Working with Square Roots

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Square Roots

Recall...

$\sqrt{4}$ $\sqrt{121}$ $\sqrt{0}$ $\sqrt{1}$ $\sqrt{36}$ $\sqrt{-81}$ *

Teacher Notes

Square Roots

All of these numbers can be written with a square. Since the square is the inverse of the square root, they "undo" each other.

$$\sqrt{144} = \sqrt{12^2} = 12$$

$$\sqrt{400} = \sqrt{20^2} = 20$$

$$\sqrt{90000} = \sqrt{300^2} = 300$$

17 What is $\sqrt{900}$?

Answer

18 Find: $2\sqrt{49}$

Answer

19 What is $-5\sqrt{100}$?

Answer

20 What is $\sqrt{625}$?

Answer

21 Find: $\sqrt{-9}$

Answer

Variables

What happens when you have variables in the radicand? To take the square root of a variable rewrite its exponent as the square of a power.

$$\sqrt{x^{12}} = \sqrt{(x^6)^2} = x^6$$

$$\sqrt{d^4} = \sqrt{(d^2)^2} = d^2$$

Variables

IMPORTANT: When taking the square root of variables, remember that answers must be positive. Even powered answers, like the last page, will be positive even if the variables are negative. The same cannot be said if the answer has an odd power. When you take a square root and the answer has an odd power, put the result inside an absolute value symbol.

$$\sqrt{x^6} = \sqrt{(x^3)^2} = \left|x^3\right|$$

$$\sqrt{x^2} = \left|x\right|$$

22 Simplify: $\sqrt{b^{16}}$

A b^{14}

B b^{8}

C b^{4}

D $\left|b^{4}\right|$

Answer

23 Simplify: $\sqrt{b^6}$

A b^4

B b^3

C $\left|b^4\right|$

D $\left|b^3\right|$

Answer

24 Simplify: $\sqrt{m^{24}}$

A m^{12}

B m^{22}

C $\left|m^{12}\right|$

D $\left|m^{8}\right|$

Answer

25 Simplify: $\sqrt{h^{14}}$

A h^7

B h^{12}

C $\left|h^7\right|$

D $\left|h^{12}\right|$

Answer

Square Roots of Fractions

For square roots of fractions, take the square root the numerator (top) and denominator (bottom) separately.

$\sqrt{\frac{4}{9}}$ $\sqrt{\frac{36}{64}}$ $\sqrt{\frac{25}{x^6}}$ $\sqrt{\frac{x^8}{100}}$

Teacher Notes

26 $\sqrt{\frac{16}{25}} =$

A $\frac{4}{10}$

B $\frac{8}{10}$

C $\frac{4}{5}$

D no real solution

Answer

27 $\sqrt{\frac{49}{100}} =$

A $\frac{7}{10}$

B $\frac{7}{50}$

C $\frac{7}{5}$

D no real solution

Answer

28 $\sqrt{\frac{16}{100}} =$

A $\frac{2}{5}$

B $\frac{4}{10}$

C $\frac{4}{5}$

D no real solution

Answer

29 $\sqrt{\frac{16}{x^8}} =$

A $\frac{8}{x^6}$

B $\frac{4}{|x^4|}$

C $\frac{4}{x^4}$

D no real solution

Answer

30 $\sqrt{\frac{-16}{25}} =$

A $\frac{4}{10}$

B $\frac{8}{10}$

C $\frac{4}{5}$

D no real solution

Answer

Irrational Roots

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Simplifying Radicals

$\sqrt{9}$ is said to be a <u>*rational*</u> number because there is a perfect square that equals the radicand. $(3^2 = 9)$

If a radicand cannot be made into a perfect square, the root is said to be <u>*irrational*</u>, like $\sqrt{24}$.

Simplifying Radicals

The commonly accepted form of a radical is called simplest radical form.

To simplify numbers that are not perfect squares, start by breaking the radicand into factors and then breaking the factors into factors and so on until only prime numbers are left. This is called prime factorization.

Prime Factorization

Examples of Prime Factorization:

$$28 = 7 \cdot 4 = 7 \cdot 2 \cdot 2 = 7 \cdot 2^2$$

$$88 = 8 \cdot 11 = 4 \cdot 2 \cdot 11 = 2 \cdot 2 \cdot 2 \cdot 11 = 2^3 \cdot 11$$

$$40 = 4 \cdot 10 = 2 \cdot 2 \cdot 2 \cdot 5 = 2^3 \cdot 5$$

Note: There is maybe more than one way to break a part
a number, but the prime factorization will always be the same.

31 Which of the following is the prime factorization of 24?

A 3(8)

B 4(6)

C 2(2)(2)(3)

D 2(2)(2)(3)(3)

Answer

32 Which of the following is the prime factorization of 72?

A 9(8)

B 2(2)(2)(2)(6)

C 2(2)(2)(3)

D 2(2)(2)(3)(3)

Answer

33 Which of the following is the prime factorization of 12?

A 3(4)

B 2(6)

C 2(2)(2)(3)

D 2(2)(3)

Answer

34 Which of the following is the prime factorization of 24 rewritten as powers of factors?

A $2^2 3^2$

B $2^3 3^2$

C $2^2 3$

D $2^3 3$

Answer

35 Which of the following is the prime factorization of 72 rewritten as powers of factors?

A $2^2 3^2$

B $2^3 3^2$

C $2^2 3$

D $2^3 3$

Answer

Simplifying Non-Perfect Square Radicands

$$\sqrt{24} = \sqrt{2 \cdot 2 \cdot 2 \cdot 3} = \sqrt{2^2 \cdot 2 \cdot 3} = \sqrt{2^2} \cdot \sqrt{2 \cdot 3} = 2\sqrt{6}$$

Find the prime factorization of the radicand, group prime factors to make perfect squares. Simplify. This is simplest radical form.

$\sqrt{72}$ $\sqrt{18}$ $\sqrt{360}$

Teacher Notes

36 Simplify: $\sqrt{80}$

A $2\sqrt{20}$

B $4\sqrt{5}$

C $16\sqrt{5}$

D already in simplified form

Answer

37 Put in simplest radical form: $\sqrt{60}$

A $2\sqrt{15}$

B $4\sqrt{15}$

C $6\sqrt{10}$

D already in simplified form

Answer

38 Put in simplest radical form: $\sqrt{30}$

A $3\sqrt{10}$

B $6\sqrt{5}$

C $2\sqrt{15}$

D already in simplified form

Answer

39 Simplify: $\sqrt{396}$

A $12\sqrt{33}$

B $6\sqrt{11}$

C $9\sqrt{22}$

D already in simplified form

Answer

40 Which of the following is not an irrational number?

A $\sqrt{40}$

B $\sqrt{60}$

C $\sqrt{80}$

D $\sqrt{100}$

Answer

Simplifying Radicals

If there is a number, or expression, on the outside of the root remember that it is held together by multiplication. To simplify, put the root in simplest radical form and multiply.

$-2\sqrt{24}$ $7\sqrt{88}$ $-6\sqrt{125}$ $18\sqrt{45}$

Teacher Notes

41 Put in simplest radical form: $-6\sqrt{20}$

A $-12\sqrt{5}$

B $24\sqrt{5}$

C $-24\sqrt{5}$

D *Solution not shown*

Answer

42 Simplify: $3\sqrt{72}$

A $18\sqrt{6}$

B $18\sqrt{2}$

C $27\sqrt{2}$

D *Solution not shown*

Answer

43 Put in simplest radical form: $-5\sqrt{200}$

A $-80\sqrt{2}$

B $-20\sqrt{2}$

C $-20\sqrt{10}$

D *Solution not shown*

Answer

44 Put in simplest radical form: $6\sqrt{32}$

A $24\sqrt{2}$

B $36\sqrt{2}$

C $12\sqrt{8}$

D *Solution not shown*

Answer

45 Put in simplest radical form: $-2\sqrt{98}$

A $-14\sqrt{7}$

B $-4\sqrt{7}$

C $-14\sqrt{2}$

D *Solution not shown*

Answer

Simplifying Radicals with Absolute Values

The same process goes for variables, but absolute value signs need to be included where appropriate.

Absolute value symbols are required when the initial exponent is even and the exponent after taking the root is odd. If the initial exponent is odd, you will not need absolute values.

Simplifying Radicals with Absolute Values

Examples:

$\sqrt{x^6y^9}$ $\sqrt{6m^3n^4p^3}$ $\sqrt{8r^8s^{10}}$ $\sqrt{24x^2y^5}$

Teacher Notes

46 Simplify: $\sqrt{36m^7n^8}$

A $6|m^3|n^4\sqrt{m}$

B $6m^3n^4\sqrt{m}$

C $6m^6n^4\sqrt{m}$

D *Answer not shown*

Answer

47 Put in simplest radical form: $3x\sqrt{32x^3y^7}$

A $48x^2\left|y^3\right|\sqrt{xy}$

B $12x^2y^3\sqrt{2xy}$

C $-12x^2y^3\sqrt{xy}$

D *Answer not shown*

Answer

48 Simplify: $\sqrt{30a^8b^{10}}$

A $5a^4\left|b^5\right|\sqrt{2}$

B $a^4b^5\sqrt{30}$

C $a^4\left|b^5\right|\sqrt{30}$

D *Answer not shown*

Answer

49 Put in simplest radical form: $-3\sqrt{48r^6s^6}$

A $-36\left|r^3s^3\right|$

B $-12r^3s^3\sqrt{3}$

C $-12\left|r^3s^3\right|\sqrt{3}$

D *Answer not shown*

Answer

50 Put in simplest radical form: $m\sqrt{25m^{14}n^{4}}$

A $5m^{8}n^{2}$

B $5m^{4}n^{2}$

C $5m^{4}\left|n^{2}\right|$

D *Answer not shown*

Answer

Cube Roots

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Cube Roots

If a square root cancels a square, what cancels a cube?

Teacher Notes

Cube Roots

$\sqrt[3]{64}$ is read "the cube root of 64"

A cube root looks like a square root. The difference is that little 3.

The 3 (or other root) is called the index of a radical. Square roots have an index of two, but it is not usually written.

An index indicates how many of each number or variable would allow taking a perfect root. A cube root is looking for groups of three, just as a square root is looking for groups of two.

Cube Roots

$$\sqrt[3]{125} = \sqrt[3]{5 \cdot 5 \cdot 5} = \sqrt[3]{5^3} = 5$$

The cube root and the cube cancel each other out.

Examples:

$\sqrt[3]{8}$ $\quad$ $\sqrt[3]{216}$ $\quad$ $\sqrt[3]{1}$ $\quad$ $\sqrt[3]{-27}$

Teacher Notes

Cube Roots

Try...

$$\sqrt[3]{x^3} \qquad \sqrt[3]{d^{12}} \qquad \sqrt[3]{m^{24}}$$

Teacher Notes

Cube Roots

Notice $\sqrt[3]{-27} = \sqrt[3]{(-3)^3} = -3$

Where as $\sqrt{-16}$ is not real.

Roots of negative numbers can only be taken if the index is odd.

82 Select all of the possible radicals that have real answers.

A $\sqrt[5]{-32}$

B $\sqrt[4]{-54}$

C $\sqrt[8]{-1}$

D $\sqrt[9]{-190}$

E $\sqrt[3]{-343}$

F $\sqrt[6]{140}$

G $\sqrt[3]{-1000}$

H $\sqrt[2]{-9}$

I $\sqrt[9]{-1}$

J $\sqrt[3]{27}$

Answer

83 Evaluate the radical: $\sqrt[3]{64}$

A 3

B 4

C 6

D 8

Answer

84 Evaluate the radical: $\sqrt[3]{216}$

A 3

B 4

C 6

D 8

Answer

85 Evaluate the radical: $\sqrt[3]{-8}$

A 2

B -2

C -4

D No real answer

Answer

86 Evaluate the radical: $\sqrt[3]{-1}$

A -1

B $-\frac{1}{3}$

C $\frac{1}{3}$

D 1

Answer

Cube Roots

Just like square roots, cube roots can also be put in simplest radical form. Instead of looking for groups of 2, just look for groups of 3!

$\sqrt[3]{72}$ $\sqrt[3]{128}$ $-4\sqrt[3]{250}$ $-6\sqrt[3]{48}$

Teacher Notes

Cube Roots

The techniques and methods for solving square roots of variables, fractions, and decimals also work with cube roots. No absolute value signs are needed when using an odd index. Therefore, the answers for cube roots will not require absolute values.

$\sqrt[3]{m^6}$ $\sqrt[3]{s^{15}}$ $\sqrt[3]{\frac{-8}{27}}$

Teacher Notes

Cube Roots

Put in simplest radical form:

$\sqrt[3]{16m^5n^4}$ $\qquad$ $\sqrt[3]{-48p^{13}q}$ $\qquad$ $\sqrt[3]{\dfrac{24j^{12}}{k^{10}}}$ $\qquad$ $2\sqrt[3]{-a^7b^3}$

Teacher Notes

87 Simplify: $\sqrt[3]{x^9 y^3}$

A $x^6 y$

B $x^2 y$

C $x^3 y$

D not possible

Answer

88 Simplify: $\sqrt[3]{27m^7n^6}$

A $3m^6n^6$

B $3m^3n^2\sqrt{3}$

C $3m^2n^2\sqrt{m}$

D not possible

Answer

89 Simplify: $\sqrt[3]{\dfrac{8p^4}{125m^3}}$

A $\dfrac{2p}{5m}$

B $\dfrac{2p\sqrt[3]{p}}{25m}$

C $\dfrac{4p\sqrt[3]{p}}{25m}$

D not possible

Answer

90 Simplify: $\sqrt[3]{\frac{27x^3y^{10}}{z^9}}$

A $\frac{3xy}{z}$

B $\frac{3xy^3\sqrt[3]{y}}{z^3}$

C $\frac{3xy^3}{z^3}$

D not possible

Answer

91 Simplify: $-4\sqrt[3]{24a^5b^6}$

A $-8ab^2\sqrt[3]{3a^2}$

B $-16ab^2\sqrt[3]{6a^2}$

C $8ab\sqrt{3a}$

D not possible

Answer

92 Put in simplest radical form: $\sqrt[3]{-27r^6s^{12}t^{15}}$

A $-3r^2s^4t^5$

B $-3r^2s^4t^7$

C $3r^3s^6t^7\sqrt[3]{t}$

D not possible

Answer

nth Roots

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Absolute Values

In general, $\sqrt[n]{t^n} = t$.
Absolute value signs are necessary

if n is even, the initial exponent is even and the variable has an odd powered exponent after taking the root.

$\sqrt[8]{x^8}$ $\sqrt[7]{m^7}$ $\sqrt[6]{t^{12}}$ $\sqrt[4]{16}$ $\sqrt[4]{m^{12}}$

Teacher Notes

nth Roots

Try...

$$\sqrt[4]{\frac{x^{16}}{625}}$$

$$\sqrt[4]{0.0081}$$

Teacher Notes

93 Simplify: $\sqrt[5]{32}$

A 2

B 6.4

C 4

D 8

Answer

94 Simplify: $\sqrt[4]{.0001}$

A 1

B $.1$

C $.01$

D $.001$

Answer

95 Simplify: $\sqrt[5]{t^5}$

A t

B $|t|$

C t^0

D t^2

Answer

96 Simplify: $\sqrt[8]{t^8}$

A t

B $|t|$

C t^0

D t^2

Answer

97 Simplify: $\sqrt[4]{\frac{n^4}{81}}$

A $\frac{n}{3}$

C $\frac{n}{9}$

B $\frac{|n|}{3}$

D $\frac{|n|}{9}$

Answer

98 Simplify: $\sqrt{\dfrac{n^{-2}m^{6}}{n^{4}}}$

A $\dfrac{m^3}{n^3}$

B $\left|\dfrac{m^3}{n^3}\right|$

C $\dfrac{n^{-1}m^3}{n^2}$

D $\dfrac{\left|n^{-1}m^3\right|}{n^2}$

Answer

99 Simplify: $\sqrt[3]{(x-1)^3}$

A $x-1$

B $|x-1|$

C $|x|-|1|$

D *simplified*

Answer

100 Simplify: $\sqrt[4]{(x-1)^4}$

A $x-1$

B $|x-1|$

C $|x|-|1|$

D *simplified*

Answer

Simplest Radical Form of Variables

Divide the index into the exponent. The number of times the index goes into the exponent becomes the power on the outside of the radical and the remainder is the power of the radicand.

$$\sqrt[3]{x^7} = \sqrt[3]{x^6 x} = x^2\sqrt[3]{x} \qquad \sqrt[5]{x^{19}} = \sqrt[5]{x^{15}x^4} = x^3\sqrt[5]{x^4}$$

Absolute Value Signs

As always, what about absolute value signs?

An absolute value sign is needed if the index is even, the starting power of the variable is even and the answer outside the radical is an odd power.

Examples of when absolute values are needed:

$$\sqrt[4]{x^{14}} = \left|x^3\right|\sqrt[4]{x^2}$$

$$\sqrt[6]{m^8} = |m|\sqrt[6]{m^2}$$

n^{th} Roots

Try...

$\sqrt{8x^5y^6z^4}$ $\quad$ $5y^2\sqrt[3]{16x^7y^4z}$ $\quad$ $-2\sqrt[4]{m^{13}n^{22}}$

Teacher Notes

101 Simplify: $\sqrt[4]{x^5}$

A $x\sqrt{x}$

B $|x|\sqrt{x}$

C $x\sqrt[4]{x}$

D $|x|\sqrt[4]{x}$

Answer

102 Simplify: $\sqrt[4]{x^6}$

A $x\sqrt{x^2}$

B $|x|\sqrt{x^2}$

C $x\sqrt[4]{x^2}$

D $|x|\sqrt[4]{x^2}$

Answer

103 Simplify: $\sqrt[4]{m^{12}n^{14}p^{16}}$

A $m^3 n^3 p^4 \sqrt[4]{n^2}$

B $|m^3| n^3 p^4 \sqrt[4]{n^2}$

C $m^3 |n^3| p^4 \sqrt[4]{n^2}$

D $|m^3 n^3| p^4 \sqrt[4]{n^2}$

Answer

104 Simplify: $\sqrt[4]{32a^2b^5c^6}$

A $2|ab|c\sqrt[4]{2bc^2}$

B $2|b|c\sqrt[4]{2a^2bc^2}$

C $2|bc|\sqrt[4]{2a^2bc^2}$

D $2b|c|\sqrt[4]{2a^2bc^2}$

Answer

105 Simplify: $\sqrt[8]{64r^{16}s^{8}}$

A $2r^{2}s$

B $2r^{2}|s|$

C $2r^{8}|s|$

D $2r^{2}s\sqrt[8]{2}$

Answer