The Rational Zeros Theorem:

Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

with integer coefficients. There is a limited number of possible roots or zeros.

- Integer zeros must be factors of the constant term, a_0.
- Rational zeros can be found by writing and simplifying fractions where the numerator is an integer factor of a_0 and the denominator is an integer fraction of a_n.

RATIONAL ZEROS THEOREM

Make list of POTENTIAL rational zeros and test them out.

$$3x^3 + 4x^2 - 5x - 2 = 0$$

Potential List:

$$= \frac{\pm factors\ of\ constant\ term}{\pm factors\ of\ lead\ coefficient} = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{1}{3}, \pm\frac{2}{3}$$

↓

$$= \pm 1\ , \pm 2, \pm\frac{1}{3}, \pm\frac{2}{3}$$

Hint: To check for zeros, first try the smaller integers -- they are easier to work with.

Using the Remainder Theorem, we find that 1 is a zero:

$$3(1)^3 + 4(1)^2 - 5(1) - 2 = 0$$

therefore (x -1) is a factor of the polynomial.
Use POLYNOMIAL DIVISION to factor out.

$$\begin{array}{r} 3x^2+7x+2 \\ x-1\overline{)3x^3+4x^2-5x-2} \end{array}$$

$$\Downarrow$$

$$(x-1)(3x^2+7x+2) = 0$$

$$(x-1) = 0 \quad \text{or} \quad (3x+1) = 0 \quad \text{or} \quad (x+2) = 0$$

$$\boxed{x = 1} \quad \text{or} \quad \boxed{x = -\frac{1}{3}} \quad \text{or} \quad \boxed{x = -2}$$

This polynomial has three distinct real zeros: -2, -1/3, and 1, each with a multiplicity of 1.
There are no imaginary zeros.

Teacher Notes

Find the zeros using the Rational Zeros Theorem, showing the multiplicities, of the following polynomial.

$$x^3+7x^2+15x+9=0$$

Potential List:

$$=\frac{\pm factors\ of\ constant\ term}{\pm factors\ of\ lead\ coefficient}=\pm\frac{3}{1}\quad\pm\frac{9}{1}\quad\pm\frac{1}{1}$$

⇩

$$=\pm 3\quad \pm 9\quad \pm 1$$

Hint: since all of the signs in the polynomial are +, only negative numbers will work. Try -3:

$$(-3)^3+7(-3)^2+15(-3)+9=0$$

-3 is a distinct zero, therefore (x + 3) is a factor.
Use POLYNOMIAL DIVISION to factor out.

$$\begin{array}{r} x^2+4x+3 \\ x+3\overline{)x^3+7x^2+15x+9} \end{array}$$

$$(x+3)(x^2+4x+3)=0$$

$(x+3)=0$ or $(x+3)=0$ or $(x+1)=0$

$\boxed{x=-3}$ or $\boxed{x=-3}$ or $\boxed{x=-1}$

This polynomial has two distinct real zeros: -3, and -1.
-3 has a multiplicity of 2 (there are 2 factors of x + 3).
-1 has a multiplicity of 1.
There are no imaginary zeros.

111 Which of the following is a zero of

$$2x^4 - 9x^2 + 7$$?

A x = -1

B x = 1

C x = 7

D x = -7

Answer

112 Find the zeros of the polynomial equation, including multiplicities, using the Rational Zeros Theorem

$x^3 + x^2 - 5x + 3.$

A x = 1, multiplicity 1

B x = 1, mulitplicity 2

C x = 1, multiplicity 3

D x = -3, multiplicity 1

E x = -3, multiplicity 2

F x = -3, multiplicity 3

Answer

113 Find the zeros of the polynomial equation, including multiplicities, using the Rational Zeros Theorem

$$x^3 + 4x^2 + 5x + 2.$$

A x = -2, multiplicity 1

B x = -2, multiplicity 2

C x = -2, multiplicity 3

D x = -1, multiplicity 1

E x = -1, multiplicity 2

F x = -1, multiplicity 3

Answer

114 Find the zeros of the polynomial equation, including multiplicities, using the Rational Zeros Theorem

$$6x^3 - 17x^2 - 4x + 3.$$

A x = 1, multiplicity 1

B x = -1, multiplicity 1

C x = 3, multiplicity 1

D x = -3, multiplicity 1

E x = $\frac{1}{3}$, multiplicity 1

F x = $-\frac{1}{3}$, multiplicity 1

G x = $\frac{1}{2}$, multiplicity 1

H x = $-\frac{1}{2}$, multiplicity 1

Answer

115 Use the Rational Zeros Theorem to find the zeros of the polynomial equation, including multiplicities.

$$4x^4 - 20x^3 + 13x^2 + 30x + 9 = 0$$

A x = 3, mulitplicity 1

B x = 2, mulitplicity 2

C x = 3, multiplicity 2

D x = -2, multiplicity 1

E x = $\frac{1}{2}$, multiplicity 1

F x = $-\frac{1}{2}$, multiplicity 2

Answer

116 Find the zeros of the polynomial equation.

$$x^4 + 7x^2 - 18 = 0$$

A $x = 2$

B $x = -2$

C $x = 3$

D $x = -3$

E $x = 3i$

F $x = -3i$

G $x = \sqrt{2}$

H $x = -\sqrt{2}$

Answer