NAME _____ DATE _____ PERIOD _____

7-5 Study Guide and Intervention *(continued)*

Operations with Radical Expressions

Operations with Radicals When you add expressions containing radicals, you can add only like terms or **like radical expressions.** Two radical expressions are called *like radical expressions* if both the indices and the radicands are alike.

To multiply radicals, use the Product and Quotient Properties. For products of the form $(a\sqrt{b} + c\sqrt{d}) \cdot (e\sqrt{f} + g\sqrt{h})$, use the FOIL method. To rationalize denominators, use **conjugates.** Numbers of the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$, where a, b, c, and d are rational numbers, are called conjugates. The product of conjugates is always a rational number.

Example 1 **Simplify $2\sqrt{50} + 4\sqrt{500} - 6\sqrt{125}$.**

$$\begin{aligned} 2\sqrt{50} + 4\sqrt{500} - 6\sqrt{125} &= 2\sqrt{5^2 \cdot 2} + 4\sqrt{10^2 \cdot 5} - 6\sqrt{5^2 \cdot 5} && \text{Factor using squares.} \\ &= 2 \cdot 5 \cdot \sqrt{2} + 4 \cdot 10 \cdot \sqrt{5} - 6 \cdot 5 \cdot \sqrt{5} && \text{Simplify square roots.} \\ &= 10\sqrt{2} + 40\sqrt{5} - 30\sqrt{5} && \text{Multiply.} \\ &= 10\sqrt{2} + 10\sqrt{5} && \text{Combine like radicals.} \end{aligned}$$

Example 2 **Simplify $(2\sqrt{3} - 4\sqrt{2})(\sqrt{3} + 2\sqrt{2})$.**

$$\begin{aligned} &(2\sqrt{3} - 4\sqrt{2})(\sqrt{3} + 2\sqrt{2}) \\ &= 2\sqrt{3} \cdot \sqrt{3} + 2\sqrt{3} \cdot 2\sqrt{2} - 4\sqrt{2} \cdot \sqrt{3} - 4\sqrt{2} \cdot 2\sqrt{2} \\ &= 6 + 4\sqrt{6} - 4\sqrt{6} - 16 \\ &= -10 \end{aligned}$$

Example 3 **Simplify $\dfrac{2 - \sqrt{5}}{3 + \sqrt{5}}$.**

$$\begin{aligned} \frac{2 - \sqrt{5}}{3 + \sqrt{5}} &= \frac{2 - \sqrt{5}}{3 + \sqrt{5}} \cdot \frac{3 - \sqrt{5}}{3 - \sqrt{5}} \\ &= \frac{6 - 2\sqrt{5} - 3\sqrt{5} + (\sqrt{5})^2}{3^2 - (\sqrt{5})^2} \\ &= \frac{6 - 5\sqrt{5} + 5}{9 - 5} \\ &= \frac{11 - 5\sqrt{5}}{4} \end{aligned}$$

Exercises

Simplify.

1. $3\sqrt{2} + \sqrt{50} - 4\sqrt{8}$

2. $\sqrt{20} + \sqrt{125} - \sqrt{45}$

3. $\sqrt{300} - \sqrt{27} - \sqrt{75}$

4. $\sqrt[3]{81} \cdot \sqrt[3]{24}$

5. $\sqrt[3]{2}\left(\sqrt[3]{4} + \sqrt[3]{12}\right)$

6. $2\sqrt{3}(\sqrt{15} + \sqrt{60})$

7. $(2 + 3\sqrt{7})(4 + \sqrt{7})$

8. $(6\sqrt{3} - 4\sqrt{2})(3\sqrt{3} + \sqrt{2})$

9. $(4\sqrt{2} - 3\sqrt{5})(2\sqrt{20} + 5)$

10. $\dfrac{5\sqrt{48} + \sqrt{75}}{5\sqrt{3}}$

11. $\dfrac{4 + \sqrt{2}}{2 - \sqrt{2}}$

12. $\dfrac{5 - 3\sqrt{3}}{1 + 2\sqrt{3}}$