Dividing Polynomials

Return to Table of Contents

Division of Polynomials

Here are 3 different ways to write the same quotient:

$$\left(2x^2 - 4x + 3\right) \div (x + 3)$$

$$\left(2x^2 - 4x + 3\right)(x + 3)^{-1}$$

$$\frac{2x^2 - 4x + 3}{x + 3}$$

To divide a polynomial by a monomial, write each term of the polynomial as a fraction with a common denominator. Then simplify each fraction.

$$\frac{12km^3 - 20k^2m + 18m^2}{4km}$$

$$\frac{12km^3}{4km} - \frac{20k^2m}{4km} + \frac{18m^2}{4km}$$

$$3m^2 - 5k + \frac{9m}{2k}$$

Examples

Click to Reveal Answer

$$\frac{10c^3 + 16c - 20}{2c}$$

$$\frac{27de^3 - 24de - 21d}{9de}$$

$$\left(4m^2 + 12m - 16\right) \div (2m)$$

$$(-6r^3t^3 + 4r^2t^2 - 2rt)(-2r^2t^2)^{-1}$$

43 Simplify $\dfrac{12m^{12} - 18m^{6} + 24m^{2}}{6m^{3}}$

A $2m^{4} - 3m^{2} + 4m^{-1}$

B $2m^{9} - 3m^{2} + 4m$

C $2m^{9} - 3m^{3} + 4m^{-1}$

D $2m^{9} - 3m^{3} + \dfrac{4}{m}$

Answer

44 Simplify $(27abc+9ab+10ac)(6a^2bc)^{-1}$

A $\frac{9}{2a}+\frac{3}{2ac}+\frac{5}{3ab}$

B $-162a^3b^2c^2-54a^3b^2c-60a^3bc^2$

C $-\frac{9}{2a}-\frac{3}{2ac}-\frac{5}{3ab}$

D $\frac{9a}{2}+\frac{3a}{2c}+\frac{5a}{3b}$

Answer

45 The set of polynomials is closed under division.

True

False

Answer

Long Division of Polynomials

To divide a polynomial by 2 or more terms, long division can be used.

Recall long division of numbers, such as 8693 ÷ 41.

$$
\begin{array}{r}
212\ R1 \\
41\overline{)8693} \\
-82 \\
\hline
49 \\
-41 \\
\hline
83 \\
-82 \\
\hline
1
\end{array}
\quad \text{or} \quad 212\frac{1}{41}
$$

- Multiply
- Subtract
- Bring down
- Repeat
- Write Remainder over divisor

Here is an example:

$$2x - 10 + \frac{33}{x+3}$$

$$x + 3 \overline{\big) 2x^2 - 4x + 3}$$

$$\underline{-2x^2 + -6x}$$

$$-10x + 3$$

$$\underline{+10x + 30}$$

$$33$$

- Multiply
- Subtract
- Bring down
- Repeat
- Write Remainder over divisor

On the next several slides, we will break this down step by step....

Start by looking at the first terms. Think "what do I multiply by x to get $2x^2$?"

(Or think "$2x^2$ / x")

Put the answer here

$$x+3\overline{\smash{)}2x^2-4x+3}$$

Continue to next slide.

Next, multiply $2x$ by x + 3, and put the product under the dividend. Then subtract.

$$\begin{array}{r} 2x \\ x+3\overline{)2x^2-4x+3} \\ -() \\ \hline \end{array}$$

Continue to next slide.

Bring down the + 3. Repeat the whole process. This time, ask "what do I multiply by x to get -10x, or what is -10x / x?"

Put the answer here

$$\begin{array}{rl} & 2x \\ x+3 & \overline{\big)\,2x^2-4x+3} \\ & -\left(2x^2+6x\right)\quad\downarrow \\ \hline & -10x \end{array}$$

Continue to next slide.

Multiply and subtract.

$$
\begin{array}{r r}
 & 2x - 10 \\
x+3 & \overline{)\,2x^2 - 4x + 3} \\
 & -\underline{\left(2x^2 + 6x\right)} \\
 & -10x + 3 \\
 & -\underline{\left(-10x - 30\right)} \\
 & 33
\end{array}
$$

Since x doesn't divide into 33, we can't divide further.

33 is the remainder, which we write as a fraction.

$$
\begin{array}{r}
2x - 10 + \dfrac{33}{x+3} \\
x + 3 \overline{\big)\, 2x^2 - 4x + 3} \\
-\left(2x^2 + 6x\right) \\
\hline
-10x + 3 \\
-\left(-10x - 30\right) \\
\hline
33
\end{array}
$$

Examples

$$a-1\overline{)4a^2+3a-2}$$

$$2a+5\overline{)6a^2-3a+2}$$

Teacher Notes

The Remainder Theorem

If a polynomial function $f(x)$, of degree ≥ 1, is divided by $x - a$, then the remainder is equal to $f(a)$.

Example: $f(x) = x^4 + 3x^3 - 2x^2 + 5x + 1$

Find $\dfrac{f(x)}{x-2}$

Complete the division, then calculate $f(2)$.

$$x - 2 \overline{\big)\, x^4 + 3x^3 - 2x^2 + 5x + 1}$$

Answer

Example: $f(m) = 3m^3 + 7m^2 - 4m + 2$

Find the quotient of $(3m^3 + 7m^2 - 4m + 2) \div (m + 2)$

Answer

Find $f(-2)$. What do you notice?

Example: In this example there are "missing terms". Fill in those terms with zero coefficients before dividing.
Then find f(-1).

$$f(t) = (4t^4 - 2)(t + 1)^{-1}$$

click

Answer

Example

$$(4b^3 - 2b + 2) \div (2b + 4)$$

Answer

46 Simplify. $(x^2-3x-41)(x+5)^{-1}$

A $x+10$

B $x-8-\dfrac{1}{x+5}$

C $x-12$

D $x-12-\dfrac{2}{x+5}$

Answer

47 Simplify. $(m^2 - 5m + 12) \div (m - 1)$

A $m - 2 - \dfrac{4}{m-1}$

B $m - 4 + \dfrac{8}{m-1}$

C $m - 3$

D $m - 3 - \dfrac{4}{m-1}$

Answer

48 Divide. $(2x^2 + 18x + 40) \div (x + 5)$

A $2x + 8$

B $2x + 5 + \dfrac{1}{x+5}$

C $2x + 4 + \dfrac{1}{x+5}$

D $2x + 5 - \dfrac{1}{x+5}$

Answer

49 Divide.

$$(n^3 - 6n - 12) \div (n - 4)$$

Answer

50 Divide the polynomial.

$$(2x^3 - 3x^2 - 5x - 12) \div (x - 3)$$

Answer

51 If $(x^2+3x-26)\div(x+7)$ = $x-4+\dfrac{2}{x+7}$,

what is f(-7)?

Answer

52 If $f(1) = 0$ for the function, $f(x) = x^3 + ax^2 - 4x + 3$, what is the value of a?

Answer

53 If $f(3) = 27$ for the function $f(x) = x^3 + ax^2 - 4x + 3$, what is the value of a?

Answer