Solve Quadratic Equations by Using the Quadratic Formula

Return to Table of Contents

Solving Quadratics

At this point you have learned how to solve quadratic equations by:

- graphing
- factoring
- using square roots and
- completing the square

Many quadratic equations may be solved using these methods. Though completing the square works for any quadratic equation, it can be cumbersome to repeatedly use the algorithm.

Today we will be given a tool to solve ANY quadratic equation, and it ALWAYS works!

Completing the Square

Now try Completing the Square on the standard form of a quadratic equation. $ax^2 + bx + c = 0$

Step 1 - Rewrite Equation

Step 2 - Find $(b/2)^2$

Step 3 - Add the result to both sides

Step 4 - Factor & Simplify

Step 5 - Take Square Root of both sides

Step 6 - Write 2 Equations & Solve

Teacher Notes

Completing the Square

$$ax^2 + bx + c = 0$$

Step 1 - Rewrite Equation and factor out *a*

$$a\left(x^2 + \frac{bx}{a}\right) = -c$$

Steps 2 and 3 - Find (b/2) , Add the result to both sides, simplify

$$a\left(x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2\right) = -c + a\left(\frac{b}{2a}\right)^2$$

$$a\left(x^2 + \frac{bx}{a} + \frac{b^2}{4a^2}\right) = -c + a\left(\frac{b^2}{4a^2}\right)$$

Step 4 - Factor & Simplify

$$a\left(x + \frac{b}{2a}\right)^2 = -c + \left(\frac{b^2}{4a}\right)$$

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a}$$

Step 5 - Take Square Root of both sides

$$\left(x + \frac{b}{2a}\right)^2 = \left(\frac{1}{a}\right)\left(\frac{-4ac + b^2}{4a}\right) = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

Step 6 - Solve for x

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Quadratic Formula

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Teacher Notes

Quadratic Formula

Example 1: Solve $2x^2 + 3x - 5 = 0$

Once you identify the values of a, b, and c, simply substitute into the quadratic formula and simplify as much as possible.

$a = 2\ b = 3\ c = -5$

Answer

How can you check your answers?

Quadratic Formula

Example 2: Solve: $2x = x^2 - 3$

- To use the Quadratic Formula, the equation must be in standard form ($ax^2 + bx + c = 0$).
- Identify a, b, and c, then substitue into the formula and simplify.
- Don't forget to check your results!

Answer

44 Solve the following equation using the quadratic formula:

$$x^2 - 5x + 4 = 0$$

Answer

A	-5	F	1
B	-4	G	2
C	-3	H	3
D	-2	I	4
E	-1	J	5

45 Solve the following equation using the quadratic formula:

$$x^2 = x + 20$$

Answer

A -5

B -4

C -3

D -2

E -1

F 1

G 2

H 3

I 4

J 5

46 Solve the following equation using the quadratic formula:

$$2x^2 + 12 = 11x$$

Answer

A -5

B -4

C -3

D -2

E -1.5

F 1.5

G 2

H 3

I 4

J 5

Quadratic Formula

Example 3: Solve using the quadratic formula, and simplify the result. $x^2 - 2x - 4 = 0$

Answer

47 Find the larger solution to the equation.

$$x^2 + 6x - 1 = 0$$

Answer

48 Find the smaller solution to the equation.

$$x^2 + 6x - 1 = 0$$

Answer

Which Method

Work in small groups to solve the quadratic equation using the following different methods.

$$2x^2 + 5x - 3 = 0$$

Factoring	Quadratic Formula	Completing the Square	Graphing

Teacher Notes

Which Method

Work in small groups to solve the quadratic equation using the following different methods.

$$2x^2 + 6x + 7 = 0$$

Factoring	Quadratic Formula	Completing the Square	Graphing

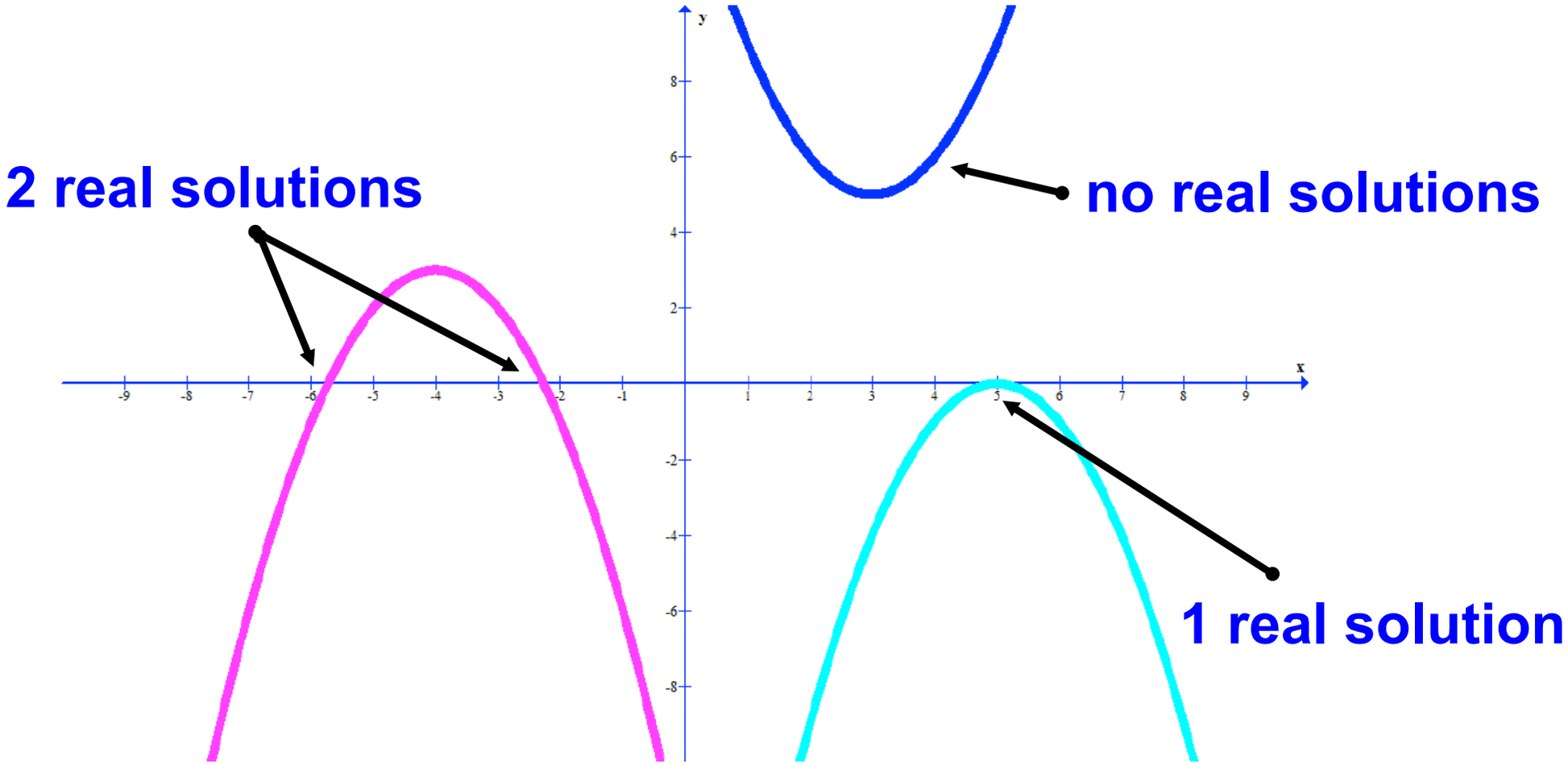
Teacher Notes

The Discriminant

Return to Table of Contents

Solutions

Recall what it means to have 0, 1, or 2 solutions/zeros/roots



The Discriminant

At times, it is not necessary to solve for the zeros or roots of a quadratic function, but simply to know how many roots exist (zero, one, or two).

The quickest way to determine how many solutions a quadratic has, algebraically, is to calculate what'scalled the discriminant.

It may look familiar, as the discriminant is a part of the quadratic formula.

The Discriminant

Discriminant: the radicand in the Quadratic Formula (the piece of the equation under the radical sign). Note, it does NOT include the radical sign.

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant: $b^2 - 4ac$

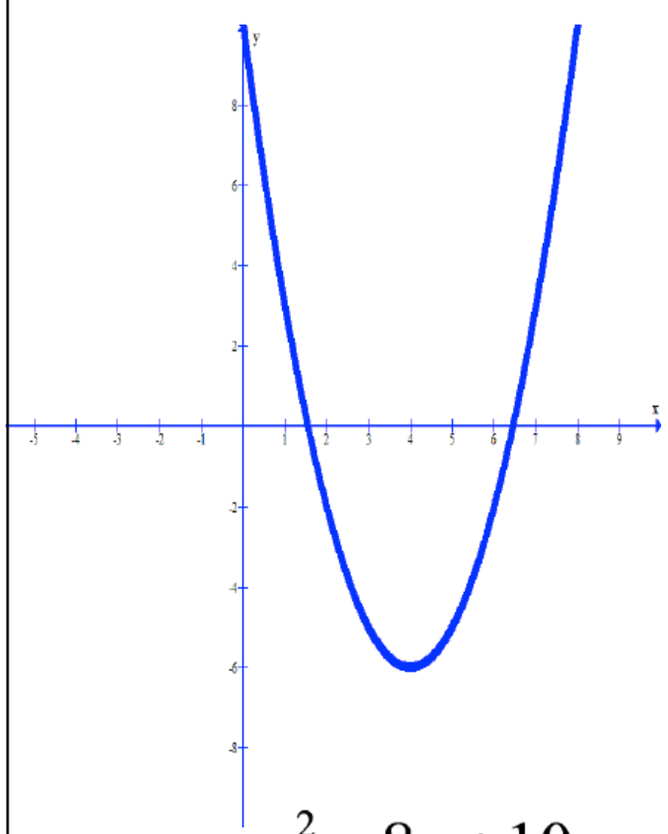
The Discriminant

Other important tips before practice:

- The square root of a positive number has two solutions.
- The square root of zero is 0.
- The square root of a negative number has no real solution.

The Discriminant

Examine the following graphs and determine the relationship between the discriminant of a quadratic and its graph.



$y = x^2 - 8x + 10$

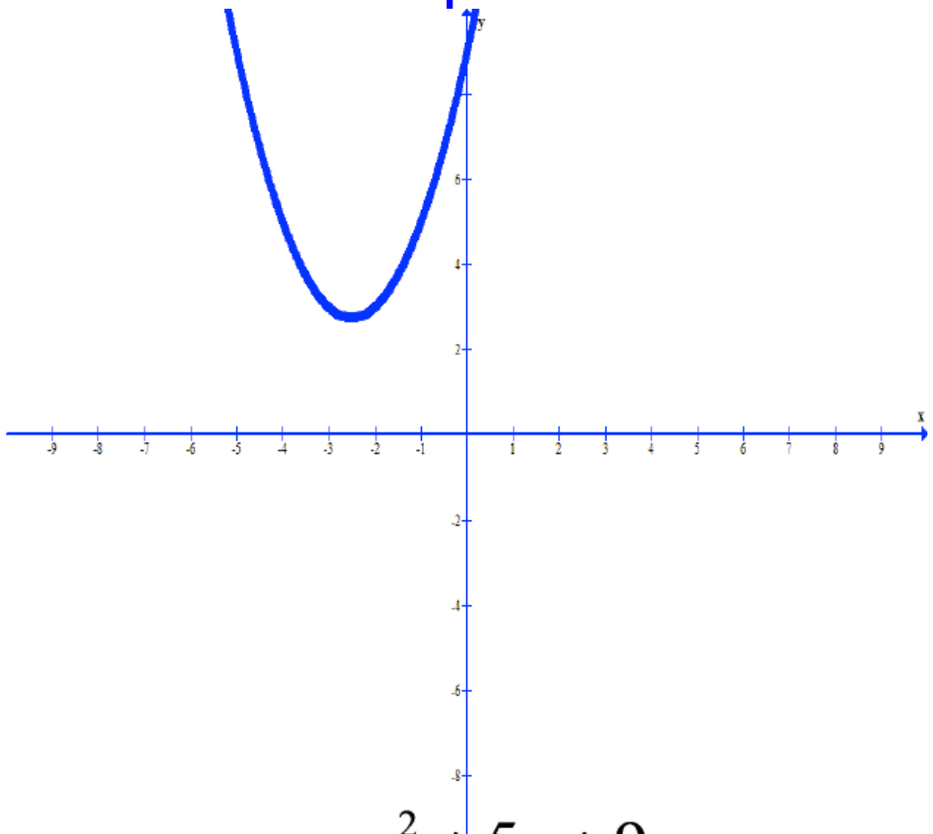
Discriminant

$b^2 - 4ac$

$= (8)^2 - 4(1)(10)$

$= 64 - 40$

$= 24$



$y = x^2 + 5x + 9$

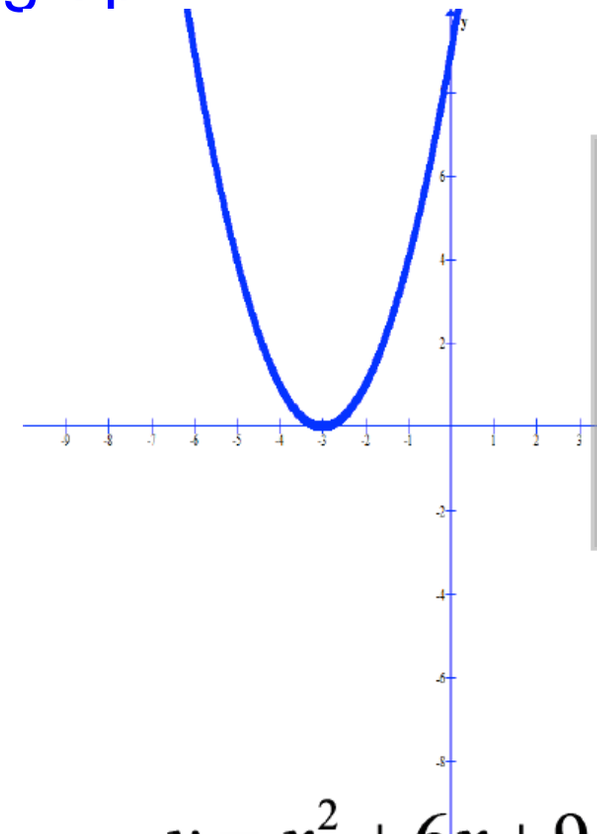
Discriminant

$b^2 - 4ac$

$= (5)^2 - 4(1)(9)$

$= 25 - 36$

$= -11$



$y = x^2 + 6x + 9$

Discriminant

$b^2 - 4ac$

$= (6)^2 - 4(1)(9)$

$= 36 - 36$

$= 0$

Teacher Notes

The Discriminant

CONCLUSION:

click to reveal

49 What is the value of the discriminant of $2x^2 - 2x + 3 = 0$?

Answer

50 Use the discriminant to find the number of solutions for $2x^2 - 2x + 3 = 0$

A 0

B 1

C 2

Answer

51 What is the value of the discriminant of $x^2 - 8x + 4 = 0$?

Answer

52 Use the discriminant to find the number of solutions for $x^2 - 8x + 4 = 0$

A 0

B 1

C 2

Answer