Law of Sines

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Law of Sines

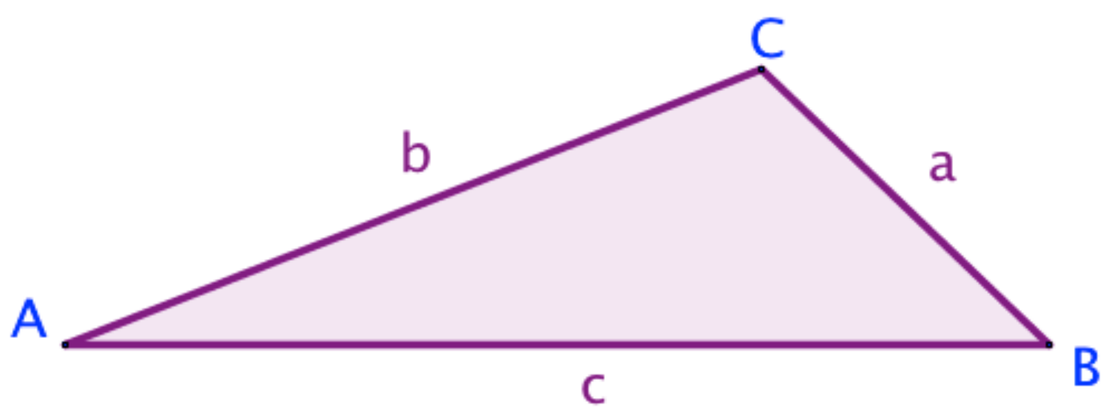
So far we've been working with right triangles, but what about other triangles?

If you know the measures of enough sides and angles of a triangle, you can solve the triangle.

The *Law of Sines* can be used when two angles and the length of any side are known (AAS or ASA) or when the lengths of two sides and an angle opposite one of the sides are known (SSA).

Law of Sines

Law of Sines $\Rightarrow$ $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



The Law of Sines relates the sines of the angles of a triangle to the lengths of the opposite sides.

Note that any one of the angles can be obtuse, and so far we've only found trig values of acute angles. For now we will use the calculator to find the trig values of obtuse angles. In the next unit we will learn how these values are calculated.

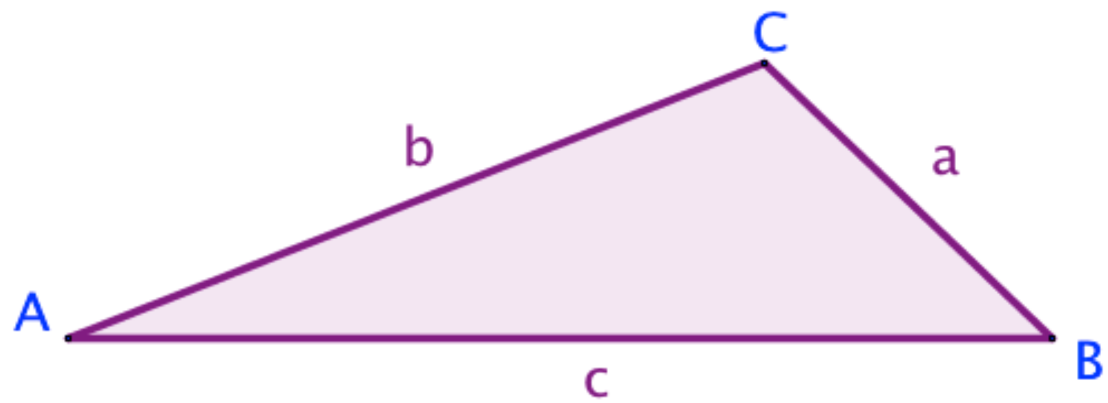
Law of Sines

Use Law of Sines when we know:

- Angle-Side-Angle (two angles and the included side
- Angle-Angle-Side (two angles and the side opposite one of them)
- Side-Side-Angle (two sides and the angle opposite one of them)

Caution - may result in 0, 1 or 2 triangles (more info to follow)

Law of Sines



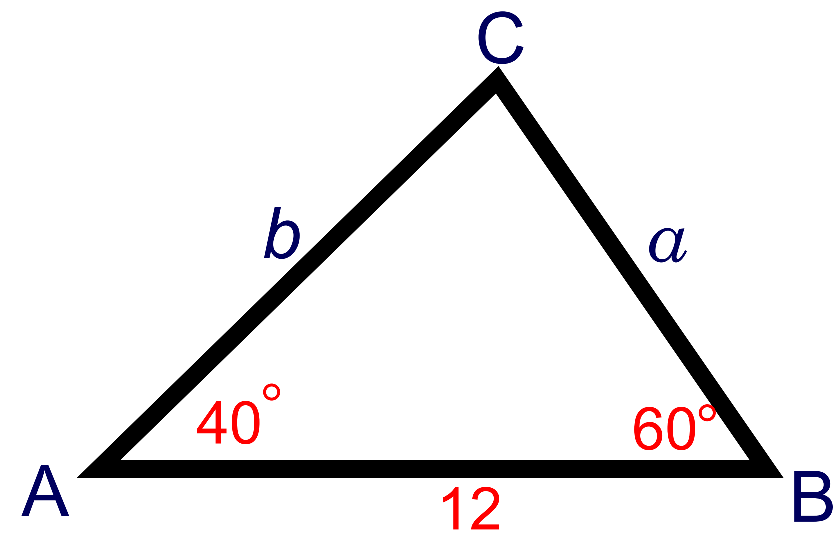
For clarity and convenience note that $\overline{CB}$, opposite $\angle A$, has length a, $\overline{AB}$, opposite $\angle C$ has length c, and $\overline{AC}$, opposite $\angle B$ has length b.

Law of Sines with ASA

Example: m∠A = 40º, m∠B = 60º, and *c* = 12

Solve triangle ABC.

Draw an approximate diagram:



By Triangle Sum Theorem, the angles of ΔABC sum to 180º, so m∠C = 80º.

$$\frac{\sin 80}{12} = \frac{\sin 60}{b}$$

$$b \sin 80 = 12 \sin 60$$

$$b = \frac{12 \sin 60}{\sin 80} \approx 10.553$$

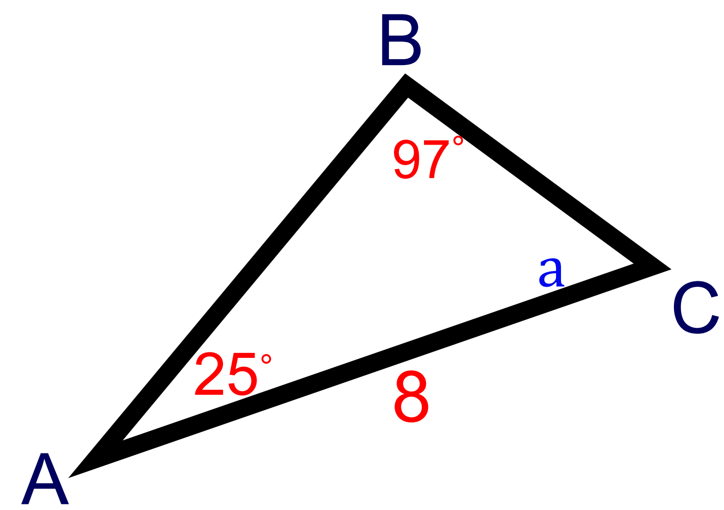
$$\frac{\sin 80}{12} = \frac{\sin 40}{a}$$

$$a \sin 80 = 12 \sin 40$$

$$a = \frac{12 \sin 40}{\sin 80} \approx 7.832$$

Law of Sines with AAS

Example: m∠A = 25º, m∠B = 97º, and *b* = 8



Solve triangle ABC.

By Triangle Sum Theorem, the angles of ΔABC sum to 180º, so m∠C = 58º.

$$\frac{\sin 97}{8} = \frac{\sin 25}{a}$$

$$a \sin 97 = 8 \sin 25$$

$$a = \frac{8 \sin 25}{\sin 97} \approx 3.406$$

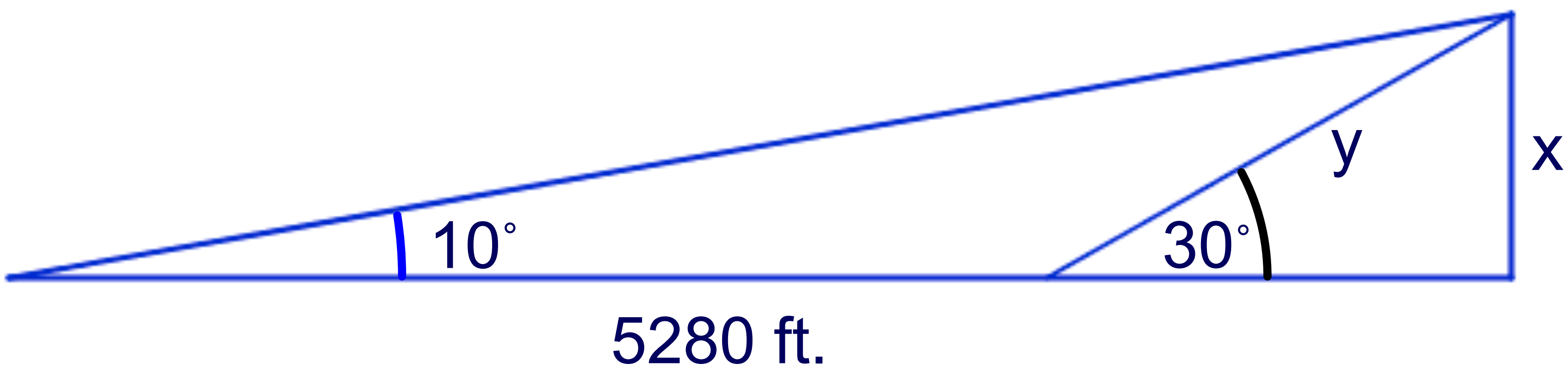
$$\frac{\sin 97}{8} = \frac{\sin 58}{c}$$

$$c \sin 97 = 8 \sin 58$$

$$c = \frac{8 \sin 58}{\sin 97} \approx 6.835$$

Law of Sines Problem

Example: As Cal C. is driving toward the Old Man of the Mountain, the angle of elevation is 10º. He drives another mile and the angle of elevation is 30º. How tall is the mountain?



Answer

24 Find *b* given $m\angle A=40^\circ$, $c=7$, and $m\angle B=75^\circ$

Answer

25 Find b given $m\angle A=35°$, $a=7$, and $m\angle B=85°$

Answer

Law of Sines with SSA - the Ambiguous Case

The ambiguous case arises from the fact that an acute angle and an obtuse angle have the same sine.

Follow these steps to determine the number of solutions for a triangle:

1. Use the Law of Sines to get a second angle of the triangle.

2. Check to see if the angle is valid - the two angles you have so far must have a sum that is less than 180º.

3. Check if there is a second angle that is valid. To do this, subtract that angle from 180º, then add this angle to the original given angle - these two angles must also have a sum that is less than 180º.

Law of Sines with SSA - the Ambiguous Case

Suppose it is given that a triangle has side lengths of 2.7 and 5 and the angle opposite the 2.7 is 30º.

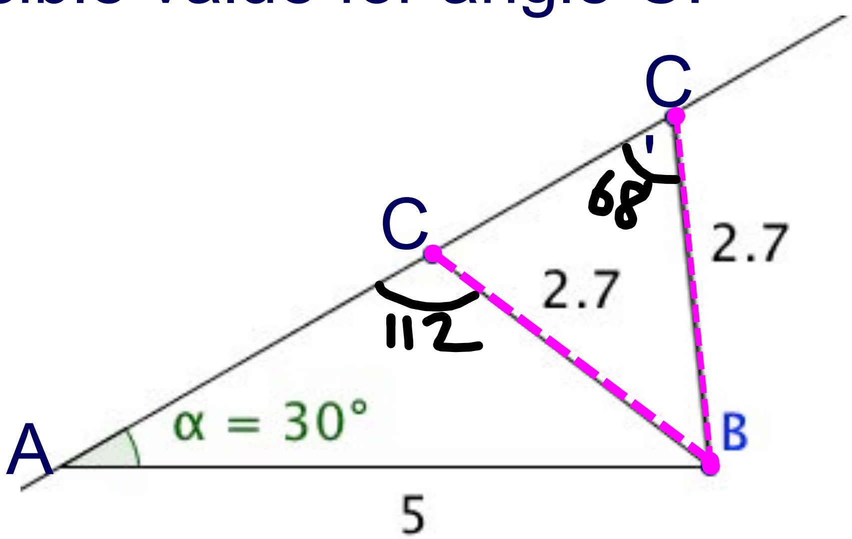
1. Use Law of Sines to find a possible value for angle C:

$$\frac{2.7}{\sin 30} = \frac{5}{\sin C}$$

$$2.7 \sin C = 5 \sin 30$$

$$\sin C = \frac{5 \sin 30}{2.7} \approx .926$$

$$\sin^{-1} .926 \approx 68^{\circ}$$



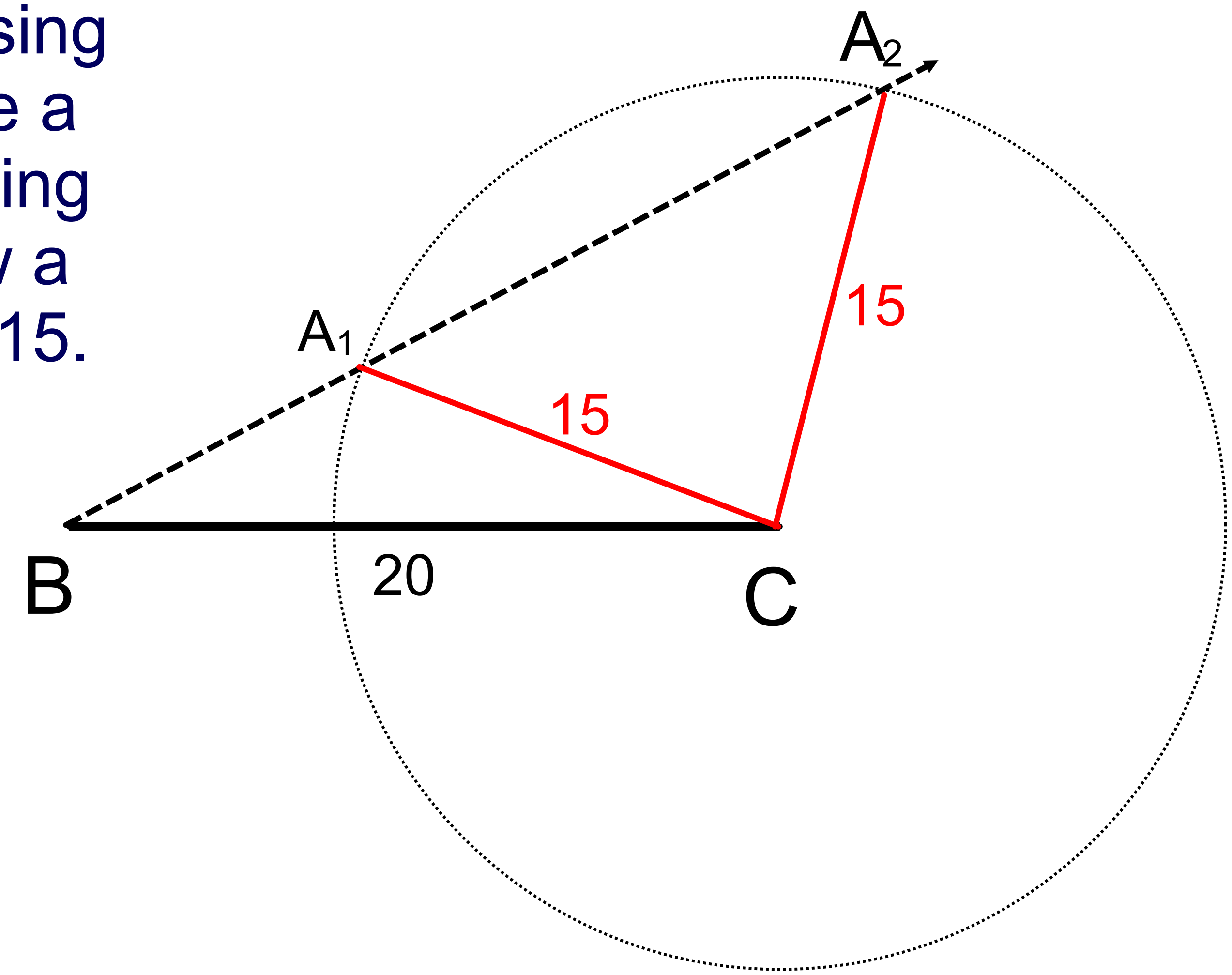
2. Check validity: 30 + 68 < 180, so 68º is a valid angle.

3. Check for a second value: 180 - 68 = 112.
   30 + 112 < 180, so 112º is a valid angle.

Law of Sines with SSA - the Ambiguous Case

Drawing a triangle: $a = 20$, $b = 15$, $m\angle B = 30°$

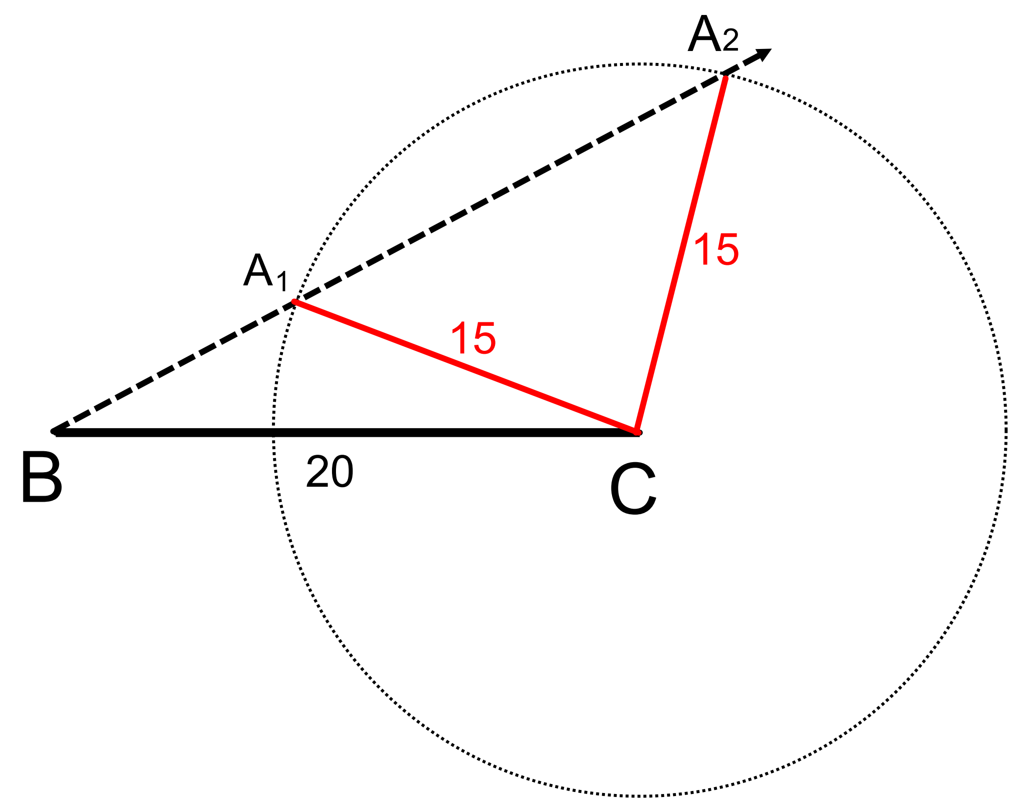
Start by drawing a segment of 20, label endpoints as B and C. Using this segment as one side, make a 30º angle with vertex B, extending the ray on the other side. Draw a circle with center C and radius 15. The two points where the circle intersects the ray are the possible positions of A (let's call them A_1 and A_2).



Law of Sines with SSA - the Ambiguous Case

The Law of Sines tells us that

$\frac{\sin A}{20} = \frac{\sin B}{15}$, $\sin B = \sin 30 = 0.5$



So 15 sin A = 10

sin A = 0.6

$\sin^{-1}(0.6) \approx 41.8°$

30 + 42 < 180, so 42° is valid.

180 - 42 = 138 and 30 + 138 < 180, so 138° is also valid.

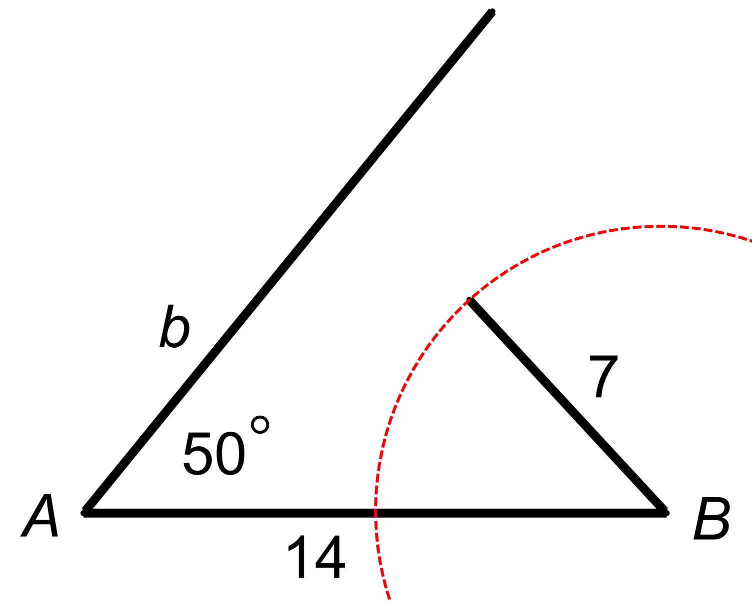
Law of Sines with SSA - the Ambiguous Case

Example: Solve ΔABC if $m\angle A = 50^\circ$, $a = 7$ and $c = 14$

Solving for sin C, we get

$$\frac{\sin 50}{7} = \frac{\sin C}{14}$$

$$\sin C = \frac{14 \sin 50}{7} \approx 1.5321$$



For any angle, θ, -1 ≤ sin θ ≤ 1.

Therefore, there is no triangle that meets these conditions. As you can see from the drawing, a and b will not meet.

26 How many solutions if $m\angle A = 40$, $a = 5$ and $c = 7$?

Answer

27 How many solutions if m$\angle$A = 40, a = 7 and c = 5?

Answer

28 How many triangles meet the following conditions?

$m\angle A = 35^\circ$, $a = 10$, and $c = 9$

Answer

29 How many triangles meet the following conditions?

$m\angle A = 25^\circ$, $a = 8$ and $c = 11$

Answer

30 Carlos has a triangular pen for his guinea pig. He has measured two sides and says that one side is 1 foot 6 inches and the other is 2 feet 3 inches, and that the angle opposite the shorter side is 75 . Is this possible and is there more than one possible shape of his pen?

A Yes, it is possible, and there is only one possible shape for the pen.

B Yes, it is possible, but there are two possible shapes for the pen.

C No, it is not possible.

Answer