# Logarithmic Functions

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# **Why do we need logarithms** (logs)?

**Logarithms (logs)** are used in many applications and explain much about how we perceive the world.

This is because the world is full of such extreme ranges that only logarithms make it possible to deal with them.

We perceive the world in a way that is logarithmic in nature.

Our eyes, ears, touch and sense of smell all behave in a logarithmic, not a linear, manner.

We even think logarithmically.

# Logarithmic Functions

We think using log scales

For instance, if one store charges $1 for an item and another charges $2 for the same item, that seems to be a big difference.

But for a different item, if one store charges $100 and another charges $101...it doesn't seem like such a difference.

The linear difference is the same, $1.

But the log difference between $1 and $2 is 0.3,
while the log difference between $100 and $101 is 0.004.

The log relationship more closely reflects the way we perceive these differences in price.

# Logarithmic Functions

Some examples of the use of logarithms in science and technology include:

Sound levels are measured in decibels (dB), which is a logarithmic scale.

In chemistry, pH is a logarithmic scale. This is used throughout medicine, agriculture, and other applications in science.

The Richter scale, used to measure the magnitude of earthquakes, is a logarithmic scale.

In addition, we need to use logarithms in solving equations when the exponent is the unknown.

# Inverse Operations

Logarithmic functions are an inverse of exponential functions.

So, to understand logarithms, it's important to first understand inverse operations.

Let's review the first two of these we learned, and see if we can extend that understanding to exponents.

# Inverse Operations

$5 + 4 = 9$

This equation provides the answer "9" to the addition question "what is the sum of 5 and 4?"

What are the two inverse questions that can be asked and answered based on the above addition fact?

Which mathematical operation is the inverse of addition?

DISCUSS!

# Inverse Operations

Addition and Subtraction

5 + 4 = 9

There are two subtraction questions that represent the inverse of that first addition question:

9 - 5 = 4

This equation provides the answer "4" to the subtraction question "what is 5 less than 9?"

9 - 4 = 5

This equation provides the answer "5" to the subtraction question "what is 4 less than 9?"

# Inverse Operations

Addition and Subtraction

Notice that although there are two inverse subtraction questions that can be asked from that first addition question...there is only one inverse operation.

Subtraction is the only inverse operation of addition.

# Inverse Operations

$7(4) = 28$

This equation provides the answer "28" to the multiplication question "what is the product of 7 times 4?"

What are the two inverse questions that can be asked and answered based on the above multiplication fact?

Which mathematical operation is the inverse of multiplication?

DISCUSS!

# Inverse Operations

Multiplication and Division

$7(4) = 28$

There are two division questions that represent the inverse of that first multiplication question:

$28/4 = 7$

This equation provides the answer "7" to the division question "what is 28 divided by 4?"

$28/7 = 4$

This equation provides the answer "4" to the division question "what is 28 divided by 7?"

# Inverse Operations

Multiplication and Division

Notice that although there are two inverse division questions that can be asked from that first multiplication question...there is only one inverse operation.

Division is the only inverse operation of multiplication.

# Inverse Operations

$10^2 = 100$

This equation provides the answer "100" to the exponent question "what is 10 raised to the power of 2?"

What are the two inverse questions that can be asked and answered based on the above exponent fact?

Which mathematical operation(s) are the inverse of exponentiation?

DISCUSS!

# Inverse Operations

$10^2 = 100$

The first inverse question and answer should be familiar to you

$(100)^{1/2} = 10$

This equation provides the answer "10" to the question "what is the square root of 100?" or "what is 100 raised to the 1/2 power?"

But, is there another inverse question and answer?

# Inverse Operations

$10^2 = 100$

This question and answer may not be familiar to you

$\log_{10}(100) = 2$

This equation provides the answer "2" to the question
"To what power must 10 be raised in order to get 100?"

# Inverse Operations

We know that $5 + 4 = 9$ is equivalent to $9 - 4 = 5$.

Now we see that

$$10^2 = 100$$

is equivalent to

$$\log_{10}(100) = 2$$

# Inverse Operations

Exponents, roots and logarithms

In the case of exponents, there are two inverse functions, not one.

One inverse is to take the root.
(We use this function to solve when the base is unknown.)

The other is to find the logarithm.
(We use this function to solve when the exponent is unknown.)

Given any one of these three equations, the other two equivalent equations can be determined.

# Inverse Operations

Let's practice using some base 10 logs and exponents.

Base 10 logarithms are called
Common Logarithms or Common Logs.
When the symbol "log" is used without an indicated base, it is a common log with base 10.

Write the two inverse equations to this exponential equation.

$10^4 = 10{,}000$

# Inverse Operations

Let's practice using some base 10 logs and exponents.

Base 10 logarithms are called Common Logarithms or Common Logs. When the symbol "log" is used without an indicated base, it is a common log and base 10.

Write the two inverse equations to this exponential equation.

$10^6 = 1{,}000{,}000$

# Inverse Operations

Let's practice using some base 10 logs and exponents.

Base 10 logarithms are called Common Logarithms or Common Logs. When the symbol "log" is used without an indicated base, it is a common log and base 10.

Write the two inverse equations to this exponential equation.

$10^{-2} = 0.01$

# Log of a Non-Positive Number

If we ask the question:
"To what power must 10 be raised in order to get -100?"

There is no answer, since there is no exponent for 10 which would yield a negative number.

There is no solution for these expressions:

$\mathbf{10}^{x} = -100$ or $\log(-100) = x$
$\mathbf{10}^{x} = 0$ or $\log(0) = x$

So we cannot find the log of a negative number, nor the log of zero.

49 What is the log form of: $10^5 = 100,000$
(Remember: $\log y = x$ if $10^x = y$)

A $\log(5) = 100,000$

B $\log(100,000) = 5$

C $\log(5,000) = 10$

D $\log(100,000) = 50$

Answer

50 What is the log form of: $10^3 = 1{,}000$

A $\log(3{,}000) = 10$

B $\log(3) = 1{,}000$

C $\log(1{,}000) = 3$

D $\log(1{,}000) = 5$

Answer

51 What is the log form of: $10^{-3} = 0.001$

A $\log(0.001) = -3$

B $\log(-3) = 1000$

C $\log(3{,}000) = 1$

D $\log(3) = -1000$

Answer

52 What is the exponential form of: $\log(1000) = 3$

A $10^{3} = 1000$

B $3^{10} = 1000$

C $10^{-2} = 10$

D $10^{1000} = 3$

Answer

53 What is the exponential form of: $\log(0.01) = -2$

A $10^{(.01)} = -2$

B $-2^{10} = 0.01$

C $10^{-2} = 0.01$

D $10^{-2} = 100$

Answer

# Common and Natural Logs

Any exponential equation has a related log equation.

The examples used above were for base 10,
but it works the same for any base.

In most cases, one of two bases is used.
These are found in all scientific calculators.

Common Logs: If no base is given, "log" indicates base 10

Natural Logs: The symbol for this is "ln" and indicates a base of the irrational number "e." Natural logs are very important in calculus.

# Logs with Bases Other Than 10

If a base other than 10 or e is used, it must be indicated by a subscript.

For instance, "$\log_4$" indicates the base of 4

The equation $\log_4 64 = 3$

is equivalent to both

$4^3 = 64$

and

$(64)^{1/3} = 4$

# Logs with Bases Other Than 10

Given the exponential function

$$y = b^x$$

the equivalent logarithmic function is

$$\log_b y = x$$

One definition of logarithms is given by this illustration:

exponent

$\log_b y = x$ if and only if $y = b^x$

base

argument

# Logs with Bases Other Than 10

Let's practice using some other bases.

Write the two inverse equations to this exponential equation.

$4^2 = 16$

# Logs with Bases Other Than 10

Let's practice using some other bases.

Write the two inverse equations to this exponential equation.

$$3^4 = 81$$

# Logs with Bases Other Than 10

Let's practice using some other bases.

Write the two inverse equations to this exponential equation.

$5^{-2} = 1/25$

# Logs with Bases Other Than 10

Let's practice using some other bases.

Write the two inverse equations to this exponential equation.

$$4^{-3} = 1/64$$

# Logs with Bases Other Than 10

Let's practice using some other bases.

Write the two inverse equations to this log equation.

$$\log_7 (343) = 3$$

# Logs with Bases Other Than 10

Let's practice using some other bases.

Write the two inverse equations to this log equation.

$$\log_5 (25) = 2$$

# Logs with Bases Other Than 10

Let's practice using some other bases.

Write the two inverse equations to this exponential equation.

$\log_6 (1/36) = -2$

# Logs with Bases Other Than 10

Let's practice using some other bases.

Write the two inverse equations to this log equation.

$\log_3 (1/27) = -3$

# Logs with Bases Other Than 10

Rewrite each of the following in logarithmic form:

| | | | |
|---|---|---|---|
| $5^2 = 25$ | $2^{-3} = \frac{1}{8}$ | $10^{-1} = 0.1$ | $\left(\frac{1}{16}\right)^{\frac{1}{2}} = \frac{1}{4}$ |

Answer

# Logs with Bases Other Than 10

Rewrite each of the following in exponential form.

| | | | |
|---|---|---|---|
| $\log_4 256 = 4$ | $\log 10000 = 4$ | $\log_6 \frac{1}{36} = -2$ | $\log_{\frac{1}{2}} 8 = -3$ |

Answer

54 Which of the following is the correct logarithmic form of:

$$3^4 = 81$$

A $\log_{81} 3 = 4$

B $\log_4 3 = 81$

C $\log_3 4 = 81$

D $\log_3 81 = 4$

Answer

55 Which of the following is the correct logarithmic form of:

$$7^2 = 49$$

A $\log_7 49 = 2$

B $\log_2 49 = 7$

C $\log_7 2 = 49$

D $\log_{49} 7 = 2$

Answer

56 Which of the following is the correct exponential form of:

$$\log_2 16 = 4$$

A $4^2 = 16$

B $2^4 = 16$

C $16^4 = 2$

D $4^{16} = 2$

Answer

57 Which of the following is the correct exponential form of:

$$\log_2 0.125 = -3$$

A $2^{0.125} = -3$

B $2^{-3} = 0.125$

C $-3^2 = 0.125$

D $0.125^{-3} = 2$

Answer

58 Which of the following is the correct logarithmic form of:

$$3^{-4} = \frac{1}{81}$$

A $\log_{81} 3 = -4$

B $\log_{-4} 3 = \frac{1}{81}$

C $\log_3 - 4 = \frac{1}{81}$

D $\log_3 \frac{1}{81} = -4$

Answer

# Solving Logarithmic Equations

Convert each of the following to exponential form. Then find the value of the variable. You might need to simplify the logarithmic expression as you solve.

| | | | |
|---|---|---|---|
| $\log_4 t = 3$ | $\log_3 27$ | $\log_b 81 = 4$ | $\log 0.0001$ |

Answer

# Log of a Non-Positive Number

Remember, there is no solution for expressions such as these:

$$\mathbf{2}^x = -16 \quad \text{or} \quad \log_2 (-16) = x$$
$$\mathbf{5}^x = 0 \quad \text{or} \quad \log_5 (0) = x$$

So we cannot find the log of a negative number, nor the log of zero for any base.

59 Solve for c: $\log_9 c = 0$

Answer

60 Solve for x: $\log_x 7 = \frac{1}{2}$

Answer

61 Evaluate: $\log_{16} \frac{1}{4}$

Answer

62 Evaluate: $\log_3 81$

Answer

63 Evaluate: $\log_5 125$

Answer

64 Evaluate: $\log_2 \frac{1}{16}$

Answer

65 Evaluate: $\log_3 \frac{1}{27}$

Answer

66 Evaluate: $\log_3(-4)$

Answer

67 Evaluate: $\log_2 35$

Answer