Solving Rational Equations

Return to
Table of
Contents

Solving Rational Equations

Step 1: Find LCD

Step 2: Multiply EACH TERM by LCD

Step 3: Simplify

Step 4: Solve

Step 5: Check for Extraneous Solutions

Teacher Notes

Extraneous Solutions

There is a fifth step to solving rational equations but some additional vocabulary is needed first.

An **Extraneous Solution** occurs when the solution of a problem is not valid when substituted into the original problem or causes the original problem to be undefined.

For Example:

Extraneous solution results in a false mathematical statement when substituted into the original equation, such as:

$$5 \neq 4$$

Extraneous solution results in undefined terms, such as:

$$\frac{1}{x-2} \qquad x \neq 2$$

Example

Solve the following rational equation:

$$\frac{1}{h+2}+\frac{1}{h-2}=\frac{4}{h^2-4}$$

Step 1: LCD = (h + 2)(h - 2)

Step 2: $$\frac{1}{h+2}(h+2)(h-2)+\frac{1}{h-2}(h+2)(h-2)=\frac{4}{h^2-4}(h+2)(h-2)$$

Step 3: $$(h-2)+(h+2)=4$$

click

Step 4:

Example Continued

Step 5: $$\frac{1}{2+2}+\frac{1}{2-2}=\frac{4}{2^2-4}$$

$$\frac{1}{4}+\frac{1}{0}\neq\frac{4}{0}$$

Explanation

When the solution of h = 2 is substituted into the original equation, it creates two undefined terms:

$$\frac{1}{0} \qquad \frac{4}{0}$$

This means that h = 2 is an extraneous solution and the rational equation has no solution.

Example

Solve: $\frac{3}{x+2}+\frac{4}{x-2}=\frac{5}{x^2-4}$

Step 1: $LCD: x^2-4 \; or \; (x-2)(x+2)$

Step 2: $(x-2)(x+2)\frac{3}{x+2}+(x-2)(x+2)\frac{4}{x-2}=\frac{5}{x^2-4}(x-2)(x+2)$

Step 3: $(x-2)3+(x+2)4=5$

Step 4: ***click***

Example Continued

Step 5:

$$\frac{3}{3/7+2}+\frac{4}{3/7-2}=\frac{5}{(3/7)^2-4}$$

$$\frac{3}{17/7}+\frac{4}{-11/7}=\frac{5}{-187/49}$$

$$\frac{21}{17}+\frac{-28}{11}=\frac{-245}{187}$$

$$\frac{231}{187}+\frac{-476}{187}=\frac{-245}{187}$$

$$\frac{-245}{187}=\frac{-245}{187}$$

Explanation

The solution $x=\frac{3}{7}$ results in a true mathematical statement when substituted into the original equation.

Therefore $x=\frac{3}{7}$ is a solution.

Solving Rational Equations

Example: Remember to find LCD and check all solutions.

$$\frac{3}{x} - \frac{2}{3x} = \frac{-7}{3x^2 - 6x}$$

Teacher Notes

27 Use Steps 1 - 4 to solve for *x*:

$$\frac{4}{x}+\frac{3}{7}=\frac{1}{7x}$$

A -9

B 9

C 24

D 30

Answer

28 Is the solution to the previous question valid when substituted into the original equation?

A Yes, the solution is valid.

B No, the solution creates a false mathematical statement and is therefore an extraneous solution.

C No, the solution creates an undefined term(s) and is therefore an extraneous solution.

Answer

29 Use Steps 1 - 4 to solve for *m*:

$$\frac{5}{2m}+\frac{2m}{m+1}=2$$

A -12

C 5

B -5

D 12

Answer

30 Is the solution to the previous question valid when substituted into the original equation?

A Yes, the solution is valid.

B No, the solution creates a false mathematical statement and is therefore an extraneous solution.

C No, the solution creates an undefined term(s) and is therefore an extraneous solution.

Answer

31 Use Steps 1 - 4 to solve for *x*:

(Choose all that apply)

$$\frac{-3}{x^2-5x+6}-\frac{2}{x^2-9}=-\frac{1}{x-2}$$

Answer

A -3

C 5

B -2

D 7

32 Are the solutions to the previous question valid when substituted into the original equation?

A Yes, both solutions are valid.

B No, both of the solutions create a false mathematical statement and are therefore extraneous solutions.

C No, one of the solutions creates an undefined term (s) and is therefore an extraneous solution.

Answer

33 Solve the following equation:

$$\frac{12r+19}{r^2+7r+12}-\frac{3}{r+3}=\frac{5}{r+4}$$

Answer

34 Is the solution to the previous question valid when substituted into the original equation?

A Yes, the solution is valid.

B No, the solution creates a false mathematical statement and is therefore an extraneous solution.

C No, the solution creates an undefined term(s) and is therefore an extraneous solution.

Answer

35 What is the solution of the equation

$$\frac{2m^2+3m-5}{m^2+4m-5}=4$$

Answer

From PARCC sample test

Basketball

Problem is from:

Illustrative Mathematics

Illustrations

Click for link for commentary and solution.

Chase and his brother like to play basketball. About a month ago they decided to keep track of how many games they have each won. As of today, Chase has won 18 out of the 30 games against his brother.

a) How many games would Chase have to win in a row in order to have a 75% winning record?

b) How many games would Chase have to win in a row in order to have a 90% winning record?

Basketball

Problem is from:

Illustrative Mathematics
Illustrations

Click for link for commentary and solution.

Chase and his brother like to play basketball. About a month ago they decided to keep track of how many games they have each won. As of today, Chase has won 18 out of the 30 games against his brother.

c) Is Chase able to reach a 100% winning record? Explain why or why not.

d) Suppose that after reaching a winning record of 90% in part (b), Chase had a losing streak. How many games in a row would Chase have to lose in order to drop down to a winning record below 55% again?