Factoring Summary

Before factoring any polynomial, write the polynomial in **descending order** of one of the variables.

1. Factor out the Greatest Common Factor (GCF). Look for this in **<u>every</u>** problem. This includes factoring out a -1 if it precedes the leading term.

   Example: $-3x^2 + 12x - 18 = -3(x^2 - 4x + 6)$

2. If there are **<u>FOUR TERMS</u>**, try to factor by grouping (GR).

   Example: $x^3 + 6x^2 - 2x - 12$

$\underline{x^3 + 6x^2}\ \ \underline{-2x - 12} =$	*group the first two terms, last two terms*
$x^2(x+6) - 2(x+6) =$	*factor out GCF from each grouping*
$(x+6)(x^2-2)$	*factor out the common grouping*

3. If there are **<u>TWO TERMS</u>**, look for these patterns:

x	x^2	x^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	
10	100	
11	121	
12	144	
13	169	
14	196	
15	225	

a. The difference of squares (DOS) factors into conjugate binomials:

$$a^2 - b^2 = (a-b)(a+b)$$

Example: $9x^4 - 64y^2 = (3x^2 - 8y)(3x^2 + 8y)$

Note: a variable is a perfect square if the exponent is even

b. The sum of squares does not factor:

$a^2 + b^2$ is prime

Example: $9x^4 + 64y^2$ is PRIME

c. The sum of cubes (SOC) or difference of cubes (DOC) factors by these patterns: each type contains a binomial (small bubble) times a trinomial (large bubble). Only the sign patterns differ between sum of cubes and difference of cubes.

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

Example : $(8x^3 + 27) = (2x+3)(4x^2 - 6x + 9)$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Example : $(64x^6 - 125y^3) = (4x^2 - 5y)(16x^4 + 20x^2y + 25y^2)$

Note: a variable is a perfect cube if the exponent is a multiple of three

4. If there are **<u>THREE TERMS</u>**, look for these patterns:

a. Quadratic trinomials of the form $ax^2 + bx + c$ *where* $a = 1$ ($QT\ \ a = 1$))factor into the product of two binomials (double bubble) where the factors of c must add to b.

Example: $x^2 - 4x - 12 = (x-6)(x+2)$

b. Quadratic trinomials of the form $ax^2 + bx + c$ *where* $a \neq 1$ ($QT\ \ a \neq 1$) eventually factor into the product of two binomials (double bubble), but you must first find the factors of *ac* that add to *b*, rewrite the original replacing *b* with these factors of *ac*, then factor by grouping to finally get to the double bubble.

Example:

$9x^2 + 15x + 4 \qquad ac = (9)(4) = 36$

$\qquad\qquad factors\ \ of\ \ 36\ \ that\ \ add\ \ to\ \ 15:\ \ 12\ \ and\ \ 3$

$9x^2 + 12x + 3x + 4 =$

$3x(3x+4) + 1(3x+4) =$

$(3x+4)(3x+1)$

c. Quadratic square trinomials (*QST)* of the form $ax^2 + bx + c$ may factor into the square of a binomial. Look for the pattern where two of the terms are perfect squares, and the remaining term is twice the product of the square root of the squares:

$a^2 \pm 2ab \pm b^2 = (a \pm b)^2$

Example: $16x^2 - 40x + 25 = (4x-5)^2$

5. Factor all expressions completely. Sometimes, you will need to use two or three types of factoring in a single problem.

Example:

$-2x^4 + 32 = \qquad factor\ \ out\ the\ \ GCF\ \ of\ \ -2$

$-2(x^4 - 16) = \qquad factor\ \ the\ \ difference\ \ of\ \ squares$

$-2(x^2 - 4)(x^2 + 4) = \qquad factor\ \ the\ \ remaining\ \ difference\ \ of\ \ squares$

$-2(x-2)(x+2)(x^2+4) \quad (remember\ \ that\ \ the\ \ sum\ \ of\ \ squares\ \ is\ \ prime)$

Factoring the Sum or Difference of Two Cubes

The Sum of Cubes Factoring Formula is:

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

The Difference of Cubes Factoring Formula is:

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Factor: $8a^3 + b^3$

Factor: $8a^3 - b^3$

Factor: $8a^3 + 27b^3$

Factor: $64x^3 - 27y^3$

Factor: $24a^4 + 3ax^3$

Answers

Factor: $8a^3 + b^3$

We begin by rewriting each term as a cube and use the sum of cubes formula as a guide.

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$8a^3 + b^3 = (2a)^3 + (b)^3 = (2a + b)((2a)^2 - (2a)(b) + (b)^2)$$

So simplifying the right hand side we get:

$$8a^3 + b^3 = (2a + b)(4a^2 - 2ab + b^2) \leftarrow \text{Answer}$$

Factor: $8a^3 - b^3$

We begin by rewriting each term as a cube and use the difference of cubes formula as a guide.

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$8a^3 - b^3 = (2a)^3 - (b)^3 = (2a - b)((2a)^2 + (2a)(b) + (b)^2)$$

So simplifying the right hand side we get:

$$8a^3 - b^3 = (2a - b)(4a^2 + 2ab + b^2) \leftarrow \text{Answer}$$

Factor: $8a^3 + 27b^3$

We begin by rewriting each term as a cube and use the sum of cubes formula as a guide.

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$8a^3 + 27b^3 = (2a)^3 + (3b)^3 = (2a + 3b)((2a)^2 - (2a)(3b) + (3b)^2)$$

So simplifying the right hand side we get:

$$8a^3 + 27b^3 = (2a + 3b)(4a^2 - 6ab + 9b^2) \leftarrow \text{Answer}$$

Factor: $64x^3 - 27y^3$

We begin by rewriting each term as a cube and use the difference of cubes formula as a guide.

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$64x^3 - 27y^3 = (4x)^3 - (3y)^3 = (4x - 3y)\left((4x)^2 + (4x)(3y) + (3y)^2\right)$$

So simplifying the right hand side we get:

$$64x^3 - 27y^3 = (4x - 3y)(16x^2 + 12xy + 9y^2) \leftarrow \text{Answer}$$

Factor: $24a^4 + 3ax^3$

This problem doesn't even look like a sum or difference of cubes problem. However, there is a common factor in each of the two terms. It is $3a$. So we can write:

$24a^4 + 3ax^3 = 3a(8a^3 + x^3)$ Now notice that the factor in parentheses is a sum of two cubes and except for the x it is the same expression that was factored in the first exercise on this sheet. So we can write:

$$24a^4 + 3ax^3 = 3a(8a^3 + x^3) = 3a(2a + x)(4a^2 - 2ax + x^2) \leftarrow \text{Answer}$$