Writing a Polynomial Function from its Given Zeros

Return to Table of Contents

Goals and Objectives

- Students will be able to write a polynomial from its given zeros.

Write (in factored form) the polynomial function of lowest degree using the given zeros, including any multiplicities.

x = -1, multiplicity of 1
x = -2, multiplicity of 2
x = 4, multiplicity of 1

Work backwards from the zeros to the original polynomial.

$x = -1$ or $x = -2$ or $x = -2$ or $x = 4$

For each zero, write the corresponding factor.

$x + 1 = 0$ or $x + 2 = 0$ or $x + 2 = 0$ or $x - 4 = 0$

$$f(x) = (x+1)(x+2)(x+2)(x-4)$$

$$f(x) = (x+1)(x+2)^2(x-4)$$

117 Write the polynomial function of lowest degree using the zeros given.

x = -.5, multiplicity of 1
x = 3, multiplicity of 1
x = 2.5, multiplicity of 1

A $f(x) = (x - .5)(x + 3)(x - 2.5)$

B $f(x) = (x - .5)(x - 3)(x - 2.5)$

C $f(x) = (x + .5)(x + 3)(x + 2.5)$

D $f(x) = (x + .5)(x - 3)(x - 2.5)$

Answer

118 Write the polynomial function of lowest degree using the zeros given.

x = 1/3, multiplicity of 1
x = -2, multiplicity of 1
x = 2, multiplicity of 1

A $f(x) = (x + \frac{1}{3})(x^2 - 4)$

B $f(x) = (x - \frac{1}{3})(x^2 - 4)$

C $f(x) = (x - \frac{1}{3})(x^2 + 4)$

D $f(x) = (x + \frac{1}{3})(x^2 + 4)$

Answer

119 Write the polynomial function of lowest degree using the zeros given.

x = 0, multiplicity of 3
x = -2, multiplicity of 2
x = 2, multiplicity of 1
x = 1, multiplicity of 1
x = -1, multiplicity of 2

A $f(x) = x^3(x-2)^2(x-2)(x-1)(x-1)^2$

B $f(x) = x^3(x+2)(x-2)^2(x-1)^2(x+1)$

C $f(x) = x^3(x+2)^2(x-2)(x-1)(x+1)^2$

D $f(x) = x^3(x-2)^2(x+2)(x+1)(x-1)^2$

E $f(x) = x^3(x+2)^2(x+2)(x+1)(x+1)^2$

Answer

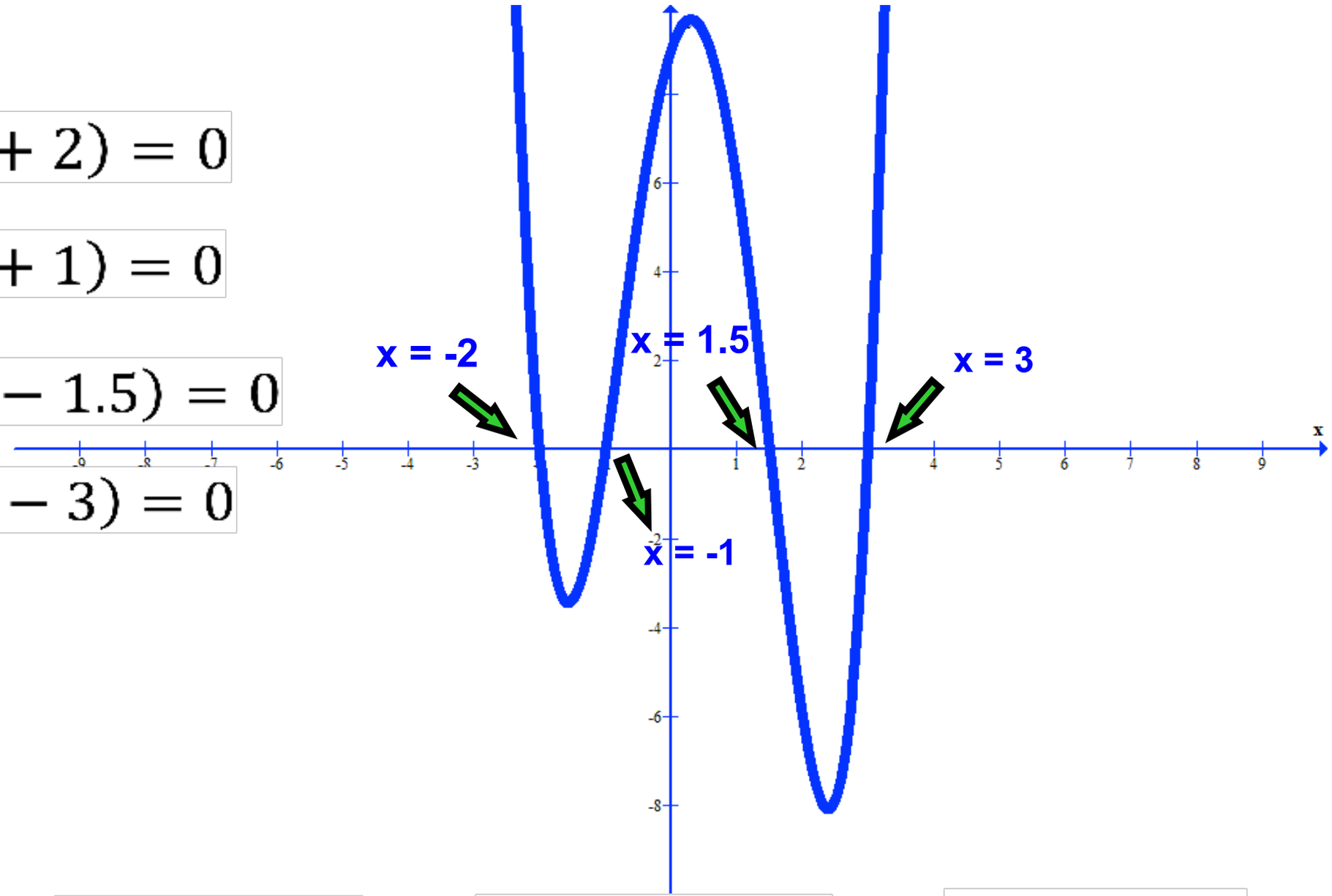
Write the polynomial function of lowest degree using the zeros from the given graph, including any multiplicities.

x = -2 ➡ $(x+2)=0$

x = -1 ➡ $(x+1)=0$

x = 1.5 ➡ $(x-1.5)=0$

x = 3 ➡ $(x-3)=0$



$(x+2)=0$ or $(x+1)=0$ or $(x-1.5)=0$ or $(x-3)=0$

$$f(x)=(x+2)(x+1)(x-1.5)(x-3)$$

120 Write the polynomial function of lowest degree using the zeros from the given graph, including any multiplicities.

A $f(x) = (x-4)(x-1)$

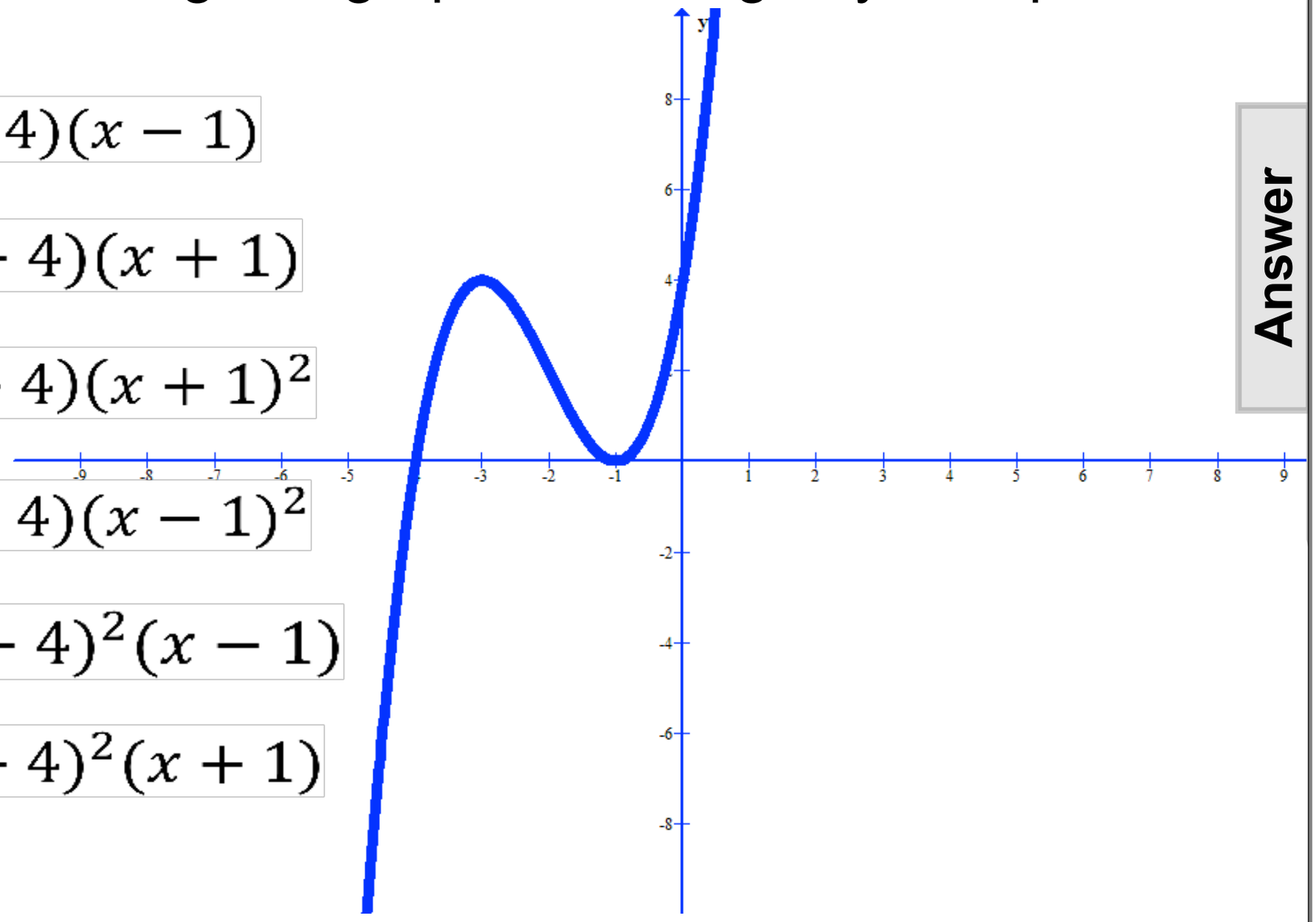
B $f(x) = (x+4)(x+1)$

C $f(x) = (x+4)(x+1)^2$

D $f(x) = (x-4)(x-1)^2$

E $f(x) = (x-4)^2(x-1)$

F $f(x) = (x+4)^2(x+1)$



Answer

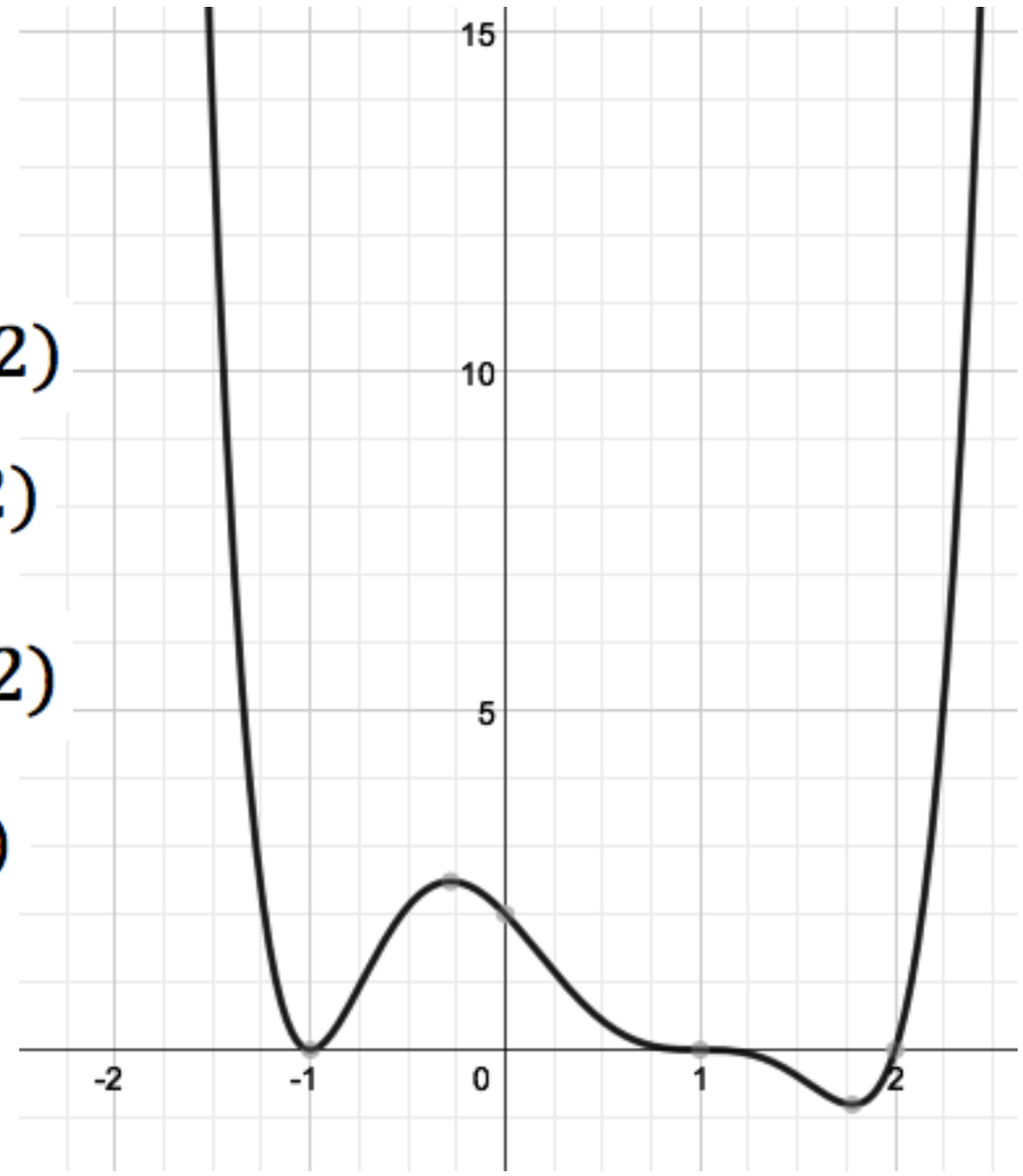
121 Write the polynomial function of lowest degree using the zeros from the given graph, including any multiplicities.

A $f(x) = (x-1)^3(x+1)^2(x-2)$

B $f(x) = (x+1)^3(x-1)^2(x+2)$

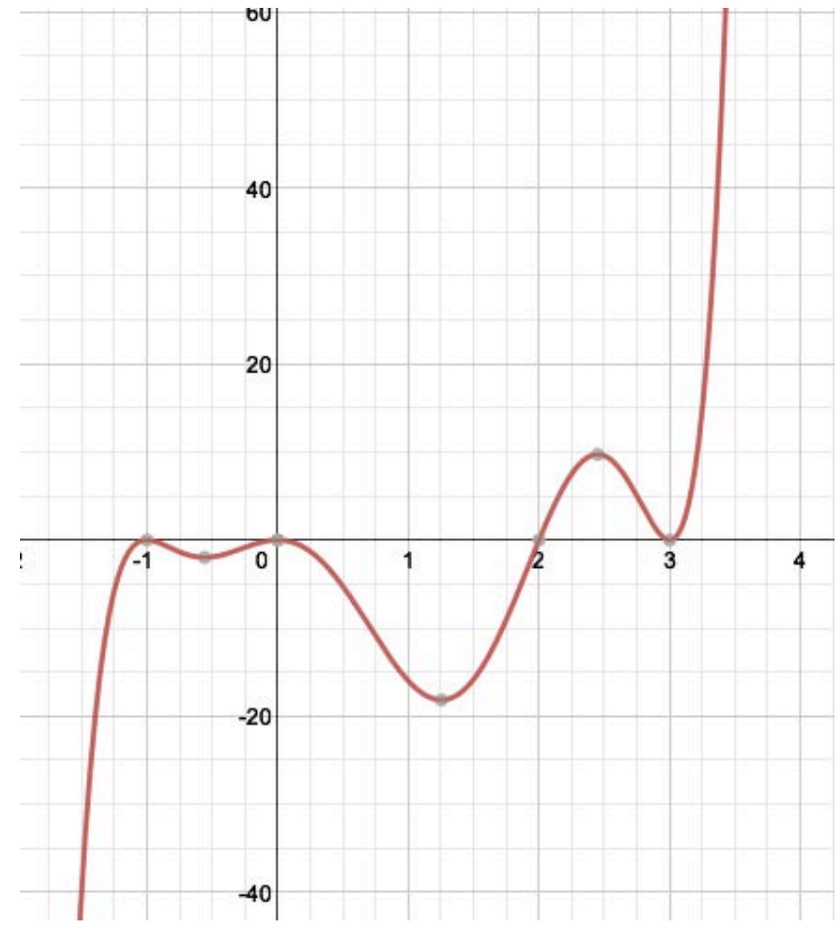
C $f(x) = (x-1)^2(x+1)(x+2)$

D $f(x) = (x+1)^2(x-1)(x-2)$



Answer

122 Which equation could be the equation of the graph below?



A $f(x) = (x+1)^2(x-2)(x-3)^2$

B $f(x) = x(x+1)(x-2)(x-3)$

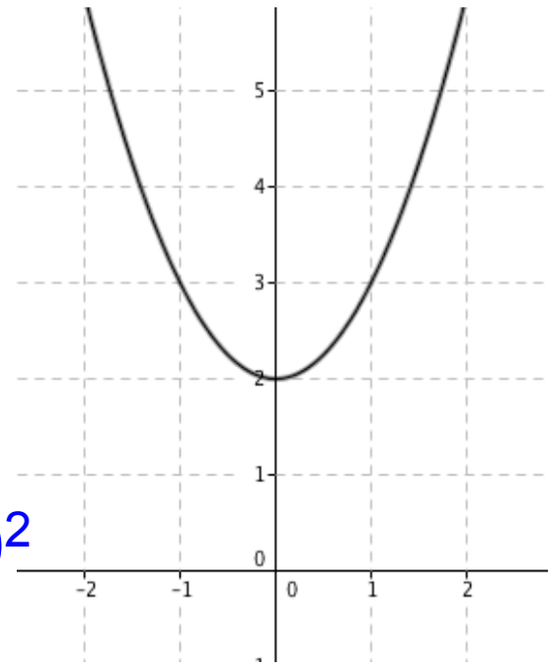
C $f(x) = x^2(x-1)^2(x+2)(x+3)^2$

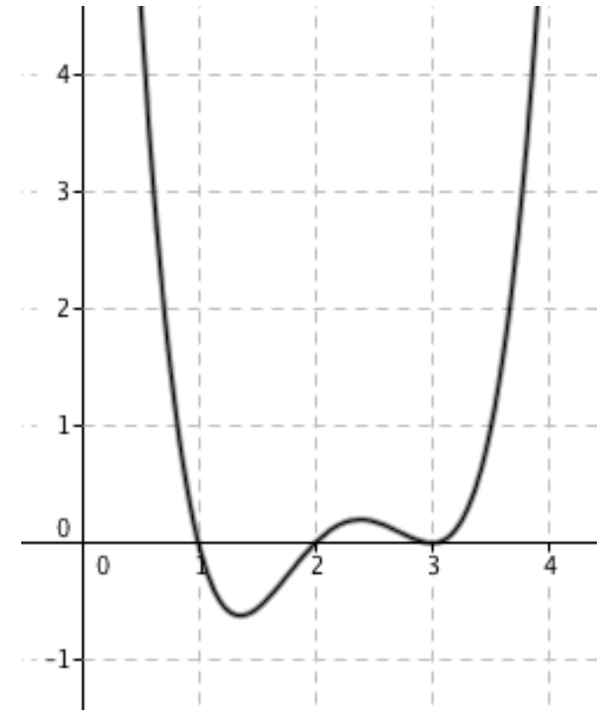
D $f(x) = x^2(x+1)^2(x-2)(x-3)^2$

Answer

Match each graph to its equation.

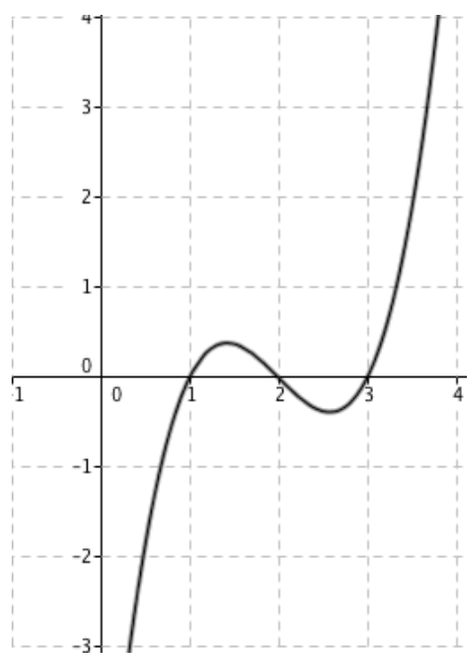
$y = x^2 + 2$

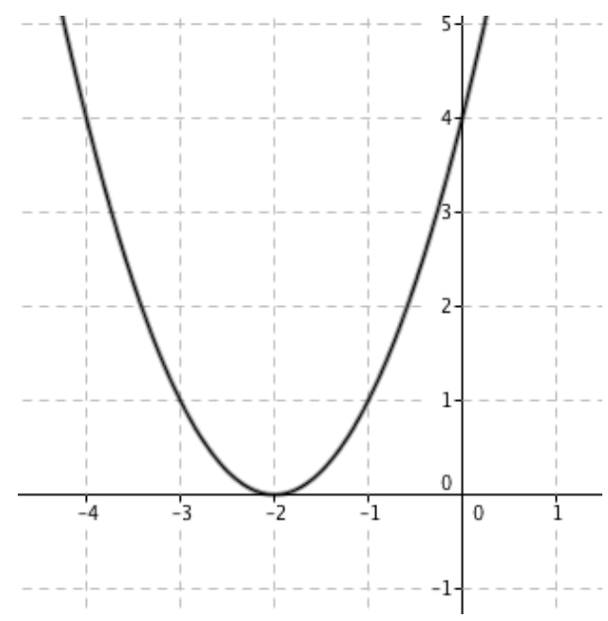




$y = (x-1)(x-2)(x-3)^2$

$y = (x-1)(x-2)(x-3)$





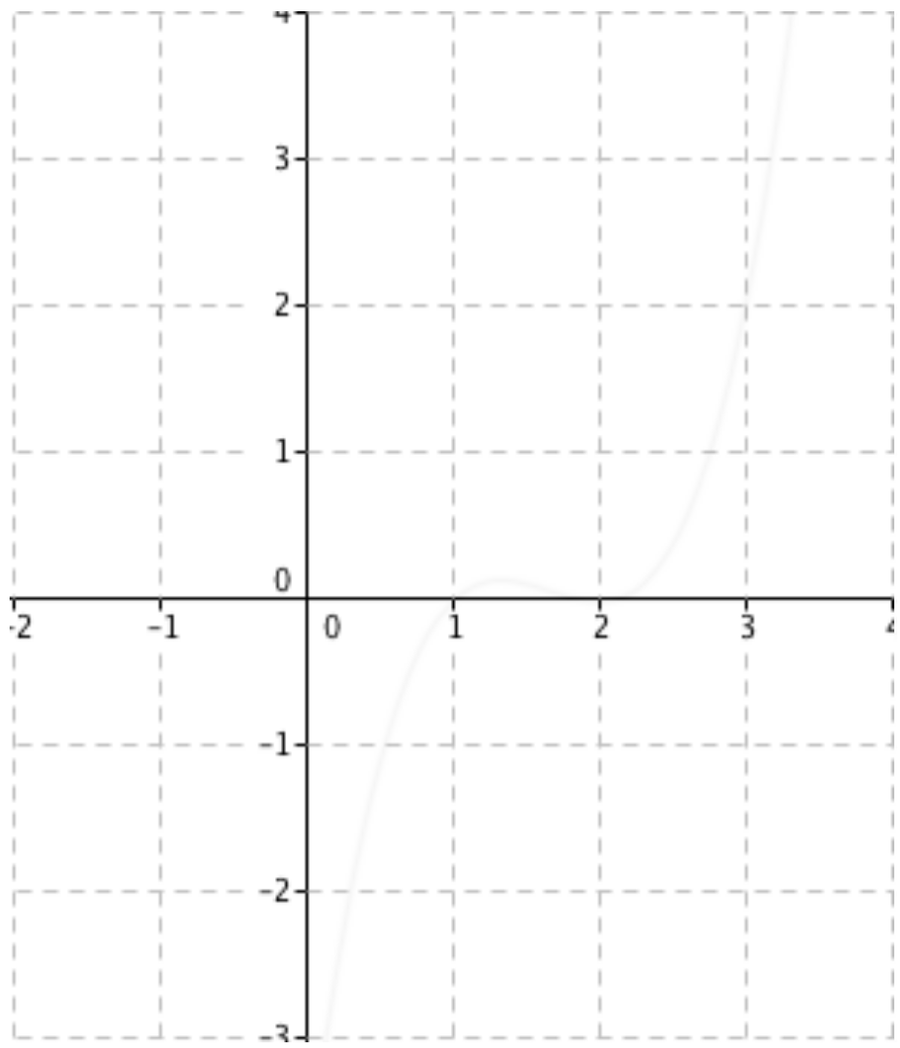
$y = (x + 2)^2$

Answer

Sketch

Sketch the graph of $f(x) = (x-1)(x-2)^2$.

After sketching, click on the graph to see how accurate your sketch is.



Analyzing Graphs using a Graphing Calculator

Enter the function into the calculator (Hit y= then type).

Check your graph, then set the window so that you can see the zeros and the relative minima and maxima. (Look at the table to see what the min and max values of x and y should be.)

Use the Calc functions (2nd TRACE) to find zeros:

Select 2: Zero Your graph should appear. The question "Left Bound?" should be at the bottom of the screen.

Use the left arrow to move the blinking cursor to the left side of the zero and press ENTER. The question "Right Bound?" should be at the bottom of the screen.

Use the right arrow to move the blinking cursor to the right side of the zero and press ENTER. The question "Guess?" should be at the bottom of the screen.

Press ENTER again, and the coordinates of the zero will be given.

Finding Minima and Maxima

Use the Calc functions (2nd TRACE) to find relative min or max:

Select 3: minimum or 4: maximum. Your graph should appear. The question "Left Bound?" should be at the bottom of the screen.

Use the left arrow to move the blinking cursor to the left side of the turning point and press ENTER. The question "Right Bound?" should be at the bottom of the screen.

Use the right arrow to move the blinking cursor to the right side of the turning point and press ENTER. The question "Guess?" should be at the bottom of the screen.

Press ENTER again, and the coordinates of the min or max will be given.

Use a graphing calculator to find the zeros and turning points of

$$f(x) = x^3 + 2x^2 - 2x - 4$$

Answer

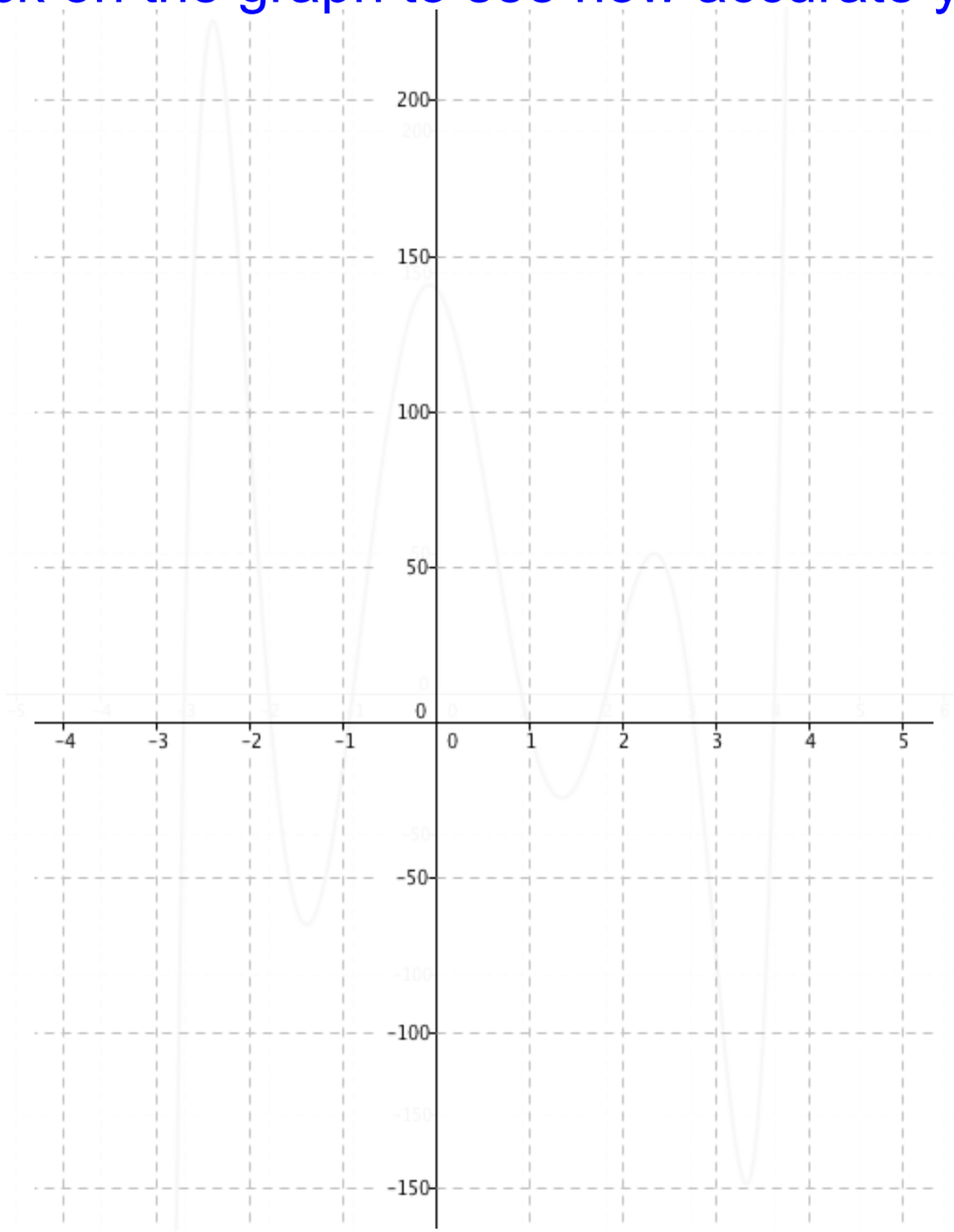
Note: The calculator will give an estimate. Rounding may be needed.

Use a graphing calculator to find the zeros and turning points of

$$f(x) = x^4 - 10x^2 + 9$$

Answer

Sketch the graph of $f(x) = (x-1)(x+1)(x-2)(x+2)(x-3)(x+3)(x-4)$. After sketching, click on the graph to see how accurate your sketch is.



The product of 4 positive consecutive integers is 175,560.

Write a polynomial equation to represent this problem.

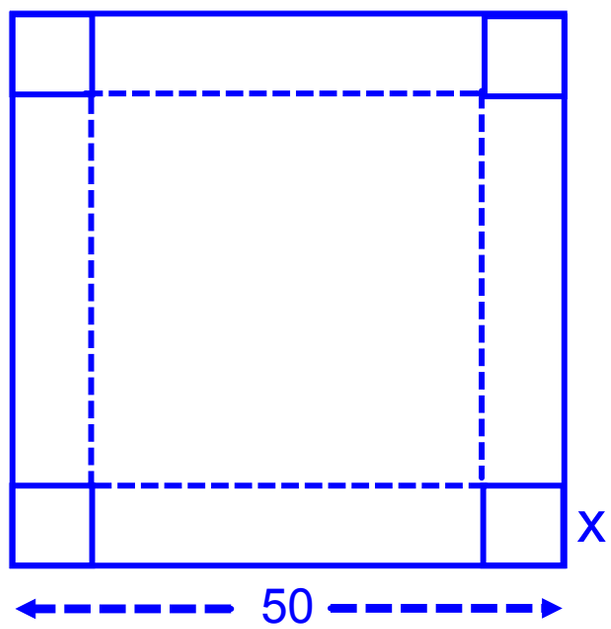
Use a graphing utility or graphing calculator to find the numbers.

Hint: set your equation equal to zero, and then enter this equation into the calculator.

How could you use a calculator and guess and check to find the answer to this problem?

Answer

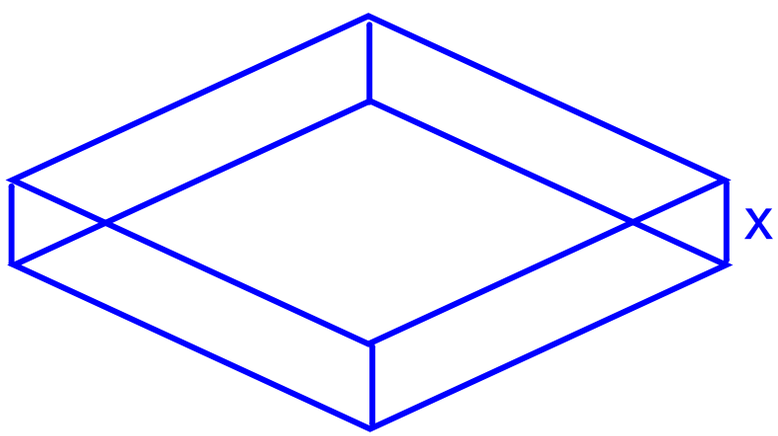
An open box is to be made from a square piece of cardboard that measures 50 inches on a side by cutting congruent squares of side-length x from each corner and folding the sides.



1. Write the equation of a polynomial function to represent the volume of the completed box.

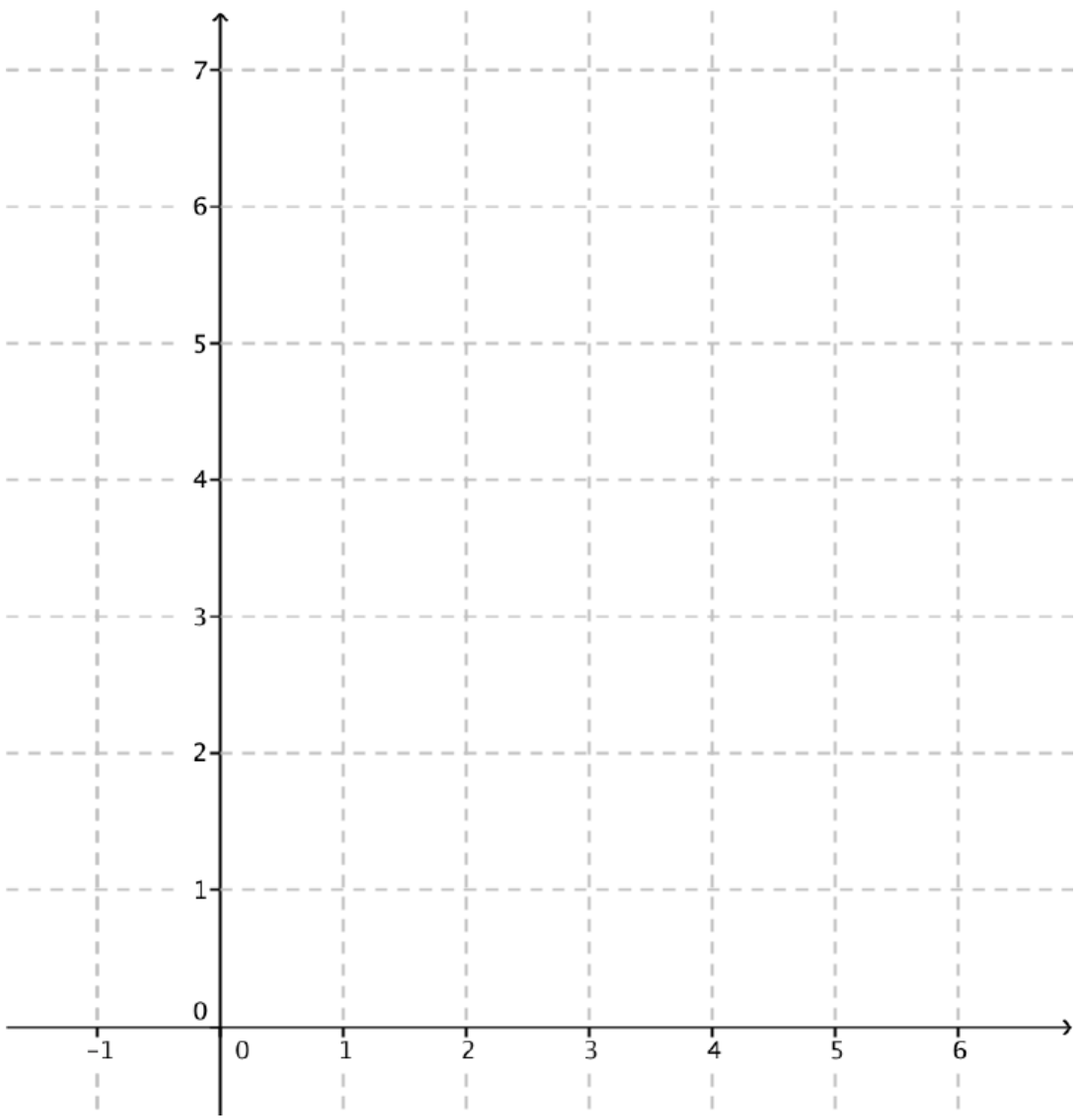
2. Use a graphing calculator or graphing utility to create a table of values for the height of the box. (Consider what the domain of x would be.) Use the table to determine what height will yield the maximum volume.

3. Look at the graph and calculate the maximum volume within the defined domain. Does this answer match your answer above? (Use the table values to determine how to set the viewing window.)



Answer

An engineer came up with the following equation to represent the height, $h(x)$, of a roller coaster during the first 300 yards of the ride: $h(x) = -3x^4 + 21x^3 - 48x^2 + 36x$, where x represents the horizontal distance of the roller coaster from its starting place, measured in 100's of yards. Using a graphing calculator or a graphing utility, graph the function on the interval $0 \leq x \leq 3$. Sketch the graph below.



Does this roller coaster look like it would be fun? Why or why not?

(Derived from engage^ny)

Answer

For what values of x is the roller coaster 0 yards off the ground? What do these values represent in terms of distance from the beginning of the ride?

Verify your answers above by factoring the polynomial

$$h(x) = -3x^4 + 21x^3 - 48x^2 + 36x$$

Answer

How do you think the engineer came up with this model?

Why did we restrict the domain of the polynomial to the interval from 0 to 3?

In the real world, what is wrong with this model at a distance of 0 yards and at 300 yards?

Teacher Notes

Consider the function $f(x) = x^3 - 13x^2 + 44x - 32$.

Use the fact that $x - 4$ is a factor to factor the polynomial.

What are the x-intercepts for the graph of f ?

At which x-values does the function change from increasing to decreasing and from decreasing to increasing?

Answer

How can we tell if a function is positive or negative on an interval between x-intercepts? Given our polynomial

$f(x) = x^3 - 13x^2 + 44x - 32$...

When $x < 1$, is the graph above or below the x-axis?

When $1 < x < 4$, is the graph above or below the x-axis?

When $4 < x < 8$, is the graph above or below the x-axis?

When $x > 8$, is the graph above or below the x-axis?

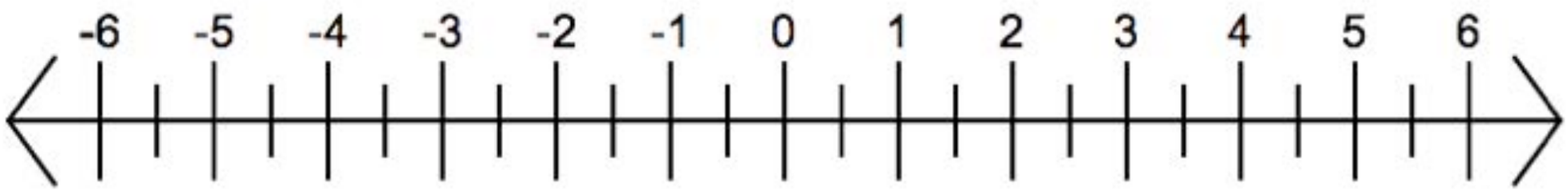
Answer

123 Consider the function $f(x)=(2x-1)(x+4)(x-2)$. What is the y-intercept of the graph of the function in the coordinate plane?

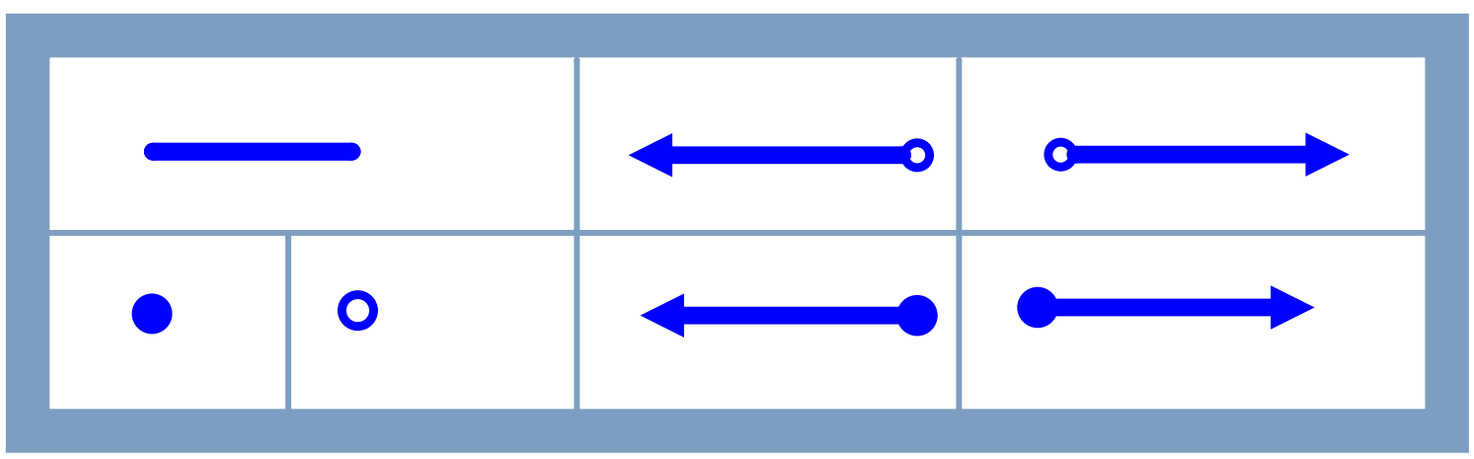
Answer

From PARCC sample test

Consider the function $f(x)=(2x-1)(x+4)(x-2)$. For what values of x is $f(x)>0$? Use the line segments and endpoint indicators to build the number line that answers the question.



Answer



From PARCC sample test

124 Consider the function $f(x)=(2x-1)(x+4)(x-2)$. What is the end behavior of the graph of the function?

A As $x \to -\infty, f(x) \to \infty$, and as $x \to \infty$, $f(x) \to \infty$.

B As $x \to -\infty, f(x) \to \infty$, and as $x \to \infty$, $f(x) \to -\infty$.

C As $x \to -\infty, f(x) \to -\infty$, and as $x \to \infty$, $f(x) \to \infty$.

D As $x \to -\infty, f(x) \to -\infty$, and as $x \to \infty$, $f(x) \to -\infty$.

Answer

From PARCC sample test

125 Consider the function $f(x)=(2x-1)(x+4)(x-2)$. How many relative maximums does the function have?

A none

B one

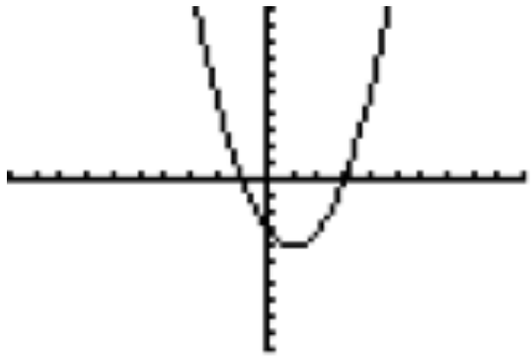
C two

D three

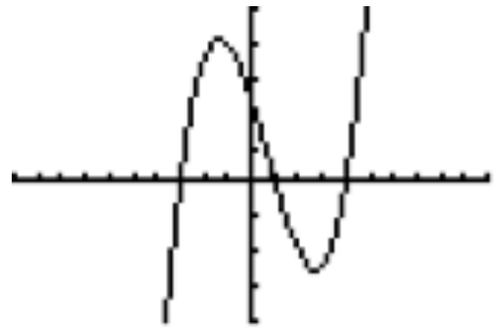
Answer

From PARCC sample test

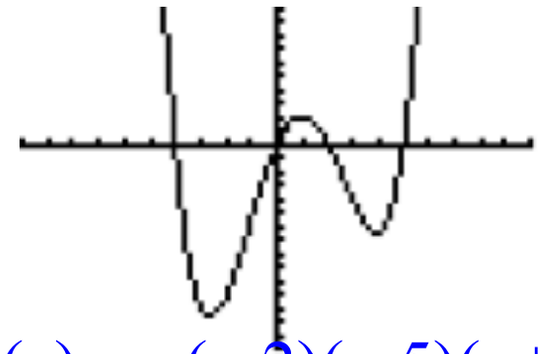
How many relative maxima and minima?



$f(x) = (x+1)(x-3)$



$g(x) = (x-1)(x+3)(x-4)$



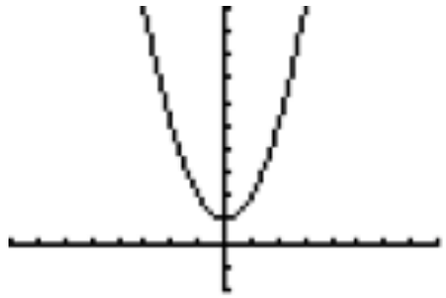
$h(x) = x\,(x-2)(x-5)(x+4)$

	$f(x)$	$g(x)$	$h(x)$
Degree:			
# x-intercepts:			
# turning points:			

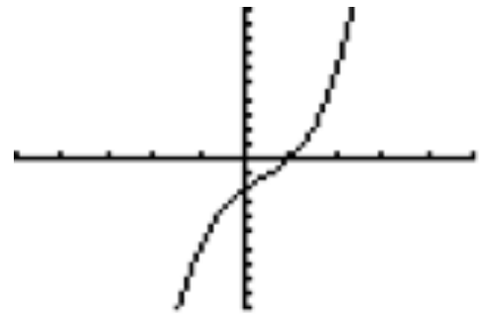
Observations:

Answer

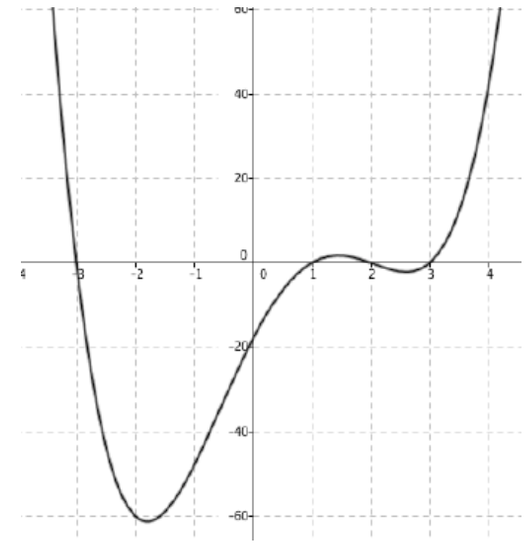
How many relative maxima and minima?



$f(x) = (x+1)(x-3)$



$g(x) = (x+3)(x-1)(x-4)$



$h(x) = (x)(x+4)(x-2)(x-5)$

Answer

	f(x)	g(x)	h(x)
Degree:			
# x-intercepts:			
# turning points:			

Observations:

Increasing and Decreasing

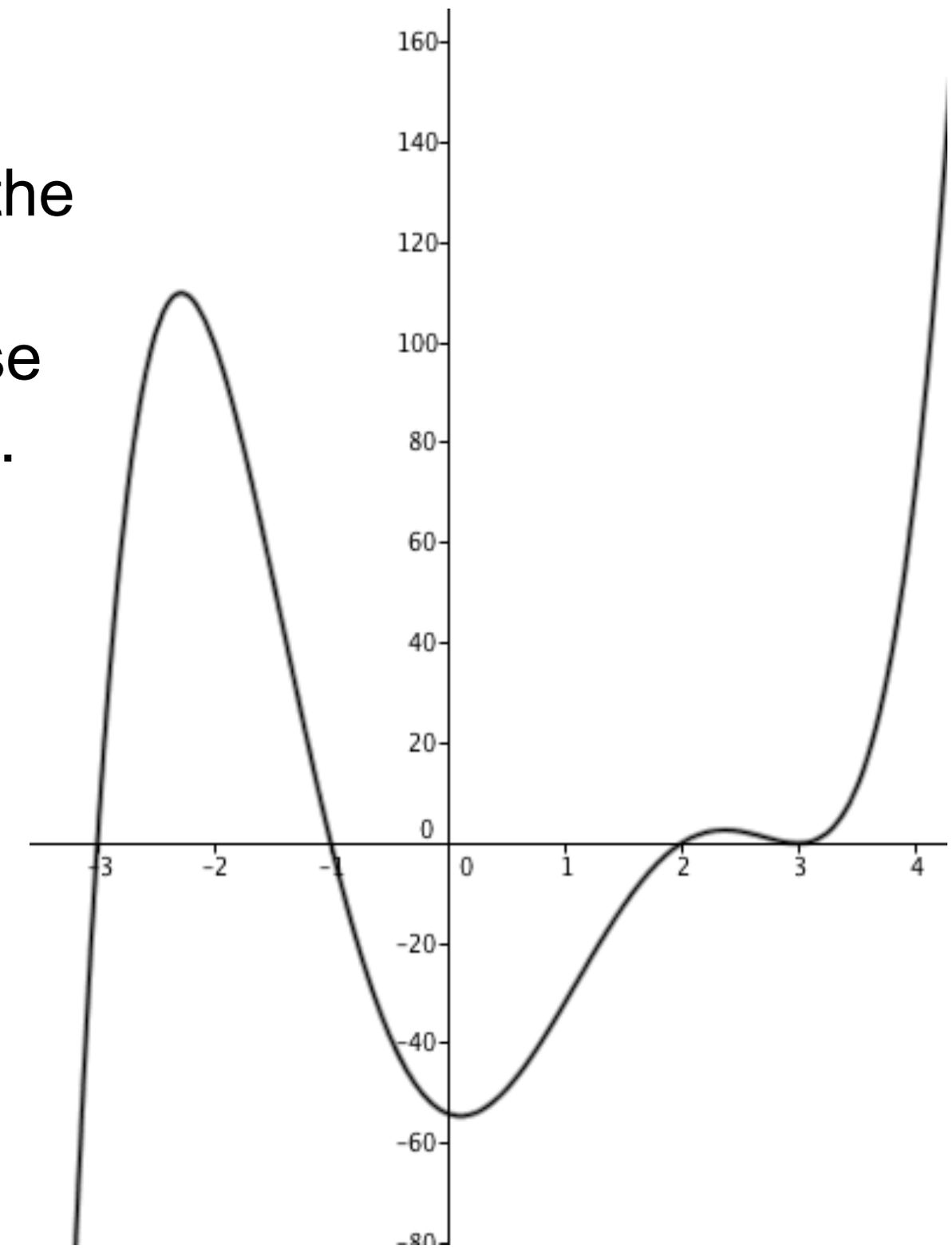
Given a function f whose domain and range are subsets of the real numbers and I is an interval contained within the domain, the function is called *increasing on the interval* if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.

It is called *decreasing on the interval* if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in I.

Restate this in your own words:

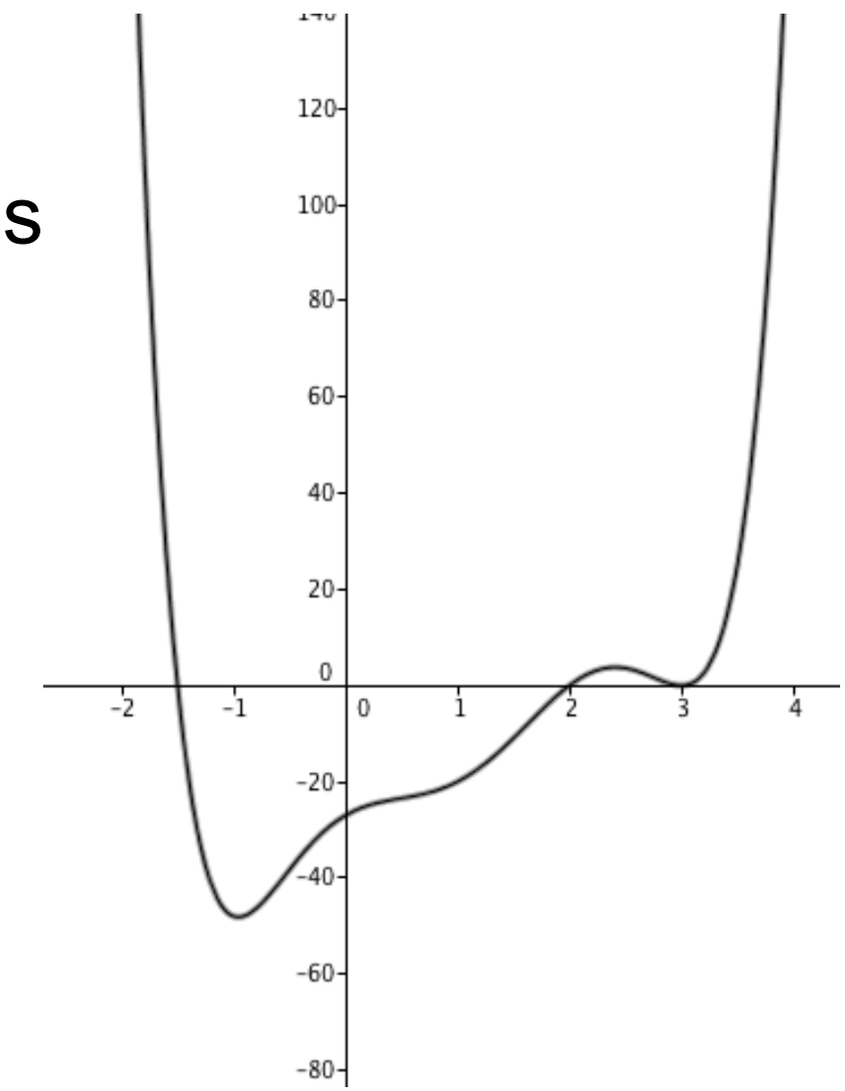
Answer

Mark on this graph and state using inequality notation the intervals that are increasing and those that are decreasing.



Answer

126 Select all of the statements that are true based on the graph provided:



A The degree of the function is even.

B There are 4 turning points.

C The function is increasing on the interval from x = -1 to x = 2.4.

D The function is increasing when x < -1.

E x - 2 and x + 3 are factors of the polynomial that defines this function.

Answer

127 Given the function $f(x) = x^7 - 4x^5 - x^3 + 4x$. Which of the following statements are true? Select all that apply.

A As $x \to \infty$, $f(x) \to \infty$.

B There are a maximum of 6 real zeros for this function.

C $x = -1$ is a solution to the equation $f(x) = 0$.

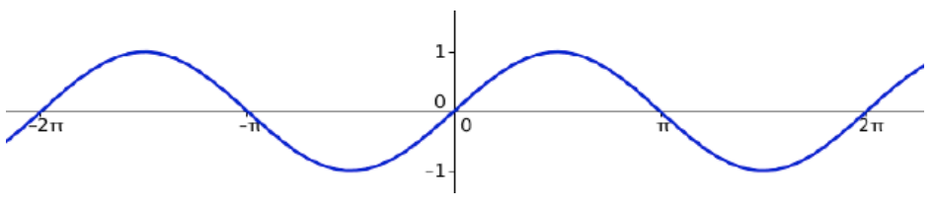
D The maximum number of relative minima and maxima for this function is 7.

Answer

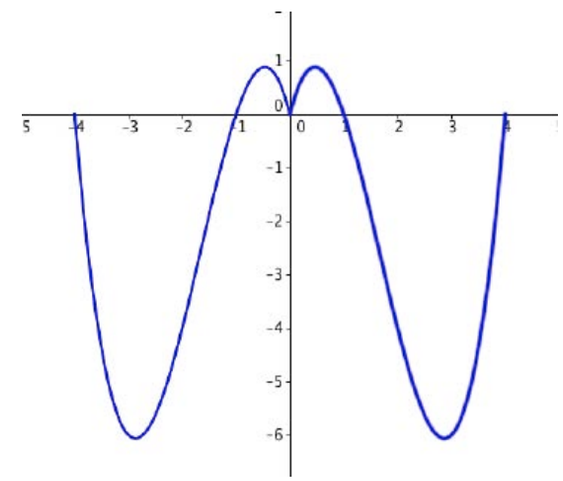
For each function described by the equations and graphs shown, indicate whether the function is even, odd, or neither even nor odd:

$f(x)=3x^2$

$g(x)=-x^3+5$



$h(x)$



$k(x)$

	Even	Odd	Neither
$f(x)$			
$g(x)$			
$h(x)$			
$k(x)$			

From PARCC sample test

Answer