The Rational Zeros Theorem:

Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

with integer coefficients. There is a limited number of possible roots or zeros.

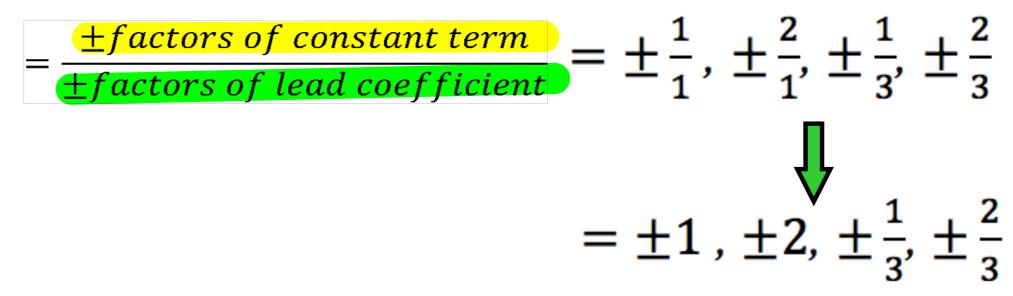
- Integer zeros must be factors of the constant term, a_0 .
- Rational zeros can be found by writing and simplifying fractions where the numerator is an integer factor of *a*₀ and the denominator is an integer fraction of *a_n*.

RATIONAL ZEROS THEOREM

Make list of POTENTIAL rational zeros and test them out.

$$3x^3 + 4x^2 - 5x - 2 = 0$$

Potential List:

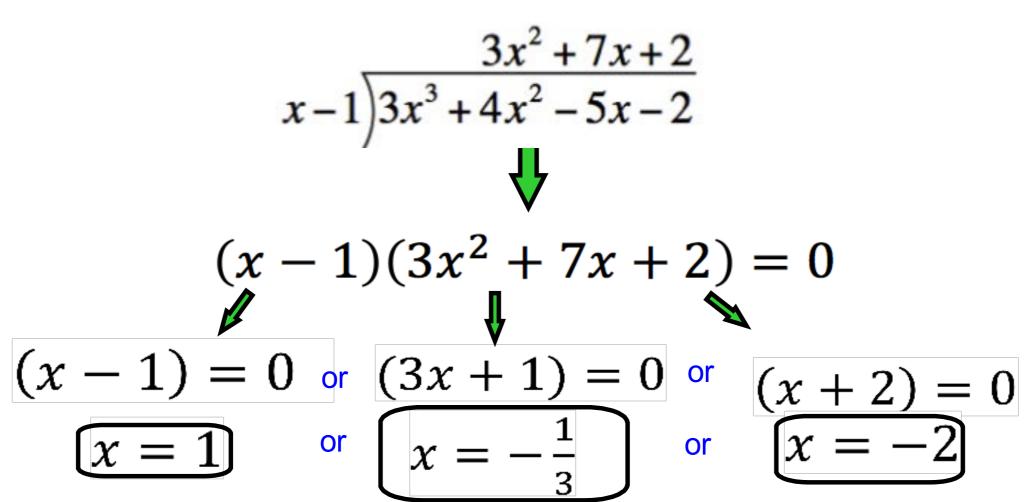


Hint: To check for zeros, first try the smaller integers -- they are easier to work with.

Using the Remainder Theorem, we find that 1 is a zero:

$$3(1)^3 + 4(1)^2 - 5(1) - 2 = 0$$

therefore (x -1) is a factor of the polynomial. Use POLYNOMIAL DIVISION to factor out.

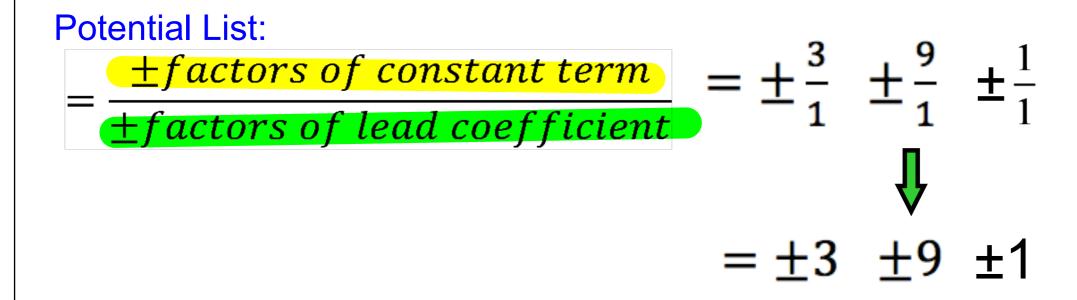


This polynomial has three distinct real zeros: -2, -1/3, and 1, each with a multiplicity of 1. There are no imaginary zeros.

Teacher Notes

Find the zeros using the Rational Zeros Theorem, showing the multiplicities, of the following polynomial.

$$x^3 + 7x^2 + 15x + 9 = 0$$



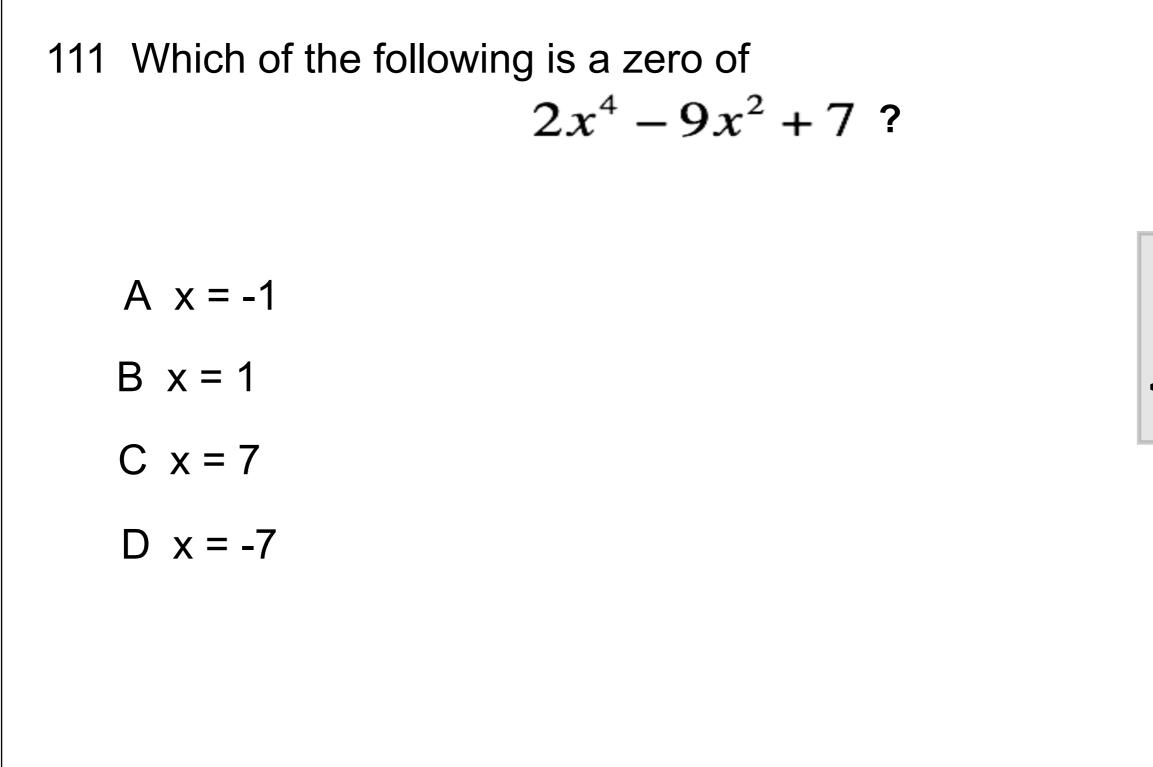
Hint: since all of the signs in the polynomial are +, only negative numbers will work. Try -3:

$$(-3)^3 + 7(-3)^2 + 15(-3) + 9 = 0$$

-3 is a distinct zero, therefore (x + 3) is a factor. Use POLYNOMIAL DIVISION to factor out.

$$\begin{array}{c}
x^{2} + 4x + 3 \\
x + 3 \\
x^{3} + 7x^{2} + 15x + 9 \\
(x + 3)(x^{2} + 4x + 3) = 0 \\
(x + 3) = 0 \text{ or } (x + 3) = 0 \text{ or } (x + 1) = 0 \\
\hline
x = -3 \text{ or } x = -3 \text{ or } x = -1
\end{array}$$

This polynomial has two distinct real zeros: -3, and -1. -3 has a multiplicity of 2 (there are 2 factors of x + 3). -1 has a multiplicity of 1. There are no imaginary zeros.



Answer

- 112 Find the zeros of the polynomial equation, including multiplicities, using the Rational Zeros Theorem $x^3 + x^2 5x + 3$.
 - A x = 1, multiplicity 1
 - B x = 1, mulitplicity 2
 - C x = 1, multiplicity 3
 - D x = -3, multiplicity 1
 - E x = -3, multiplicity 2
 - F x = -3, multiplicity 3

- 113 Find the zeros of the polynomial equation, including multiplicities, using the Rational Zeros Theorem $x^3 + 4x^2 + 5x + 2$.
 - A x = -2, multiplicity 1
 - B x = -2, multiplicity 2
 - C x = -2, multiplicity 3
 - D x = -1, multiplicity 1
 - E x = -1, multiplicity 2
 - F x = -1, multiplicity 3

114 Find the zeros of the polynomial equation, including multiplicities, using the Rational Zeros Theorem

$$6x^3 - 17x^2 - 4x + 3$$

- A x = 1, multiplicity 1 B x = -1, multiplicity 1 E x = $\frac{1}{3}$, F x = $-\frac{1}{3}$
 - C x = 3, multiplicity 1

D x = -3, multiplicity 1

E x =
$$\frac{1}{3}$$
, multiplicity 1
F x = $-\frac{1}{3}$, multiplicity 1
G x = $\frac{1}{2}$, multiplicity 1
H x = $-\frac{1}{2}$, multiplicity 1

1

Answer

115 Use the Rational Zeros Theorem to find the zeros of the polynomial equation, including multiplicities.

 $4x^4 - 20x^3 + 13x^2 + 30x + 9 = 0$

- A x = 3, mulitplicity 1
- B x = 2, mulitplicity 2
- C x = 3, multiplicity 2
- D x = -2, multiplicity 1
- E x = $\frac{1}{2}$, multiplicity 1 F x = $-\frac{1}{2}$, multiplicity 2

116 Find the zeros of the polynomial equation.

$$x^4 + 7x^2 - 18 = 0$$

A x = 2B x = -2C x = 3 D x = -3E x = 3iF x = -3iG x = $\sqrt{2}$ H x = $-\sqrt{2}$