15.2 Simplifying Radical Expressions

● SIMPLIFYING RADICAL EXPRESSIONS

A radical expression is simplified when there are no perfect square factors inside the radical; i.e., when you take as much as you can out of the radical. Simplifying can be done using the following rule:

PRODUCT RULE

$$\sqrt{a \bullet b} = \sqrt{a} \bullet \sqrt{b} \qquad \textbf{OR} \qquad \sqrt{a} \bullet \sqrt{b} = \sqrt{a \bullet b}$$

The product rule can be used to simplify radicands that are not perfect squares. Simply factor the radicand **using a perfect square as a factor**.

Sample Problem 1: ***Simplify*** $\sqrt{36}$

Solution: $\sqrt{36} = \sqrt{4 \bullet 9} = \sqrt{4} \bullet \sqrt{9} = 2 \bullet 3 = 6$

Sample Problem 2: ***Simplify*** $\sqrt{45}$

Solution: Since 45 is not a perfect square, we first factor 45 using a perfect square factor. It may help to list the first few perfect square factors, which are 1,4,9,16: In our case, we'll use 9. $\sqrt{45} = \sqrt{9 \bullet 5} = \sqrt{9} \bullet \sqrt{5} = 3 \bullet \sqrt{5} = 3\sqrt{5}$

Radical Expressions and Equations
Notes

15.1 Introduction to Radical Expressions

The symbol $\sqrt{}$ is called the **square root** and is defined as follows:

$$\sqrt{a} = c \quad \text{only if} \quad c^2 = a$$

Sample Problem: ***Simplify*** $\sqrt{16}$

Solution: $\sqrt{16} = 4$ since $4^2 = 16$.

Note that every positive number has two square roots, a positive and a negative root. For example, the square roots of 16 are 4 and -4, since $4^2 = 16$ *and* $(-4)^2 = 16$. *The* $\sqrt{}$ *symbol implies the positive root, or the* ***principal square root***. *To get the negative root, a negative sign must be used in front of the square root sign as in* $-\sqrt{}$.

Sample Problem: ***Simplify*** $-\sqrt{16}$

Solution: $-\sqrt{16} = -4$

15.3 Addition and Subtraction of Radical Expressions

To **add/subtract** two radical terms, they **MUST BE LIKE TERMS**. In other words, the radicands **MUST** be the same. If the radicands are not the same, simplify the radicands to make them match and then combine the coefficients and leave the radicand alone.

Sample Problem: ***Add*** $4\sqrt{72}+7\sqrt{8}$

Solution:

$$4\sqrt{72}+7\sqrt{8}=4\sqrt{36\bullet 2}+7\sqrt{4\bullet 2}$$
$$=4\bullet 6\sqrt{2}+7\bullet 2\sqrt{2}$$
$$=24\sqrt{2}+14\sqrt{2}$$
$$=38\sqrt{2}$$

15.4 Multiplying and Dividing Radical Expressions

When multiplying two radical expressions, recall the product rule from before which states that the product of two radical expressions is the radical of the product.

PRODUCT RULE

$$\sqrt{a}\bullet\sqrt{b}=\sqrt{a\bullet b}$$

Sample Problem: Multiply, then simplify $\sqrt{12x^3y}\bullet\sqrt{6x^5y^4}$

Solution:

$$\sqrt{12x^3y}\bullet\sqrt{6x^5y^4}=\sqrt{12x^3y\bullet 6x^5y^4}=\sqrt{72x^8y^5}=\sqrt{36\bullet 2\bullet x^8\bullet y^4\bullet y}=6x^4y^2\sqrt{2y}$$

To multiply radical expressions with more than one term, use the product rule discussed earlier along with the distributive property. Multiply the inside of the radicals together and the outside of the radicals together, **then simplify if possible**.

Sample Problem: ***Multiply*** $(2\sqrt{3}-5\sqrt{2})(3\sqrt{3}+\sqrt{2})$

Solution: (FOIL) $(2\sqrt{3}-5\sqrt{2})(3\sqrt{3}+\sqrt{2}) = 2\sqrt{3}\bullet 3\sqrt{3} + 2\sqrt{3}\bullet\sqrt{2} - 5\sqrt{2}\bullet 3\sqrt{3} - 5\sqrt{2}\bullet\sqrt{2}$

$$= 6\sqrt{9} + 2\sqrt{6} - 15\sqrt{6} - 5\sqrt{4}$$

$$= 6\bullet 3 - 13\sqrt{6} - 5\bullet 2$$

$$= 18 - 13\sqrt{6} - 10$$

$$= 8 - 13\sqrt{6}$$

● RATIONALIZING DENOMINATORS

Often times in mathematics it is useful to write a fraction without a radical in the denominator.

The process of writing a fraction with a radical in the denominator as an equivalent fraction without a radical in the denominator is called **rationalizing the denominator.** To **rationalize a denominator**, try the following:

- Multiply the numerator and denominator by a radical term that will make the bottom radicand a perfect square.

Sample Problem: ***Rationalize*** $\frac{3}{\sqrt{5}}$

Solution: $\frac{3}{\sqrt{5}}\bullet\frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{\sqrt{25}} = \frac{3\sqrt{5}}{5}$

15.5 Solving Radical Equations

To solve equations with radicals,

1. Isolate the radical on one side of the equation.
2. Raise each side to the power of the index of the radical.

If the equation still contains a radical, repeats steps 1 and 2.

3. Solve the resulting equation.
4. **Ensure that your answer works in the original equation.**

Sample Problem: *Solve for x* a. $\sqrt{x-5}+2=7$ b. $\sqrt{2x+1}+1=x$

Solution: a.

$$\sqrt{x-5}+2=7$$
$$\sqrt{x-5}=5$$
$$x-5=25$$
$$x=30$$

Check:

$$\sqrt{30-5}+2=7$$
$$\sqrt{25}+2=7$$
$$5+2=7$$

The solution is 30.

Solution: b.

$$\sqrt{2x+1}+1=x$$
$$\sqrt{2x+1}=x-1$$
$$2x+1=(x-1)^2$$
$$2x+1=x^2-2x+1$$
$$0=x^2-4x$$
$$0=x(x-4)$$
~~$x=0$~~ $x=4$

Only $x=4$ is a solution to the original equation.