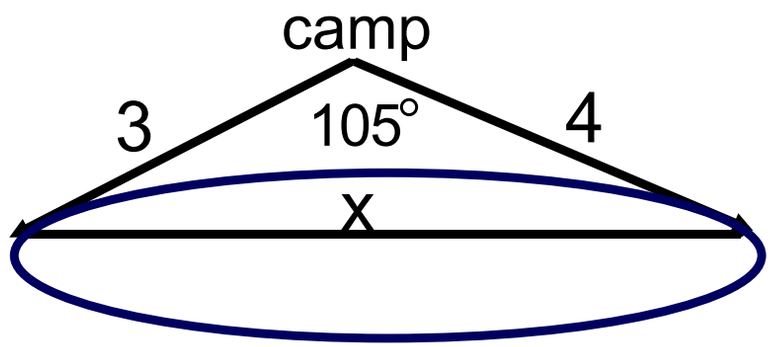


Answer

Use Law of Cosines or Law of Sines to find the other angles. You try it!

Law of Cosines

Example: Cal C. went camping. Sitting at his camp site he noticed it was 3 miles to one end of the lake and 4 miles to the other end. He determined that the angle between these two line of sites is 105 degrees. How far is it across the lake?



Teacher Notes

31 If m∠A = 35, b = 10 and c = 12, find a.

Answer

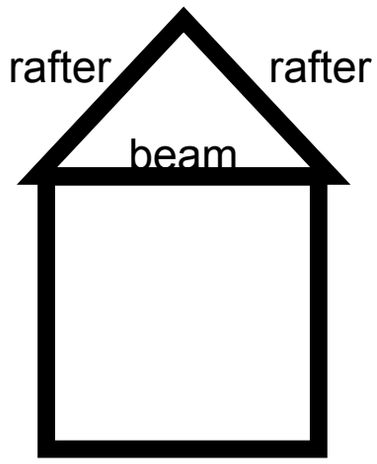
32 If a = 7, b = 10 and c = 12, find m∠A.

Answer

33 If m∠A = 95º, b = 7 and c = 11, find m∠B

Answer

34 Roof rafters of 16 feet are supported by a 20 foot beam as shown. What is the measure of the angle where the rafter meets the beam?



Answer

35 Quadrilateral Park has a walking trail in the shape of a parallelogram. The shorter sides are 0.6 miles, the longer sides are 1 mile, and the acute angles are 60º. How far apart are the vertices of the obtuse angles?

Answer