Standard Deviation and Normal Distribution

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Goals and Objectives

Students will be able to calculate the standard deviation of a data set and analyze a a normal distribution.

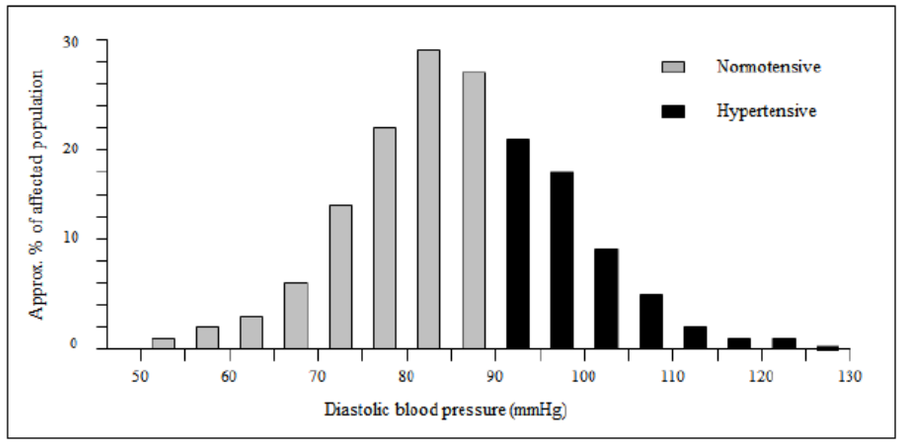
Why do we need this?

In short, the standard deviation of data represents how close the data is to its mean. It is used to report such things as results from political polls and data from medical experiments. We need to understand how these numbers are calculated to make informed decisions.

Graphs

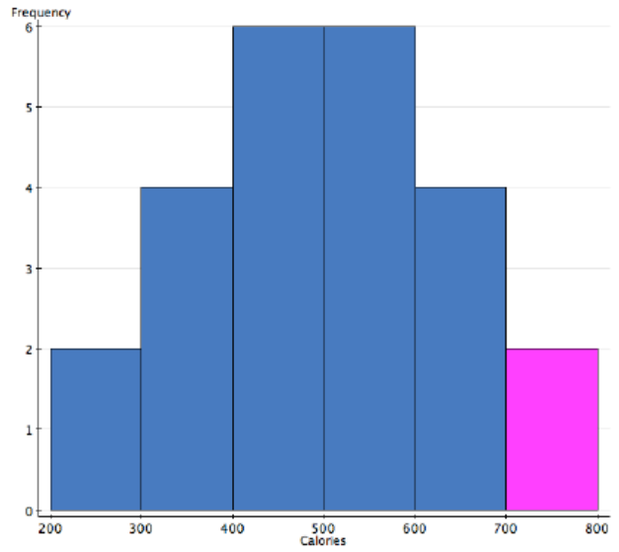
Check out the following graphs. What do they have in common?

Diastolic Blood Pressure



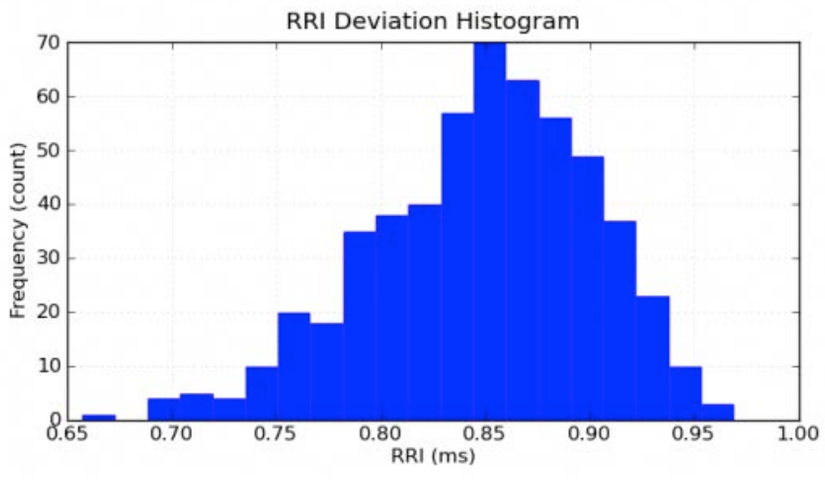
http://www.trialsjournal.com/content/6/1/5

Calories in French Fries



http://www.statcrunch.com/grabimage.php?image_id=427473

Intervals of Peaks of Heartbeats

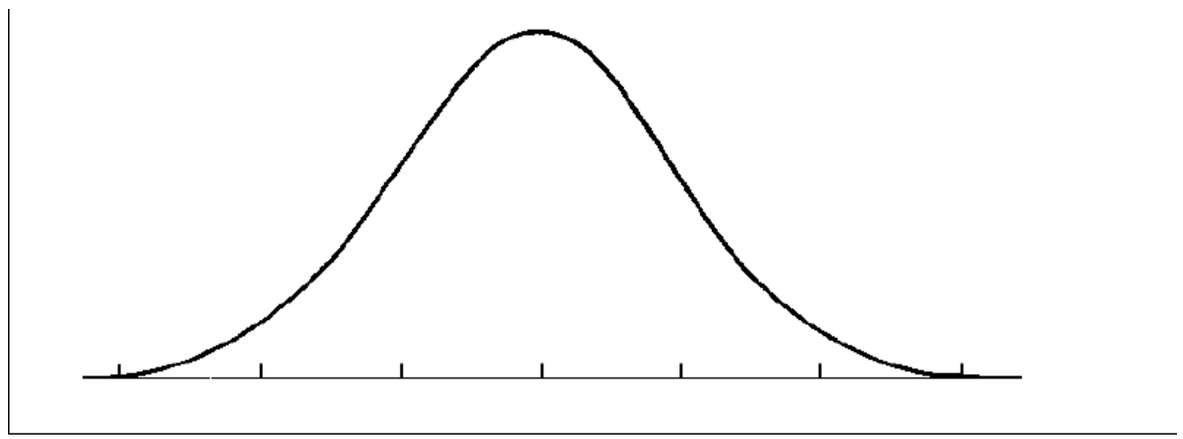


http://www.swharden.com/blog/2009-08-14-diy-ecg-machine-on-the-cheap/

Teacher Notes

Normal Distribution

Many different aspects of life, when measured and graphed, fit this type of distribution. Imagine a what the graph of height for humans, weight for bears or size of homes would look like. Most of the data would be around the same number (the mean), yet there would be some that would be larger or smaller. Finally, you would have the extremes that would be rare. This is called a Normal Distribution.

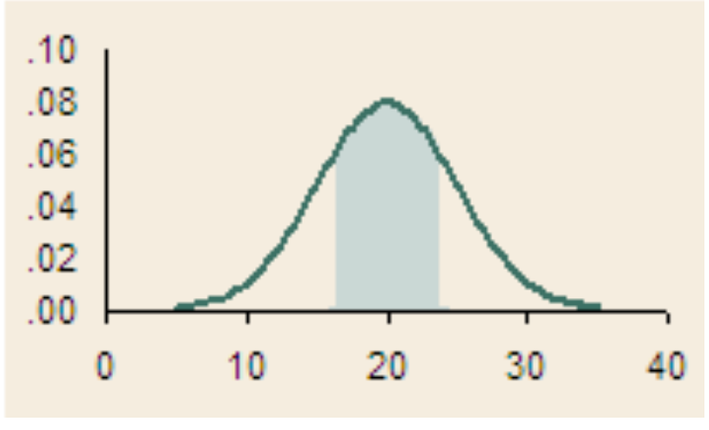


Normal Distribution

Normal Distributions are very useful when analyzing data. It allows you to calculate the probability that an event will happen as well as a percentile ranking of scores. Consider the following examples...

Normal Distribution

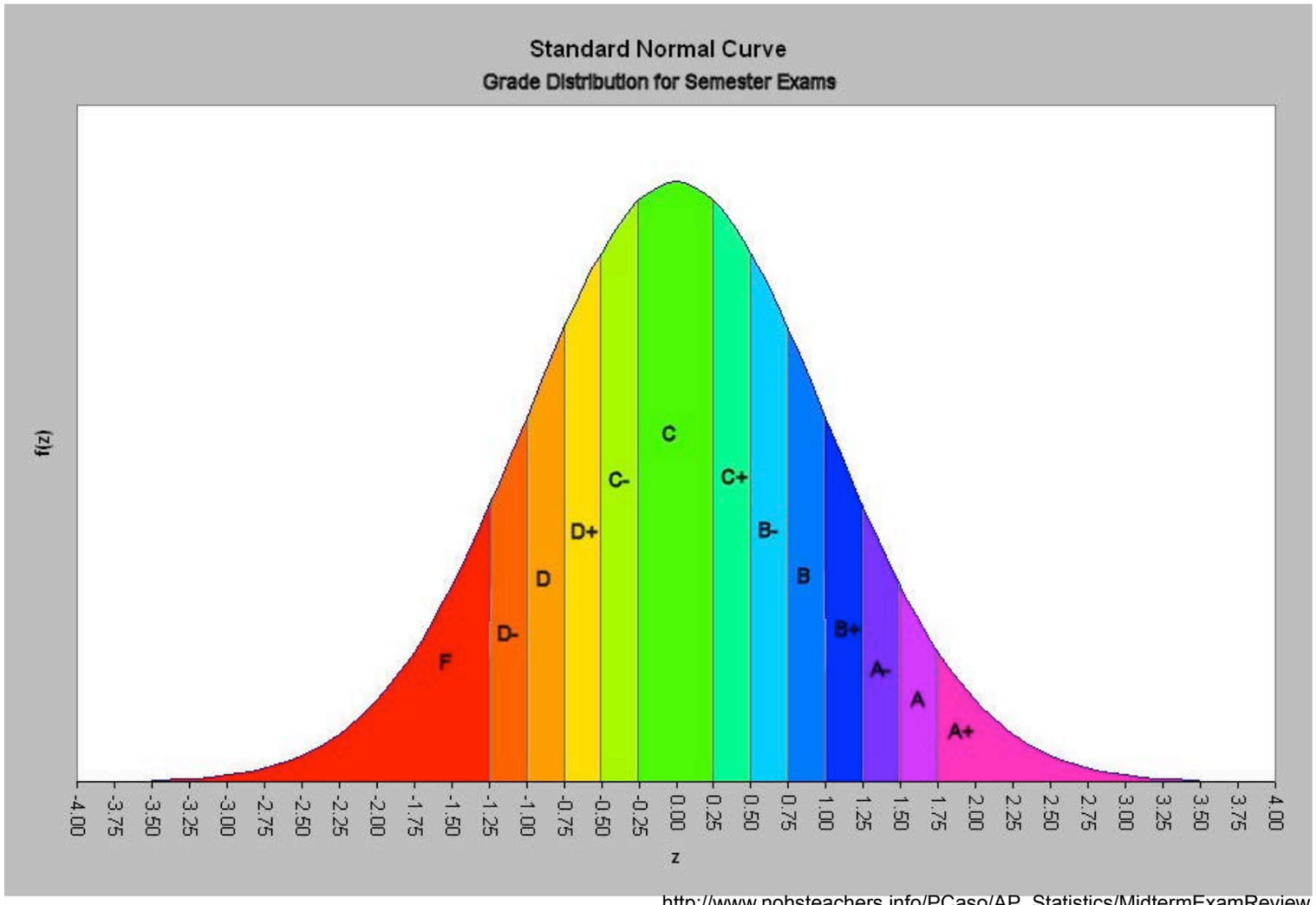
A tennis ball manufacturer measures the height their tennis balls bounce after dropping them from 5 feet off of the ground. The balls will not bounce the same height each time, but should be very close. A graph of this, after many trials, would begin to resemble a normal distribution. From here, you can calculate a mean height of the ball and use that to test other tennis balls from the factory to make sure that the quality is consistent. The blue shaded area would represent the range of acceptable heights.



Normal Distribution

A particular engineering school at a university prides itself on producing high quality engineers. Each class coming through has to take an introductory physics class. The professor uses a normal distribution to calculate grades such that only the top 5% of students get As. This ensures the course is challenging and that the best are the ones that continue on.

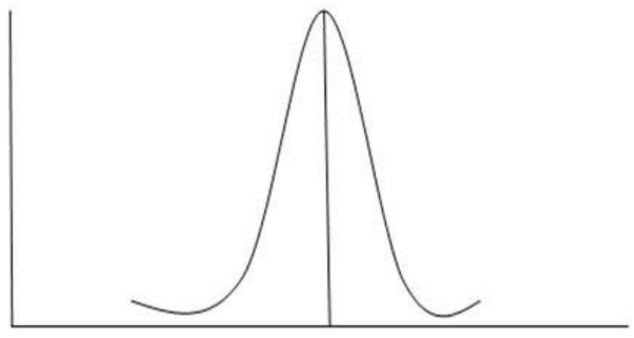
*note: graph does not represent top 5% with As.



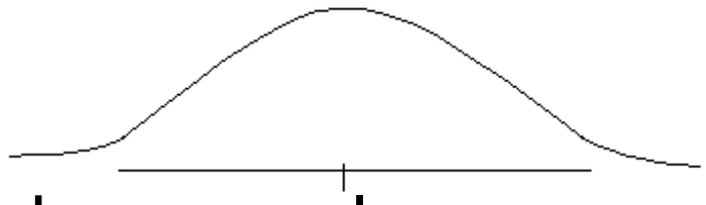
http://www.nohsteachers.info/PCaso/AP_Statistics/MidtermExamReview.htm

Normal Distribution

Using the mean and standard deviation takes into account different spreads of the graph. In fact, knowing the standard deviation of a study can tell you how reliable the study is. Small standard deviations indicate that the mean is a good representation of the information. Large standard deviations tell you that the data was actually very spread out and the mean may not be reliable.



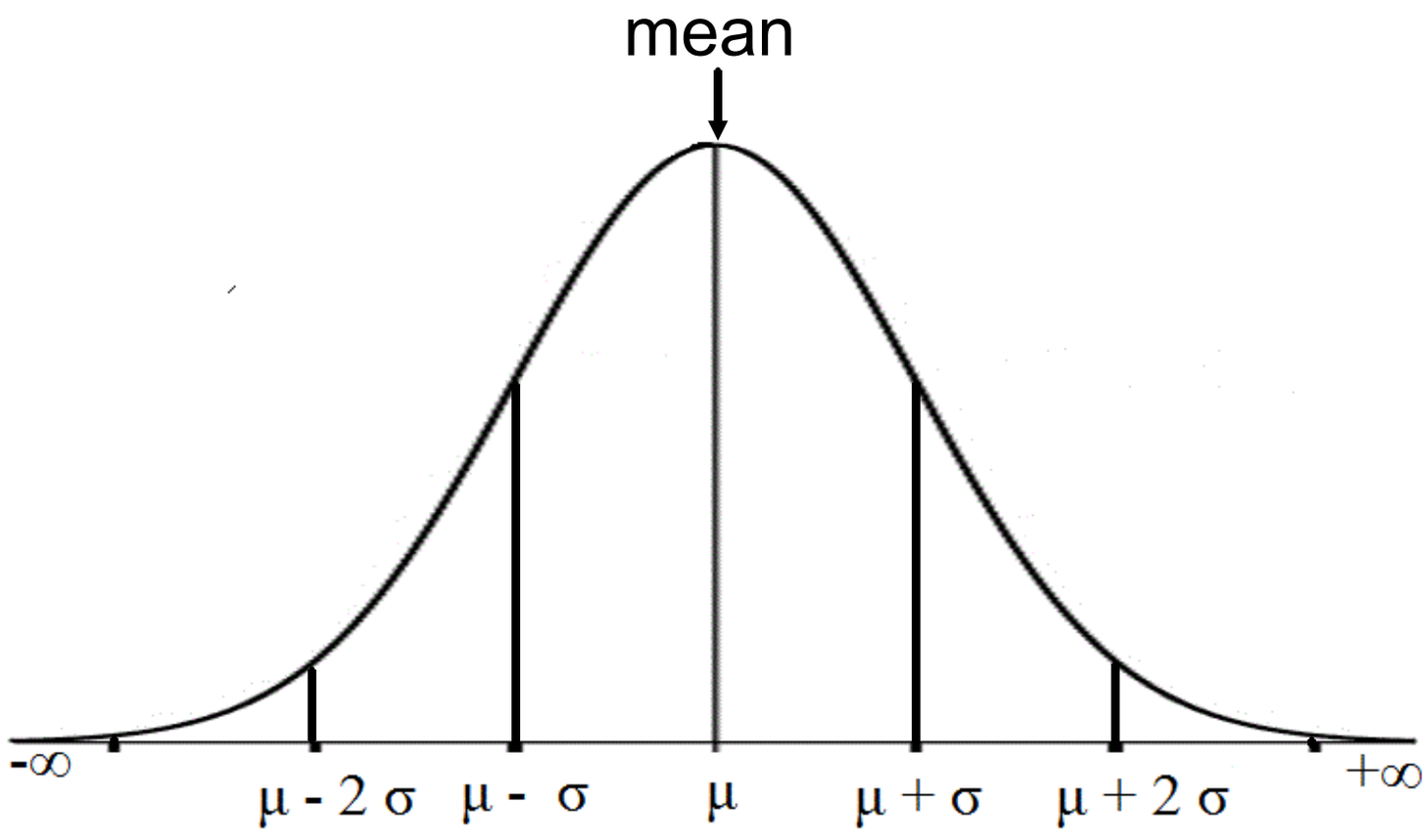
Small σ = small spread



Large σ = large spread

Normal Curve

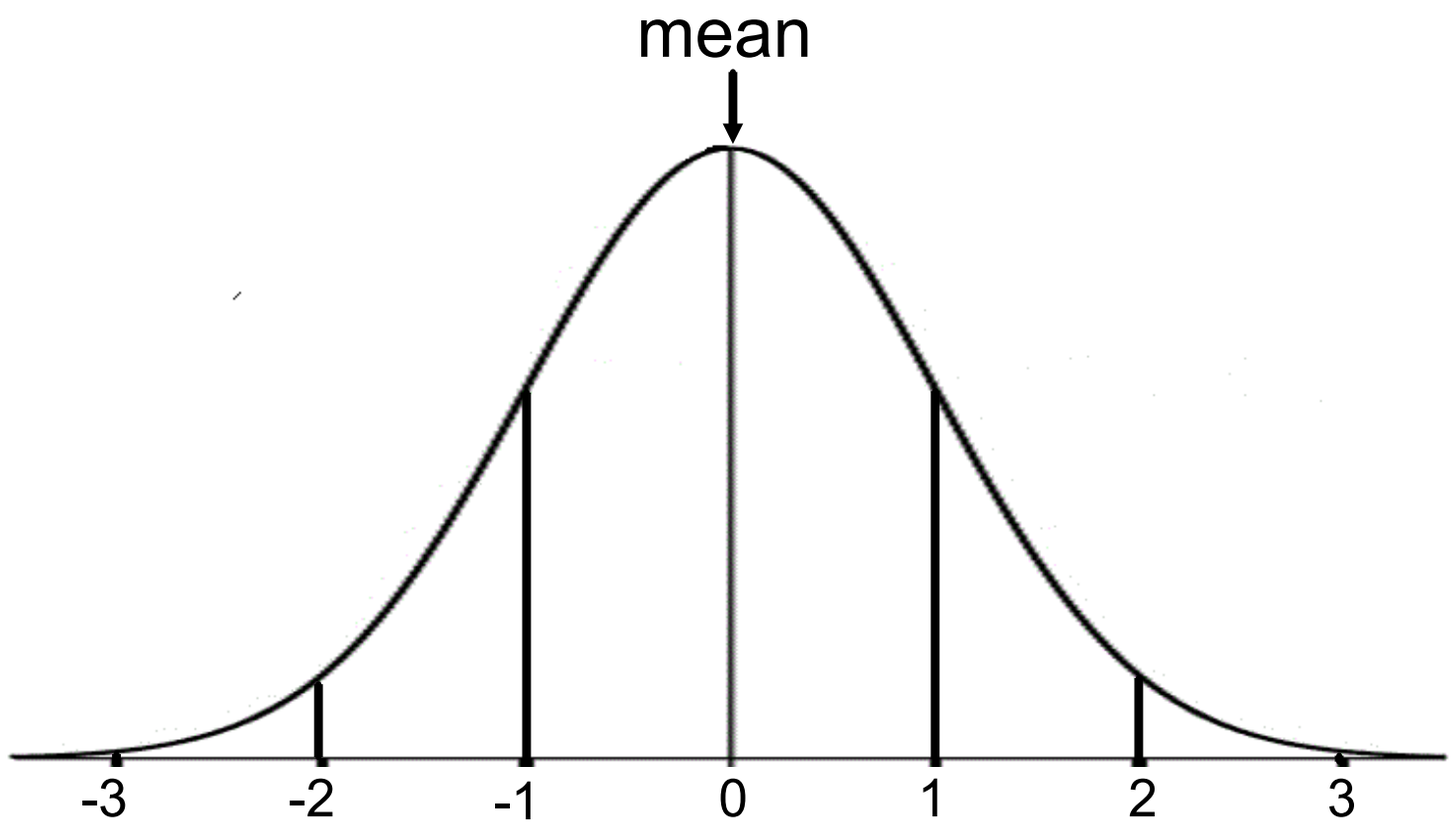
Normal Curves are created using the mean (μ or $\bar{x}$) of the data and the standard deviation (σ).



Teacher Notes

Normal Distribution

Much of the time, instead of having more complicated numbers for μ ± σ, we write 0, ±1 or ±2 representing the number of standard deviations from the mean as shown below. These numbers are also known as z-scores.



Teacher Notes

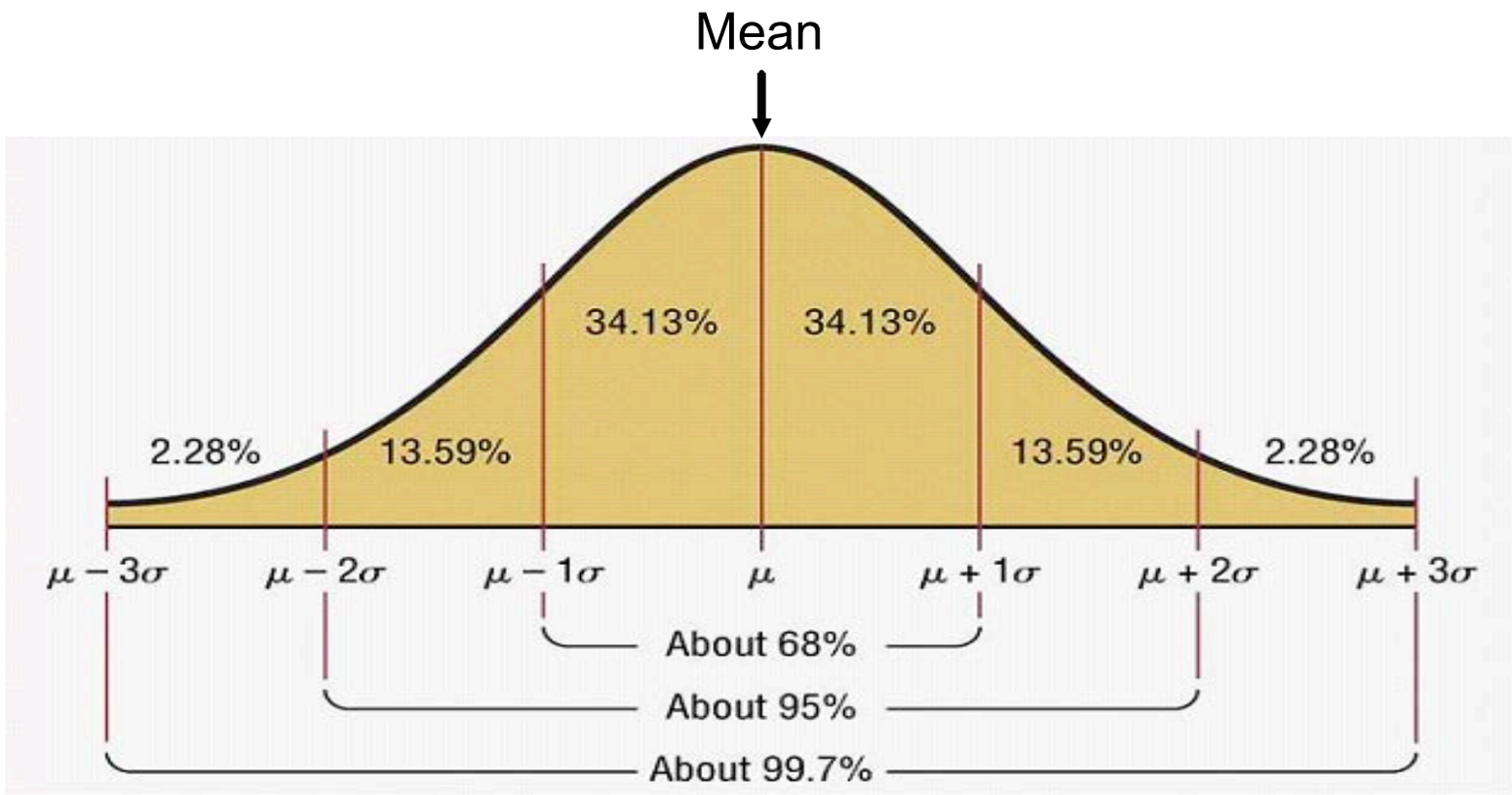
Normal Distribution

In normal distributions, the area under the curve is what is used to calculate percentages or probabilities. These numbers follow what is called the *Empirical Rule* and is the same for each distribution.

- 68% of all data will fall within 1 standard deviation of the mean.
- 95% of all data falls within 2 standard deviations of the mean.
- 99.7% of all data falls within 3 standard deviations of the mean.

The graph on the next page is an excellent illustration of this.

Normal Distribution



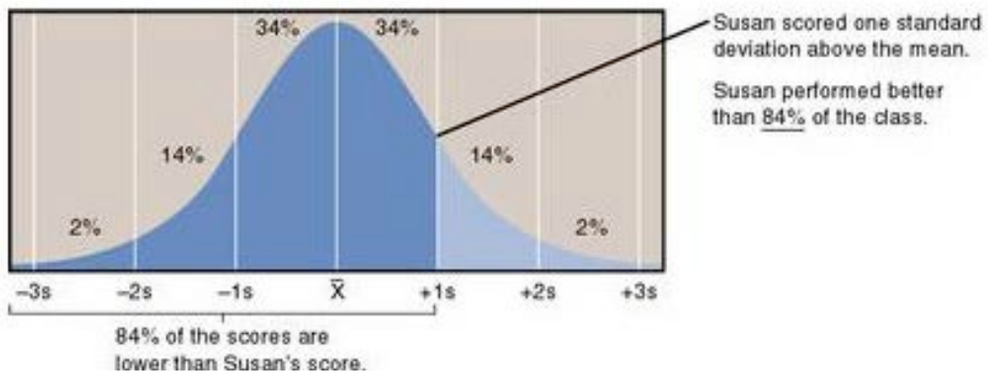
http://rchsbowman.wordpress.com/2009/11/29/statistics-notes-%E2%80%93-properties-of-normal-distribution-2/

Teacher Notes

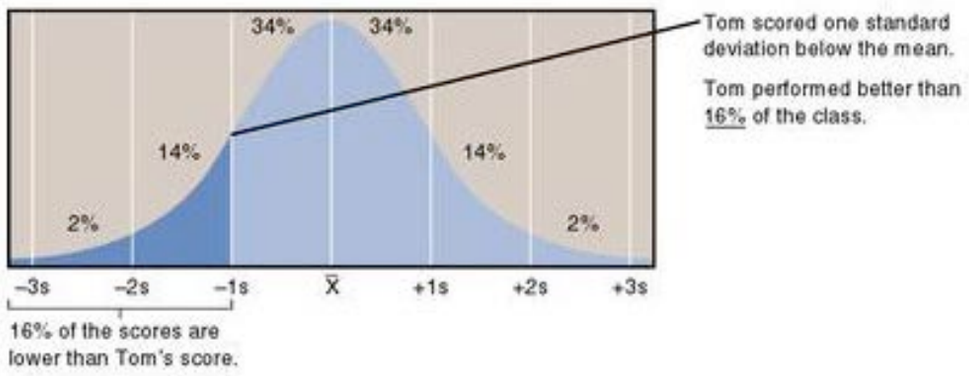
Normal Distribution

Each graph can be used differently even though there is a uniformity about their calculations.

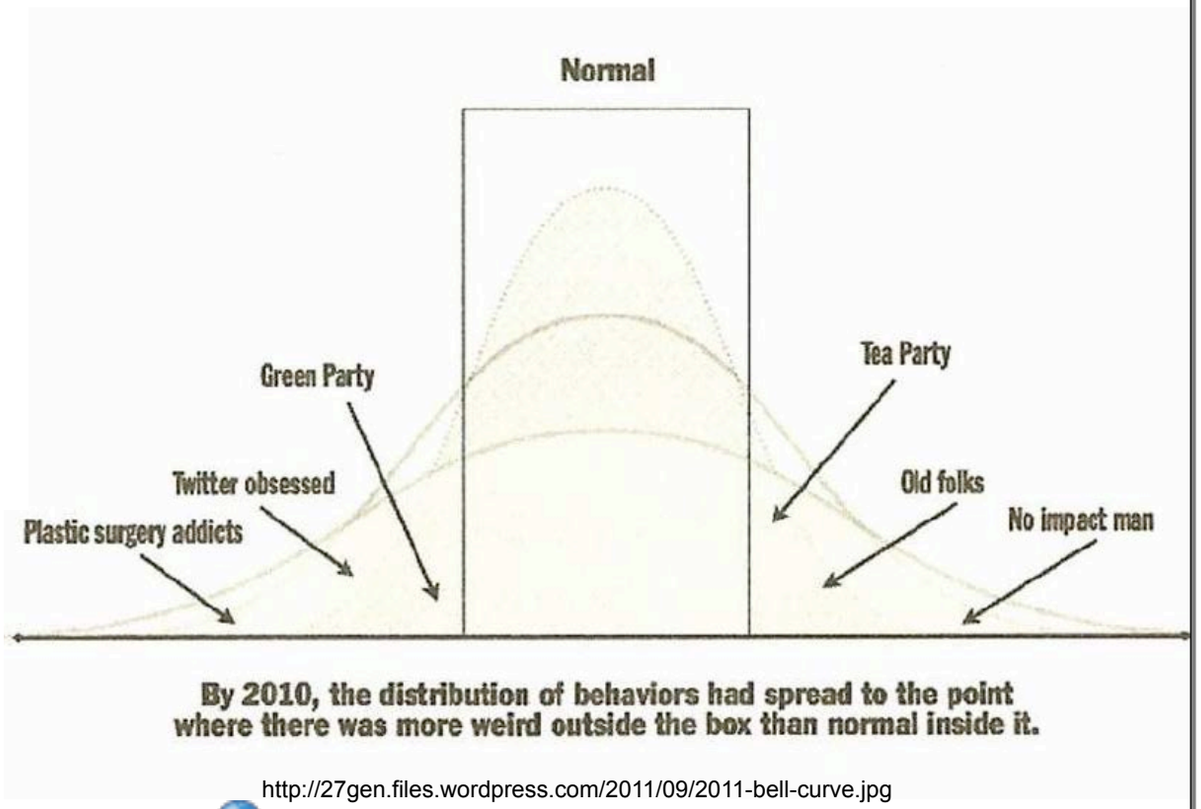
Normal Curve Interpretation 1



Normal Curve Interpretation 2



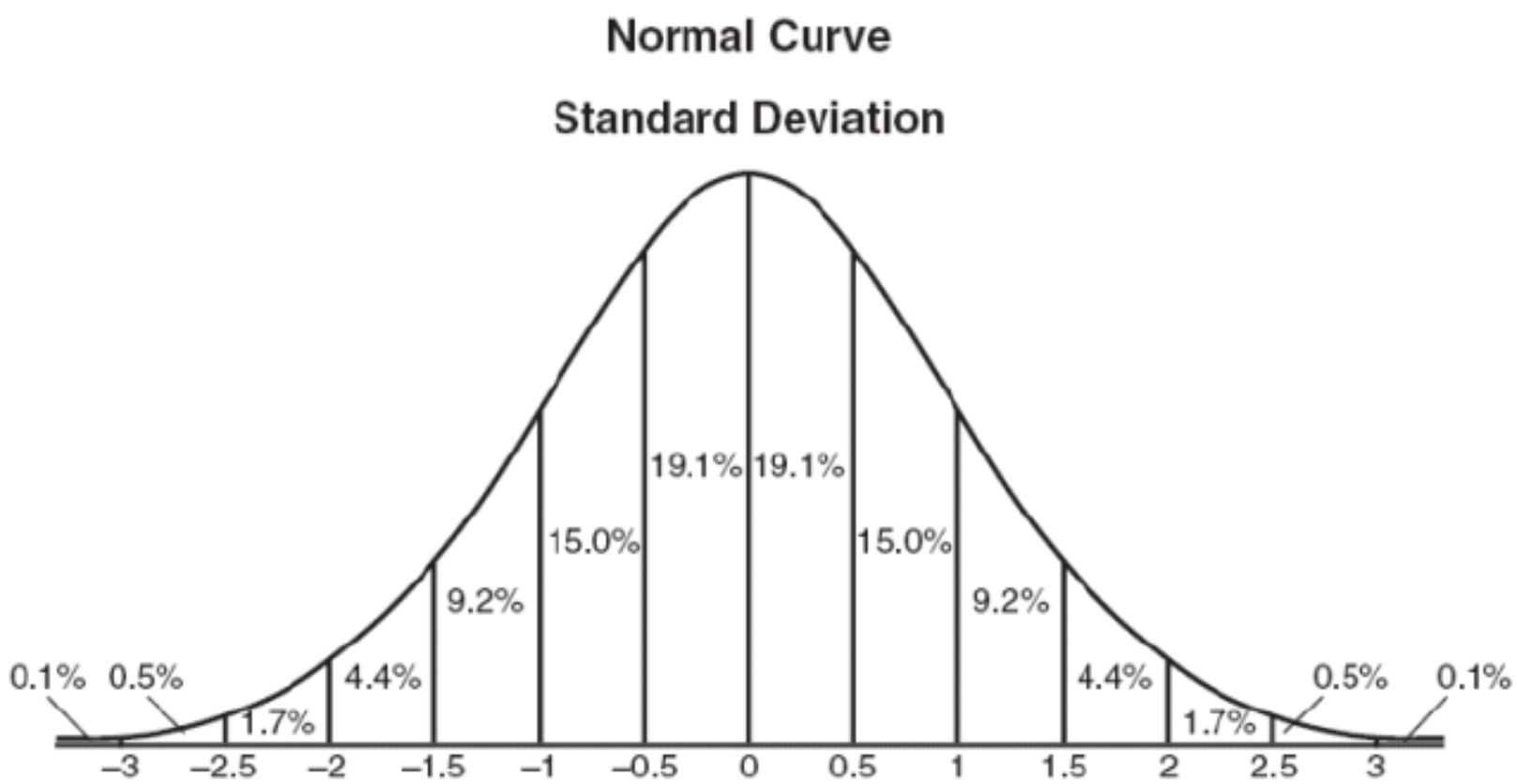
http://www.mhhe.com/socscience/intro/ibank/set4.htm



http://27gen.files.wordpress.com/2011/09/2011-bell-curve.jpg

Chart for Examples

Use this chart to answer the following questions.



http://www.regentsprep.org/Regents/math/algtrig/ATS2/NormalLesson.htm

Teacher Notes

Examples

a) John usually scores an average of 82% on his math tests with a standard deviation of 5%. What is the probability that John will get an between an 82% and an 87% on his next test?

Answer

b) At Big Mama's Gym, there is a special weight loss program that is a big hit. And, it works! At the start of the program 95.4% of all members, centered about the mean, weighed between 180 and 260 pounds. Find the average weight and the standard deviation of the data.

Example

c) A machine at Superfoods Food Factory puts a mean of 44 oz of mayonnaise in their bottles. The machine has a standard of deviation of 0.5 ounces. While filling 1000 bottles with mayonnaise, about how many times will the machine fill a bottle with 45 or more ounces?

Answer

Example

d) Scores on the final exam in Mr. Dahlberg's Precalculus classes are normally distributed. He calculates a mean to be 71% with a standard deviation of 7. What is the probability that a student in his classes will get between an 85 and a 92 on the final exam?

Answer

52 Battery lifetime is normally distributed for large samples. The mean lifetime is 500 days and the standard deviation is 61 days. To the nearest percent, what percent of batteries have lifetimes longer than 561 days?

Answer

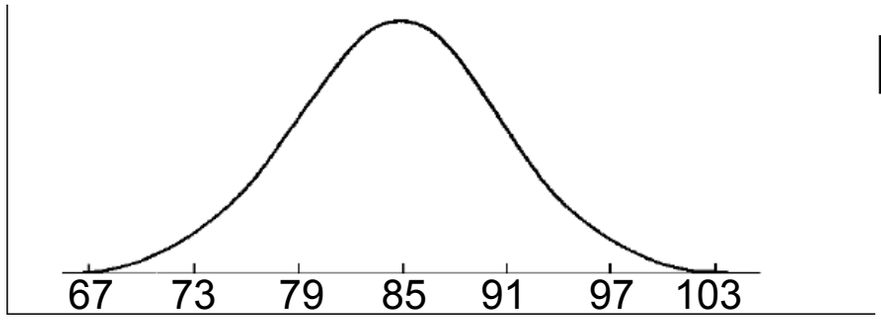
http://www.regentsprep.org/Regents/math/algtrig/ATS2/NormalPrac.htm

53 A normal distribution of a group the ages of 340 students has a mean age of 15.4 years with a standard deviation of 0.6 years. How many students are younger than 16 years? Express your answer to the nearest student.

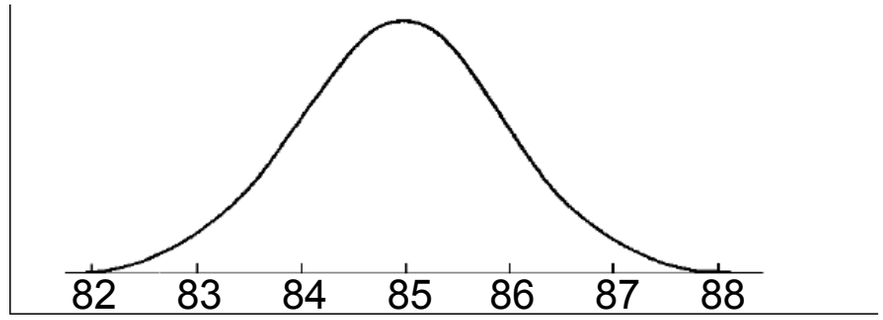
Answer

54 Which of the following curves represents a mean of 85 and a standard deviation of 6?

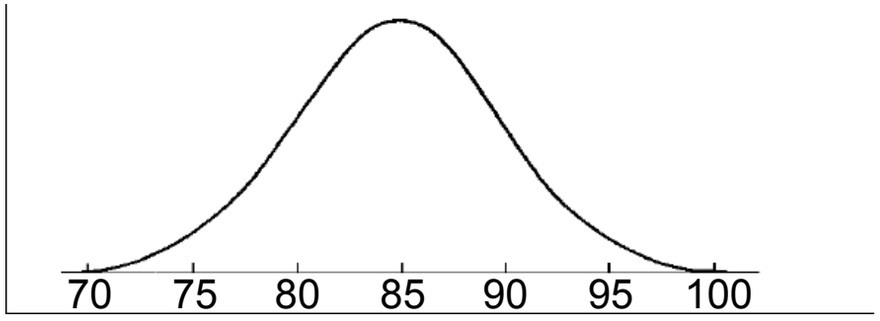
A



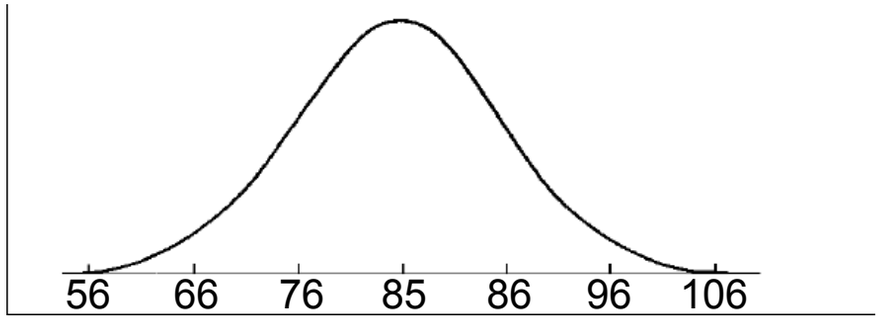
B



C



D



Answer

55 Given a mean of 27 and a standard deviation of 3 on a data set that is normally distributed, what is the number that is +2σ from the mean?

Answer

56 Given a mean of 27 and a standard deviation of 3 on a data set that is normally distributed, what is the number that is -3σ from the mean?

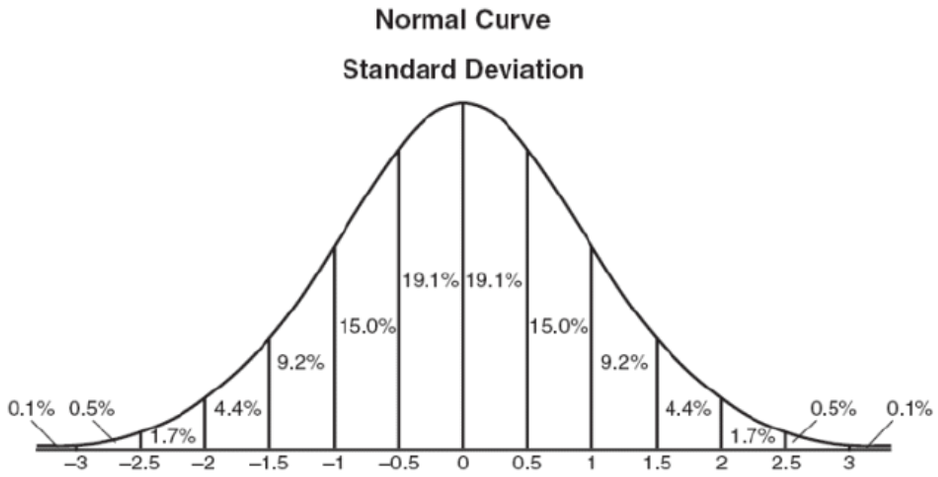
Answer

57 A set of information collected by the Department of Wildlife is normally distributed with a mean of 270 and a standard deviation of 12. What percent of the data falls between 246 and 258?

Answer

Z-Score

The graph we have been using to the right helps us find values that are multiples of 0.5 away from the mean. But what about numbers that are in between? For those, we use a formula for the z-score and a table of values.



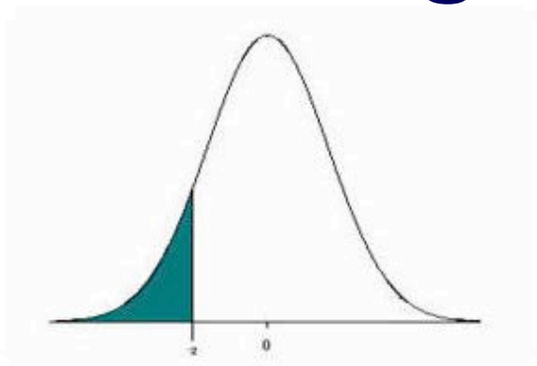
Z-Score

$$\text{z-score} = \frac{x - \mu}{\sigma}$$

$\mu = mean$
$\sigma = standard\ deviation$

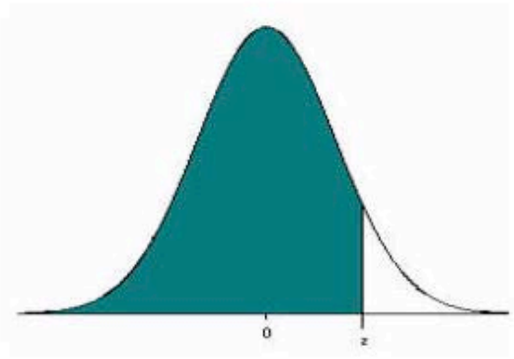
A table of z-scores is shown on the next 2 slides. Each score is associated with the amount of area under the normal curve from the score to the left.

Z-Scores: Negative



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Z-Scores: Positive



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Z-Score

Z-scores are what is used to calculate all of the percentile values that are reported for standardized tests. Remember how you are given a result of, say, the 94th percentile? This means that you have done better than 94% of the students who have taken the test. Welcome to a major use of z-scores, normal distribution and standard deviation!

$$\text{z-score} = \frac{x - \mu}{\sigma}$$

$\mu = mean$

$\sigma = standard\ deviation$

Z-Score

Example: On a test, your score was 83%. The mean of all of the tests was 79, the data was normally distributed and the standard deviation was 4.25. Find your z-score and then use the table to calculate the percentile.

Answer

$$\text{z-score} = \frac{x - \mu}{\sigma}$$

$\mu = mean$

$\sigma = standard\ deviation$

Z-Score

Your friend took the same test and got a score of 92%.
Find your friend's z-score and calculate their percentile.

Answer

58 Find the z-score for a 29 if the mean was 34 and the standard deviation is 2.3.

Answer

59 Which is the z-score and percent of area under the curve for a score of 520 in a normally distributed set of data with a mean of 565 and a standard deviation of 24.2.

A -1.86, 1.95%
B -1.86, 3.14%
C 1.86, 31.4%
D 1.86, 97.5%

Answer

60 A value has a z-score of 0.82. The mean for the data is 73 and the standard deviation is 2.16. What was the original value?

Answer

61 A student calculated a z-score of -1.25. What percentile does this score fall in?

Answer

62 Find the z-score of 10 if the data set is:

9 11 5 7 10 10 10 11 9 15

Answer