Adding and Subtracting Radicals

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Adding and Subtracting Radicals

*Note: When adding or subtracting radicals, you do not add or subtract the radicands (the inside).

Consider: $\sqrt{4+9} \neq \sqrt{4} + \sqrt{9}$

$$\sqrt{13} \neq 2+3$$

$$\sqrt{13} \neq 5$$

Adding and Subtracting Radicals

To add and subtract radicals they must be like terms.

Radicals are like terms if they have the same radicands and the same indexes.

Like Terms

$\sqrt{2}, \sqrt{2}$

$\sqrt{5}, 6\sqrt{5}$

$\sqrt[3]{7}, 4\sqrt[3]{7}$

Unlike Terms

$\sqrt{5}, \sqrt{3}$

$\sqrt{2}, 2$

$\sqrt{10}, \sqrt[3]{10}$

Adding and Subtracting Radicals

An index indicates what root you are taking. Just like square roots undo squares, cube roots undo cubes, fourth roots undo powers of four, fifth roots undo powers of 5, etc... This concept will be studied more in depth later in the unit.

$\sqrt{2^2} = 2$ $\sqrt[3]{2^3} = 2$ $\sqrt[4]{2^4} = 2$ $\sqrt[5]{2^5} = 2$

51 Identify all of the pairs of like terms:

A $\sqrt{2}, \sqrt{3}$

B $\sqrt{5}, 6\sqrt{5}$

C $7\sqrt{10}, 10\sqrt{7}$

D $3\sqrt{2}, \sqrt[3]{2}$

E $\sqrt[3]{5}, \sqrt[4]{5}$

F $4\sqrt[5]{6}, 5\sqrt[5]{6}$

Answer

Adding and Subtracting Radicals

To add or subtract radicals, only the coefficients of the like terms are combined - just like 3x + 4x = 7x.

$6\sqrt{5}+3\sqrt{5}$ $\qquad$ $5\sqrt{7}-4\sqrt{7}$ $\qquad$ $10\sqrt{2}+10\sqrt{3}$

Teacher Notes

Adding and Subtracting Radicals

Try...

$\sqrt{3}+\sqrt{3}$ $\quad$ $5\sqrt{2}-6\sqrt[3]{2}$ $\quad$ $4\sqrt{3}+5\sqrt{2}+6\sqrt{3}$ $\quad$ $3\sqrt{2}+4\sqrt{2}-5\sqrt{2}$

Teacher Notes

Adding and Subtracting Radicals

It is the same for expressions containing variables. Simplify:

$$3\sqrt{x} - 4\sqrt{x} + \sqrt{x}$$

$$10\sqrt{p} - 4\sqrt{q} + 3\sqrt{p}$$

Teacher Notes

52 Simplify: $4\sqrt{11}+5\sqrt{11}$

A $1\sqrt{22}$

B $9\sqrt{11}$

C $9\sqrt{22}$

D Already Simplified

Answer

53 Simplify: $3\sqrt{3}+2\sqrt{2}$

A $5\sqrt{6}$

B $5\sqrt{5}$

C $6\sqrt{6}$

D Already Simplified

Answer

54 Simplify: $6\sqrt{7x}-8\sqrt{7x}$

A $14\sqrt{7x}$

B $2\sqrt{7x}$

C $-2\sqrt{7x}$

D Already Simplified

Answer

55 Simplify: $6x\sqrt{3}+5x\sqrt{3}-2x\sqrt{3}$

A $13x\sqrt{3}$

B $9x\sqrt{3}$

C $11x\sqrt{3}-2\sqrt{3}$

D Already Simplified

Answer

56 Simplify: $5\sqrt{3p}-4\sqrt{2p}-2\sqrt{3p}$

A $3\sqrt{3p}-4\sqrt{2p}$

B $3\sqrt{3p}$

C $10\sqrt{3p}-4\sqrt{2p}$

D Already Simplified

Answer

Adding and Subtracting Radicals

Some irrational radicals will not be like terms, but could be put in simplest radical form. In theses cases, simplify, then collect any like terms.

$$\sqrt{12}-\sqrt{3}$$

$$\sqrt{8}+3\sqrt{2}-5\sqrt{24}$$

Teacher Notes

Adding and Subtracting Radicals

The same goes for expressions containing variables. Try:

$$4x\sqrt{x^3} - 2x^2\sqrt{x}$$

$$7y\sqrt{y} - 9\sqrt{y^3}$$

Teacher Notes

57 Simplify: $2\sqrt{3}+4\sqrt{27}$

A $3\sqrt{30}$

B $5\sqrt{3}$

C $14\sqrt{3}$

D Already in simplest form

Answer

58 Simplify: $5\sqrt{8}-4\sqrt{18}$

A $2\sqrt{2}$

B $\sqrt{10}$

C $-2\sqrt{2}$

D Already in simplest form

Answer

59 Simplify: $5\sqrt{6x^2}+3|x|\sqrt{12}-3|x|\sqrt{24}+4\sqrt{3x^2}$

A $7|x|\sqrt{3}+|x|\sqrt{6}$

B $10|x|\sqrt{3}-|x|\sqrt{6}$

C $8|x|\sqrt{3}$

D Already in simplest form

Answer

60 Simplify: $2\sqrt{3}+4\sqrt[3]{3}-3\sqrt{2}$

A $4\sqrt[3]{3}$

B $6\sqrt{3}-3\sqrt{2}$

C $3\sqrt{3}$

D Already in simplest form

Answer

61 Simplify: $\sqrt{8x^3y^4}+y^2\sqrt{128x^3}-\sqrt{98x^3y^4}-y^2\sqrt{12x^3}$

A $xy^2\sqrt{2x}$

B $3xy^2\sqrt{2x}-2xy^2\sqrt{3x}$

C $17xy^2\sqrt{2x}-2xy^2\sqrt{3x}$

D Already simplified

Answer

62 Simplify: $\sqrt{8a^4} - 3\sqrt{2a^4} + 6a\sqrt{32a^2}$

A $5a\sqrt{2a}$

B $28a\sqrt{2a}$

C $23a^2\sqrt{2}$

D Already simplified

Answer

Multiplying Radicals

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Multiplying Radicals

When multiplying radicals, you may multiply radicands.

Consider... $\sqrt{4}\cdot\sqrt{9}=\sqrt{36}$

$$2\cdot 3 = 6$$

Multiplying Radicals

Whole number times whole number and radical times radical. Never multiply a whole number and radical! Leave all answers in simplest radical form.

$$(a\sqrt{b})(c\sqrt{d}) = ac\sqrt{bd}$$

$$\left(3\sqrt{5}\right)\left(6\sqrt{7}\right)$$

$$4x\sqrt{2}(7x\sqrt{3})$$

Teacher Notes

Multiplying Radicals

Examples:

$$\left(-6\sqrt{7}\right)\left(\sqrt{10}\right)$$

$$5(4\sqrt{3})$$

Teacher Notes

Multiplying Radicals

Examples:

$\left(5\sqrt{2}\right)\left(4\sqrt{6}\right)$ $\left(10\sqrt{7}\right)\left(\sqrt{7}\right)$ $-3\sqrt{10}\cdot 4\sqrt{15}$ $\left(4\sqrt{x^2}\right)\left(3\sqrt{y^3}\right)$

Teacher Notes

63 Multiply: $\left(6\sqrt{3}\right)\left(8\sqrt{7}\right)$

A $14\sqrt{10}$

B $48\sqrt{21}$

C $42\sqrt{24}$

D $24\sqrt{42}$

Answer

64 Simplify: $\left(3\sqrt{6}\right)\left(2\sqrt{2}\right)$

A $6\sqrt{12}$

B $12\sqrt{6}$

C $12\sqrt{3}$

D $6\sqrt{2}$

Answer

65 Simplify: $\left(3\sqrt{6}\right)\left(2\sqrt{3}\right)$

A $6\sqrt{18}$

B $18\sqrt{6}$

C $6\sqrt{2}$

D $18\sqrt{2}$

Answer

66 Simplify: $\left(4y\sqrt{x^2}\right)\left(-5\sqrt{8}\right)$

A $-20xy\sqrt{8}$

B $-20|xy|\sqrt{8}$

C $-20|x|y\sqrt{8}$

D $-40|x|y\sqrt{2}$

Answer

67 Simplify: $\left(3\sqrt{6}\right)\left(6\sqrt{5}\right)$

A $18\sqrt{30}$

B $36\sqrt{15}$

C $54\sqrt{10}$

D $108\sqrt{5}$

Answer

Multiplying Polynomials with Radicals

Leave all answers in simplest radical form

$$5\sqrt{3}\left(3\sqrt{6}-4\sqrt{5}\right)$$

$$\left(2+3\sqrt{5}\right)\left(3-4\sqrt{2}\right)$$

$$\left(4+2\sqrt{3}\right)\left(4-2\sqrt{3}\right)$$

$$\left(2+\sqrt{2}\right)^2$$

Teacher Notes

68 Multiply and write in simplest form: $9\sqrt{3}\left(2-5\sqrt{6}\right)$

A $9\sqrt{6}-45\sqrt{18}$

B $18\sqrt{3}-45\sqrt{18}$

C $18\sqrt{3}-135\sqrt{2}$

D $9\sqrt{6}-135\sqrt{2}$

Answer

69 Multiply and write in simplest form: $3\sqrt{2}\left(4\sqrt{2}-5\sqrt{6}\right)$

A $12\sqrt{2}-15\sqrt{12}$

B $12\sqrt{4}-15\sqrt{12}$

C $24-15\sqrt{12}$

D $24-30\sqrt{2}$

Answer

70 Multiply and write in simplest form: $\left(2+4\sqrt{3}\right)\left(5+2\sqrt{2}\right)$

A $10+8\sqrt{6}$

B $10+20\sqrt{3}+4\sqrt{2}+8\sqrt{6}$

C $10+24\sqrt{5}+8\sqrt{6}$

D $10+32\sqrt{11}$

Answer

71 Multiply and write in simplest form: $\left(3-4\sqrt{2}\right)\left(5+3\sqrt{2}\right)$

A $15-12\sqrt{2}$

B $15-7\sqrt{2}$

C $-9+12\sqrt{2}$

D $-9-11\sqrt{2}$

Answer

72 Multiply and write in simplest form: $\left(1+6\sqrt{5}\right)^2$

A 181

B $181+14\sqrt{5}$

C $1+36\sqrt{5}$

D $181+12\sqrt{5}$

Answer

Rationalizing the Denominator

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Rationalizing the Denominator

Teacher Notes

Mathematicians don't like radicals in the denominators of fractions.

When there is one, the denominator is said to be irrational. The method used to rid the denominator is termed "rationalizing the denominator".

Which of these has a rational denominator?

$$\frac{2}{\sqrt{3}} \qquad \frac{6}{\sqrt{7}} \qquad \frac{5}{4} \qquad \frac{-6}{7} \qquad \frac{5}{-3}$$

Rational Denominator	Irrational Denominator

Rationalizing the Denominator

If the denominator is a monomial, to rationalize, just multiply top and bottom of the fraction by the root part of the denominator.

Examples:

$$\frac{5}{\sqrt{3}} \qquad \frac{14}{5\sqrt{7}} \qquad \frac{4\sqrt{2}}{3\sqrt{5}} \qquad \frac{2\sqrt{3}}{\sqrt{6}}$$

Teacher Notes

Rationalizing the Denominator

If a denominator is a binomial with a root, rationalize the denominator by multiplying top and bottom by finding itsconjugate . The conjugate of a binomial is found by negating the second term of a binomial.

Binomial:	Conjugate:
$4-2\sqrt{5}$	$4+2\sqrt{5}$
$3+\sqrt{3}$	$3-\sqrt{3}$
$1+4\sqrt{2}$	$1-4\sqrt{2}$
$2-3\sqrt{7}$	$2+3\sqrt{7}$

Rationalizing the Denominator

Multiplying by the conjugate turns an irrational number into a rational number.

Check out what happens...

$(3+\sqrt{3})(3-\sqrt{3})$ $(1+4\sqrt{2})(1-4\sqrt{2})$ $(2-3\sqrt{7})(2+3\sqrt{7})$

Teacher Notes

Rationalizing the Denominator

Do you see a pattern that let's us go from line 1 to line 3 directly?

Example

$$\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)$$

$$4+2\sqrt{3}-2\sqrt{3}-\sqrt{3^2}$$

$$4-3$$

$$1$$

Example

$$\left(4+\sqrt{5}\right)\left(4-\sqrt{5}\right)$$

$$16-4\sqrt{5}+4\sqrt{5}-\sqrt{5^2}$$

$$16-5$$

$$11$$

Example

$$\left(\sqrt{6}-\sqrt{7}\right)\left(\sqrt{6}+\sqrt{7}\right)$$

$$\sqrt{6^2}+\sqrt{42}-\sqrt{42}-\sqrt{7^2}$$

$$6-7$$

$$-1$$

Teacher Notes

Rationalizing the Denominator

Use conjugates to rationalize the denominators:

$$\frac{-5}{1-\sqrt{7}}$$

$$\frac{3}{2+\sqrt{3}}$$

Teacher Notes

Rationalizing the Denominator

Use conjugates to rationalize the denominators:

$$\frac{2-\sqrt{3}}{4-3\sqrt{2}}$$

$$\frac{5-2\sqrt{6}}{3+4\sqrt{6}}$$

Teacher Notes

73 What is conjugate of $6-2\sqrt{5}$?

A $6-2\sqrt{5}$

B $6+2\sqrt{5}$

C $-\sqrt{5}$

D $\sqrt{5}$

Answer

74 What is conjugate of $\sqrt{6}+\sqrt{5}$?

A $\sqrt{6}-\sqrt{5}$

B $\sqrt{6}+\sqrt{5}$

C $-\sqrt{30}$

D $\sqrt{30}$

Answer

75 Simplify: $\frac{2}{\sqrt{3}}$

A $\frac{2\sqrt{3}}{3}$

B $\frac{\sqrt{6}}{3}$

C $\sqrt{2}$

D Already simplified

Answer

76 Simplify: $\frac{\sqrt{2}}{\sqrt{3}}$

A $\frac{2}{3}$

B $\frac{\sqrt{6}}{3}$

C $\frac{2\sqrt{3}}{3}$

D Already simplified

Answer

77 Simplify: $\frac{\sqrt{2}}{3}$

A $\frac{2}{3}$

B $\frac{\sqrt{6}}{3}$

C $\frac{3\sqrt{2}}{3}$

D Already simplified

Answer

78 Simplify: $\dfrac{\sqrt{2}}{3+\sqrt{2}}$

A $\dfrac{-2+3\sqrt{2}}{5}$

B $3\sqrt{2}-2$

C $\dfrac{-2+3\sqrt{2}}{7}$

D Already simplified

Answer

79 Simplify: $\dfrac{6-\sqrt{3}}{5+\sqrt{3}}$

A $\dfrac{30-6\sqrt{3}}{22}$

B $\dfrac{33-11\sqrt{3}}{22}$

C $\dfrac{3-\sqrt{3}}{28}$

D Already simplified

Answer

80 Simplify: $\dfrac{3-4\sqrt{2}}{1-3\sqrt{2}}$

A $\dfrac{27-5\sqrt{2}}{19}$

B $\dfrac{-21-13\sqrt{2}}{-17}$

C $\dfrac{21-5\sqrt{2}}{17}$

D Already simplified

Answer

81 Simplify: $\dfrac{2-\sqrt{3}}{1-\sqrt{3}}$

A $\dfrac{1+\sqrt{3}}{2}$

B $\dfrac{1-\sqrt{3}}{2}$

C $-\dfrac{\sqrt{3}}{2}$

D Already simplified

Answer

Rationalizing n^{th} roots of Monomials

Remember that $\sqrt[n]{x^n} = x$, given an n^{th} root in the denominator, it will need to be rationalized. To rationalize, find the complement if the n^{th} root that will create a perfect root in the denominator. Multiply top and bottom by the complement. Simplify.

Examples:

$$\frac{1}{\sqrt[5]{x^3}} \qquad \sqrt[6]{\frac{4}{9}}$$

Teacher Notes

Rationalizing nth roots of Monomials

Try:

$$\frac{\sqrt[7]{2p^4}}{\sqrt[7]{p^2}}$$

$$\sqrt[5]{\frac{n^4}{4m}}$$

Teacher Notes

106 Rationalize: $\dfrac{1}{\sqrt[3]{2}}$

A $\dfrac{\sqrt[3]{2}}{2}$

B $\dfrac{\sqrt[3]{4}}{2}$

C $\dfrac{\sqrt{2}}{2}$

D $\sqrt[3]{4}$

Answer

107 Rationalize: $\dfrac{6}{\sqrt[4]{27}}$

A $\dfrac{6\sqrt[4]{3}}{3}$

B $\dfrac{2\sqrt[4]{3}}{3}$

C $2\sqrt{3}$

D $2\sqrt[3]{3}$

Answer

108 Rationalize: $\dfrac{6}{\sqrt[5]{9x^4}}$

A $\dfrac{\sqrt[5]{27x}}{3x}$

B $\dfrac{2\sqrt[5]{27x}}{x}$

C $2\sqrt{27x}$

D $2\sqrt[5]{27}$

Answer

109 Rationalize: $\dfrac{4}{\sqrt[3]{12}}$

A $\dfrac{\sqrt[3]{12}}{12}$

B $\dfrac{\sqrt[3]{48}}{12}$

C $\dfrac{2\sqrt[3]{18}}{3}$

D $\dfrac{\sqrt[3]{18}}{3}$

Answer

110 Simplify: $\sqrt[4]{\dfrac{2}{25}}$

A $\dfrac{\sqrt[4]{50}}{2}$

B $\sqrt[4]{25}$

C $\sqrt[4]{10}$

D $\dfrac{\sqrt[4]{50}}{5}$

Answer

111 Rationalize: $\frac{\sqrt[3]{5x^2}}{\sqrt[3]{25x}}$

A $\sqrt[3]{5x}$

B $\frac{\sqrt[3]{25x}}{5x}$

C $\frac{x\sqrt[3]{25x}}{5}$

D $\frac{\sqrt[3]{25x}}{5}$

Answer

112 Simplify: $\sqrt[5]{\dfrac{x^2}{2y^3}}$

A $\dfrac{\sqrt[5]{16x^2y^2}}{2y}$

C $\dfrac{\sqrt[5]{8x^2y^2}}{y}$

B $\dfrac{\sqrt[5]{x^2y^2}}{2y}$

D $4\sqrt[5]{x^2y^2}$

Answer