Trigonometry of the Right Triangle

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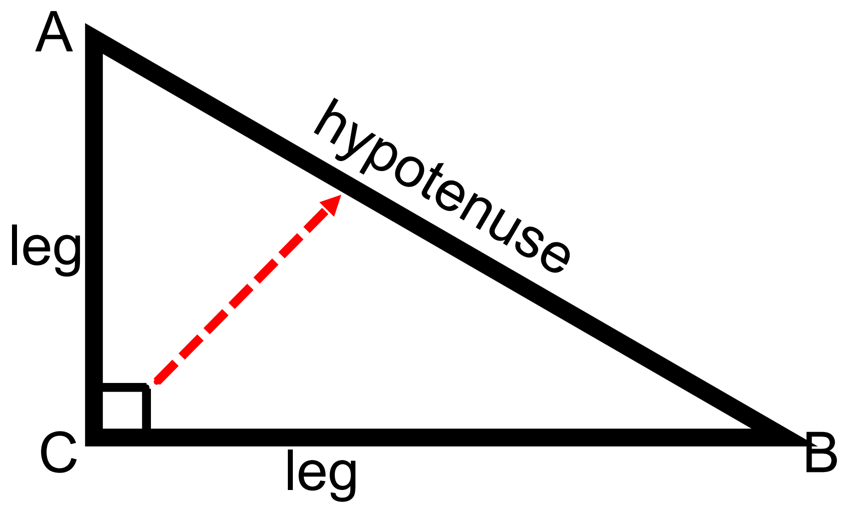
Trigonometry

Trigonometry means "measurement of triangles". In its earliest applications, it dealt with triangles and the relationships between the lengths of their sides and the angles between those sides.

Historically trig was used for astronomy and geography, but it has been used for centuries in many other fields. Today, among many other fields, it has applications in music, financial market analysis, electronics, probability, biology, medicine, architecture, economics, engineering and game development.

Recall the Right Triangle



The sum of the measures of the angles is 180°.

The **hypotenuse** is the longest side and opposite the right angle. The other two sides are called **legs**.

In any right triangle, the **Pythagorean Theorem** tell us that:

$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$.

Pythagorean Triples

Pythagorean Triples (these are helpful to know)

A **Pythagorean Triple** is a set of "whole numbers",
a, b and c that fits the rule:

$$a^2 + b^2 = c^2$$

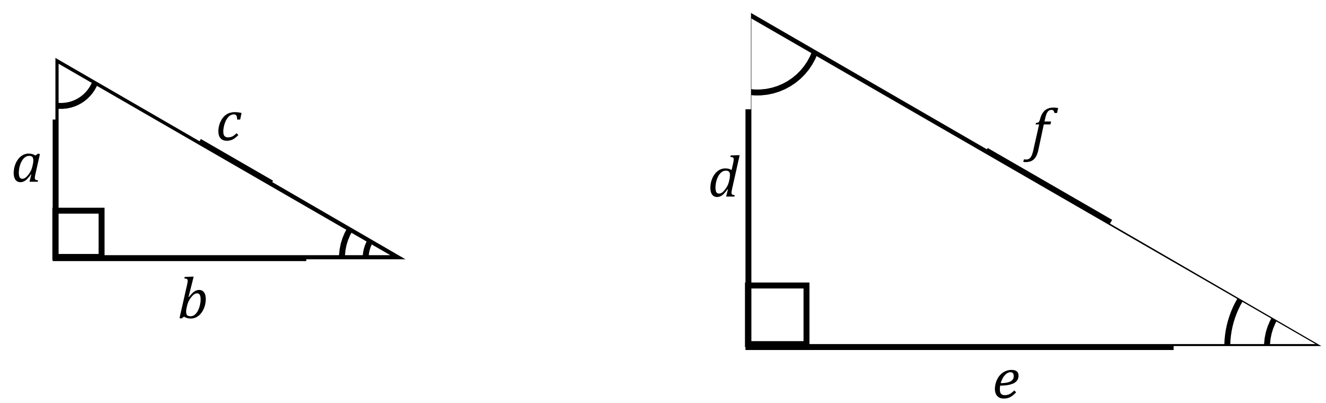
Recognizing these numbers can save time and effort
in solving trig problems.
Here are the first few:

3, 4, 5	5, 12, 13	7, 24, 25
8, 15, 17	9, 40, 41	11, 60, 61

Also, any multiple of a triple is another triple:
6, 8, 10   10, 24, 26 and so on

Similar Triangles

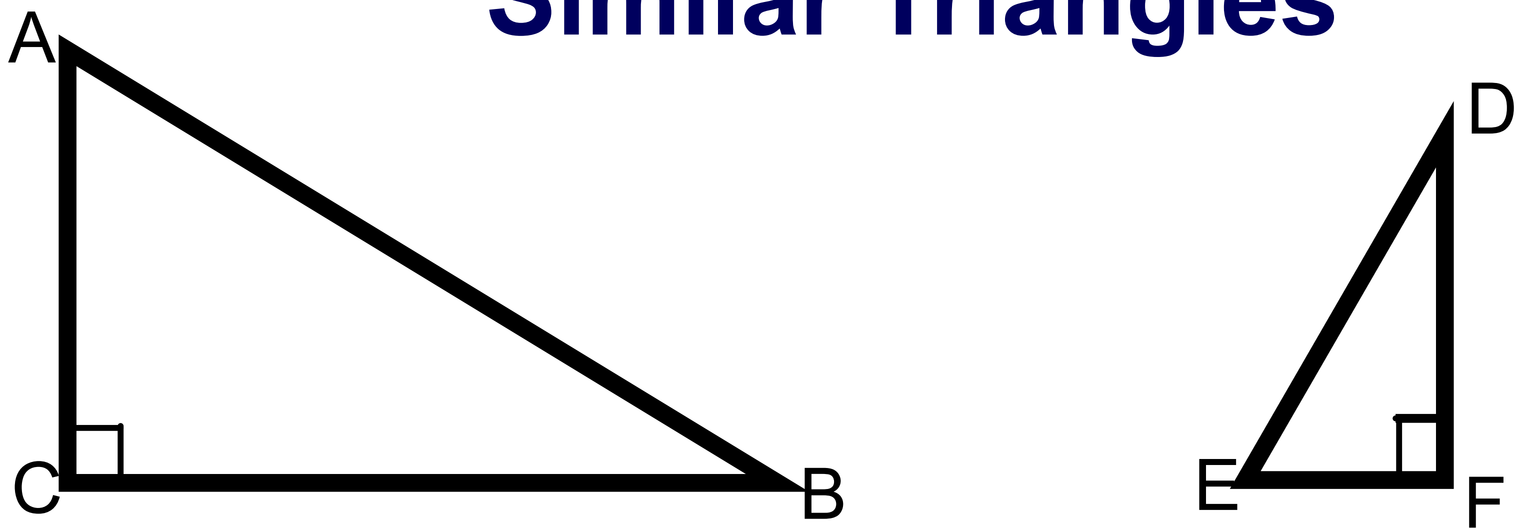
If the two acute angles of two right triangles are congruent, then the triangles are **similar** and the sides are **proportional**.



$$\frac{a}{b} = \frac{d}{e} \qquad \frac{a}{c} = \frac{d}{f} \qquad \frac{b}{c} = \frac{e}{f}$$

The ratios of the sides are the **trig ratios**.

Similar Triangles



If $\triangle ABC \sim \triangle DEF$, drag the measurements into the proportions:

$$\frac{AB}{AC} = \frac{\_\_\_}{\ } \qquad \frac{AC}{\ } = \frac{\_\_\_}{DF} \qquad \frac{AB}{\ } = \frac{\_\_\_}{DF}$$

EF *DE* *BC* *EF* *BC* *DE*

Trigonometric Ratios

The fundamental trig ratios are:

Sine abbreviated as "sin"
(pronounced like "sign")

Cosine abbreviated as "cos", but
pronounced "cosine"

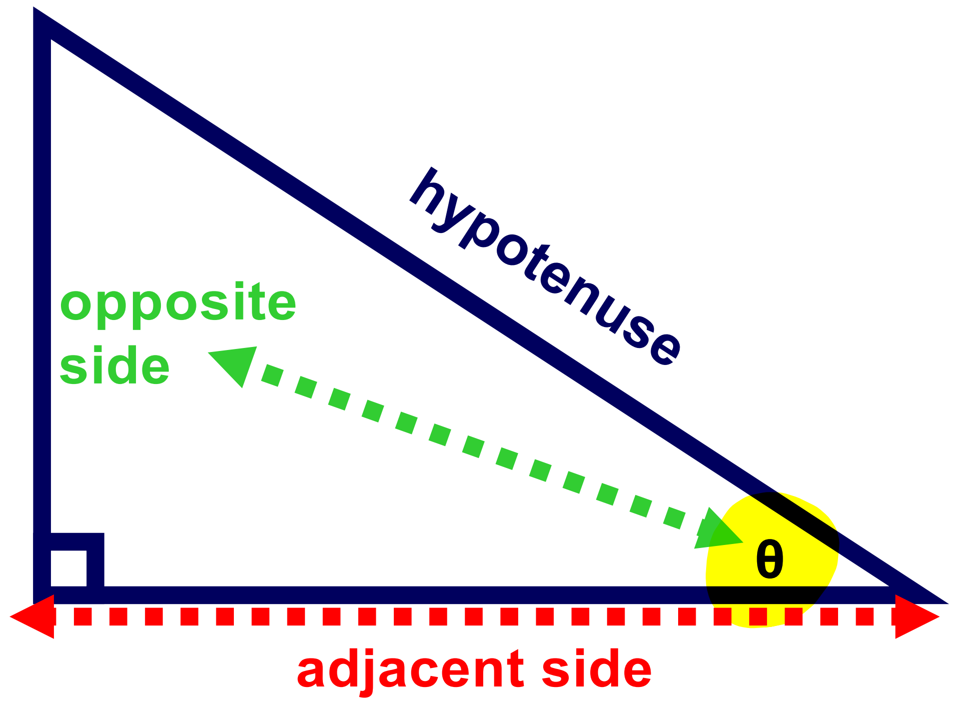
Tangent abbreviated as "tan", but
pronounced "tangent"

Greek letters like **θ,** "**theta**", and **β**, "**beta**", are often used to represent angles. Upper-case letters are also used.

sin *θ* means "the sine of the angle θ"
cos *θ* means "the cosine of the angle θ"
tan *θ* means "the tangent of the angle θ"

Trigonometric Ratios

In order to name the trig ratios,
you need a reference angle, θ.



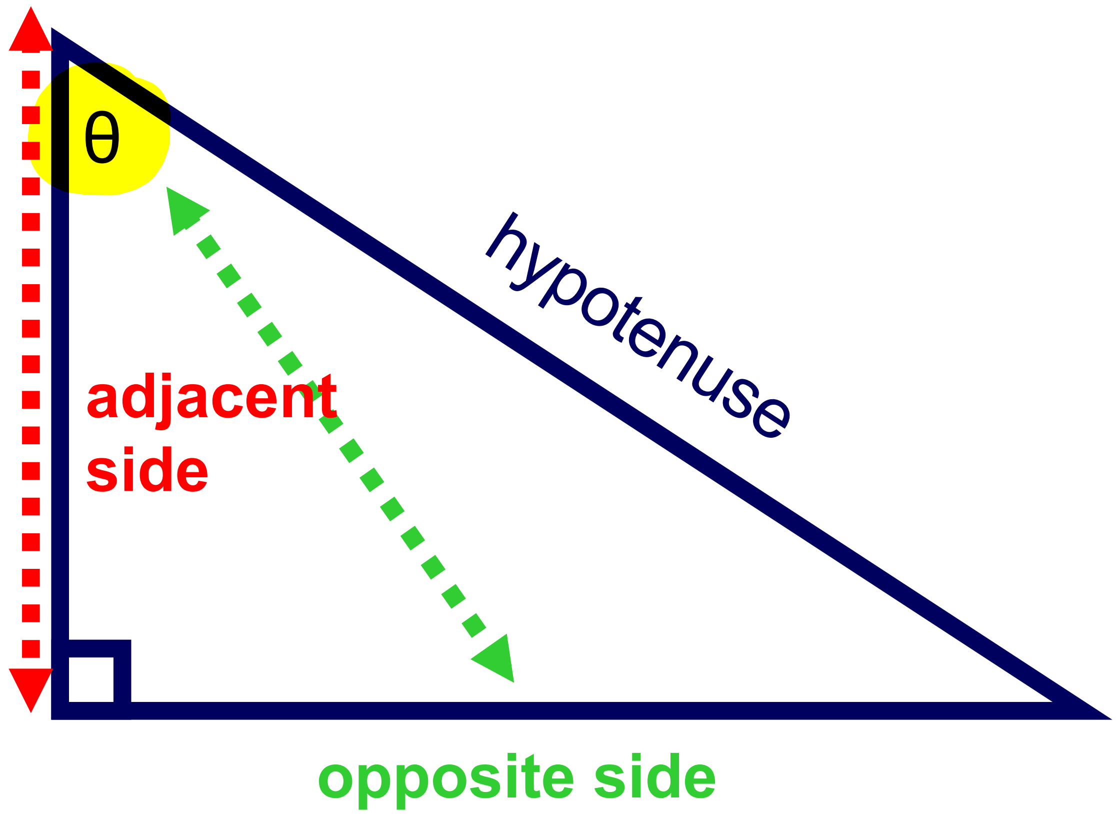
If θ is the reference angle, then

- the leg opposite θ is called the opposite side. (You have to cross the triangle to get to the opposite side.)

- the adjacent side is one of the sides of θ, but not the hypotenuse

- the side opposite the right angle is the hypotenuse

Trigonometric Ratios

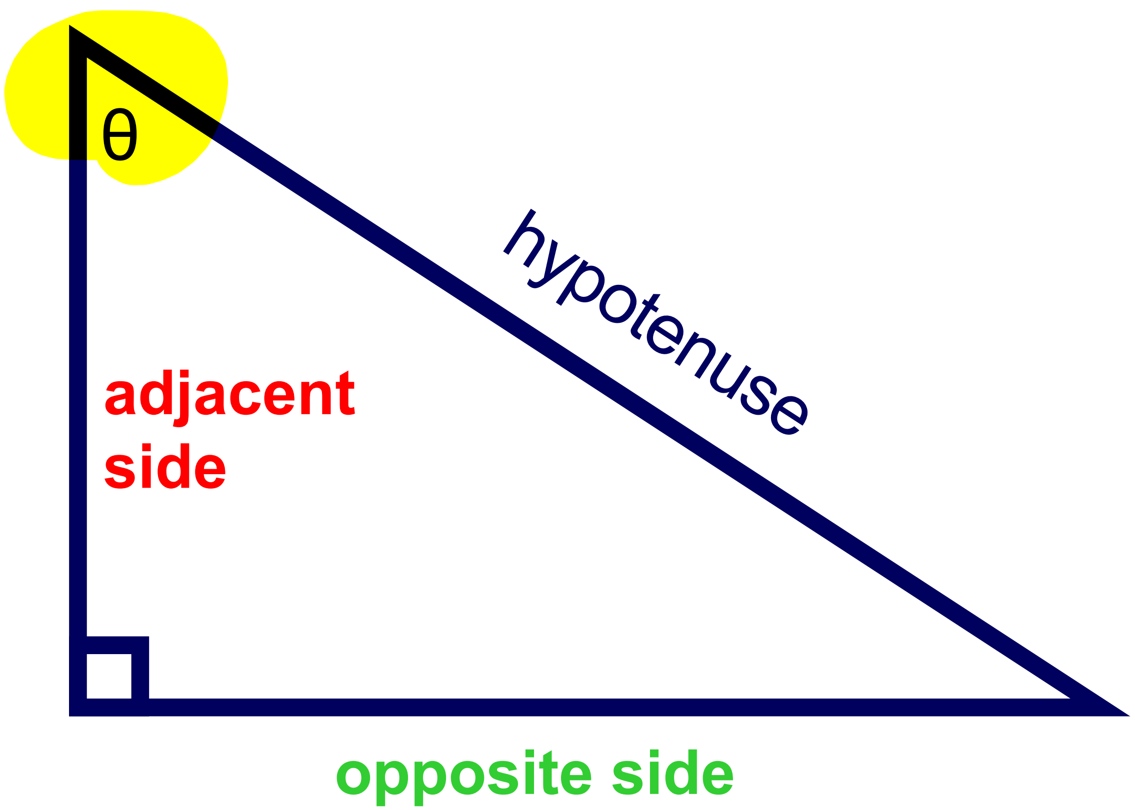


Notice what happens when θ is the other angle.

If the other angle is our reference angle θ, then the sides labeled as opposite and adjacent switch places.

The hypotenuse is always the hypotenuse.

Trigonometric Ratios



$$\sin\,\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{\text{opp}}{\text{hyp}}$$

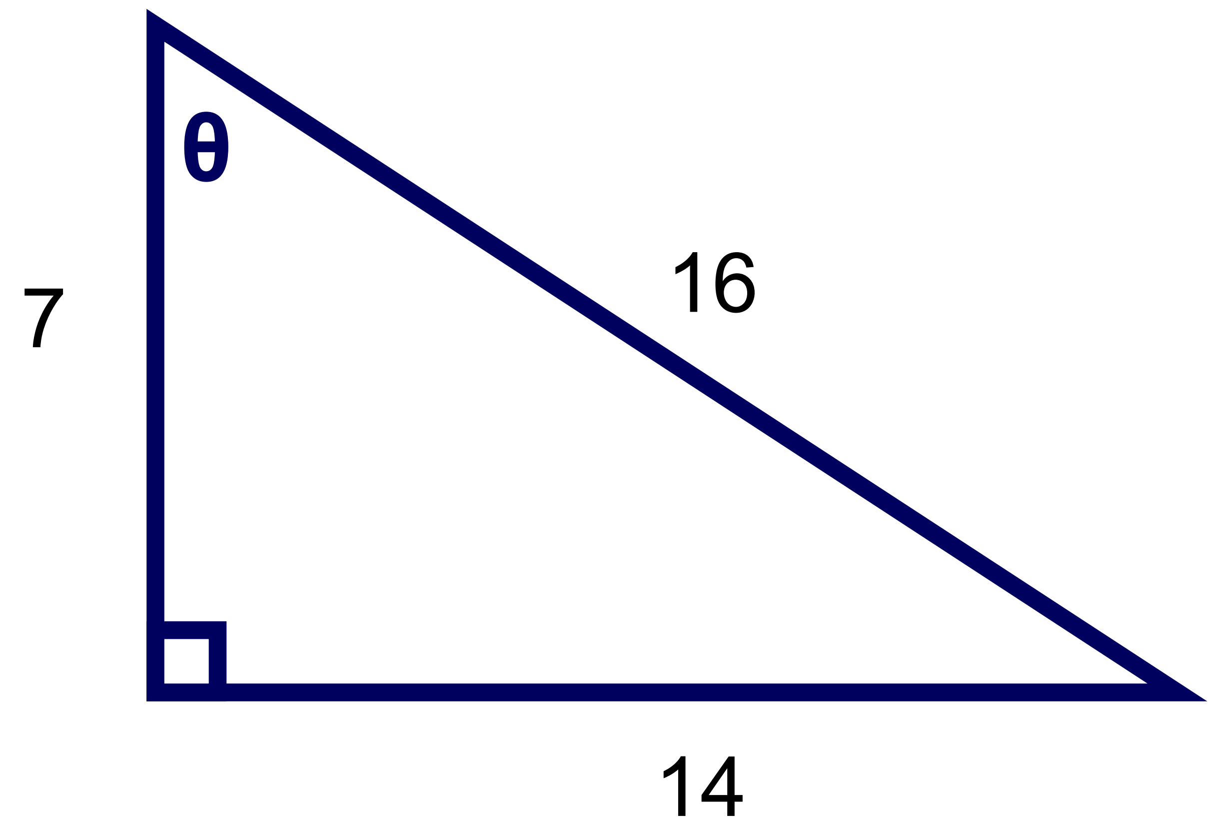
$$\cos\,\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\,\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{\text{opp}}{\text{adj}}$$

SOH-CAH-TOA: use this acronym to remember the trig ratios

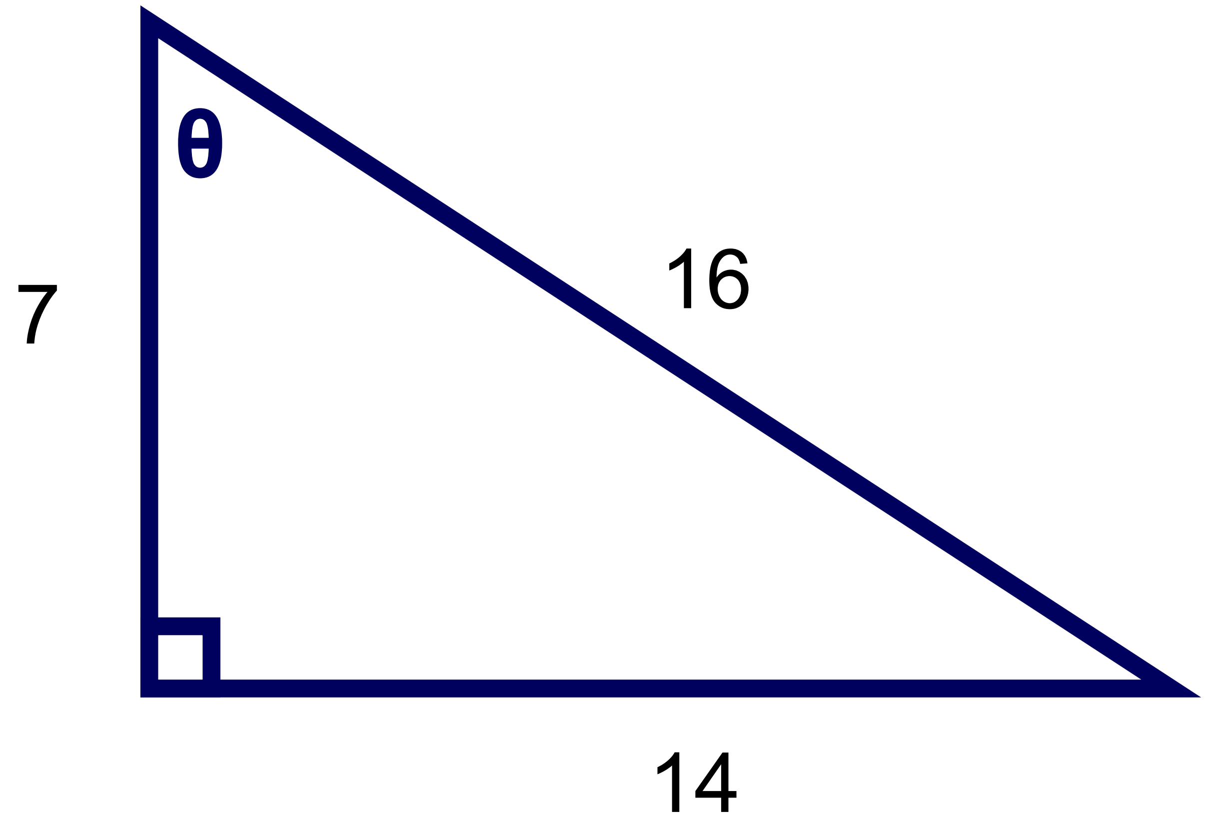
For any right triangle with angle θ the ratios will be equal.

1 What is the ratio for sin θ?



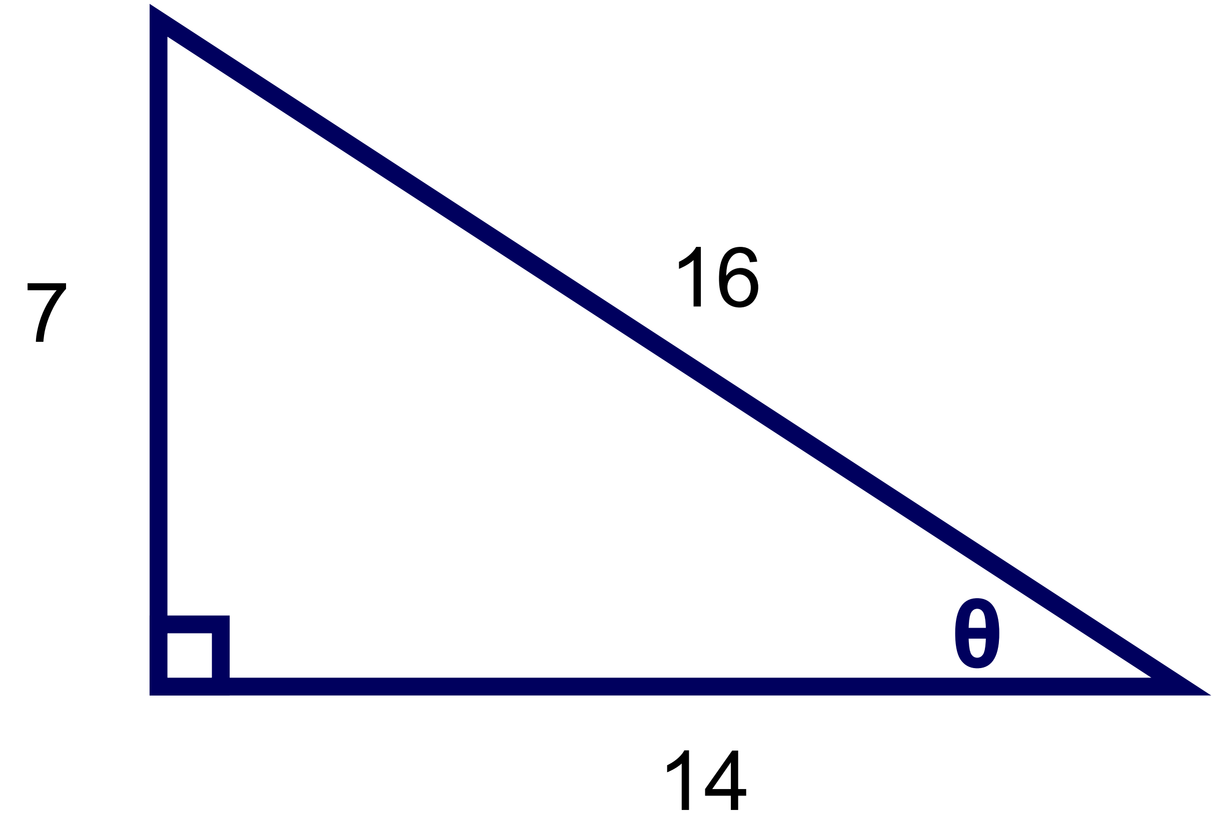
Answer

2 What is the ratio for cos θ ?



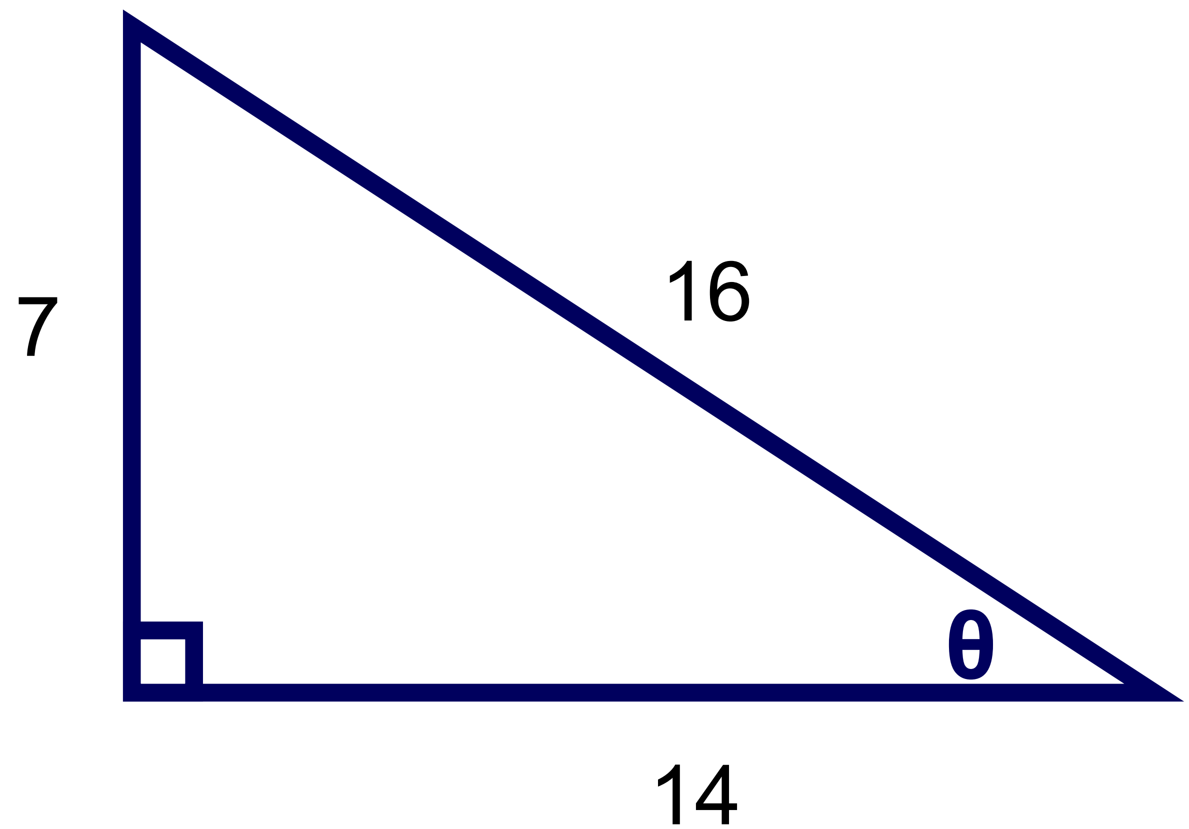
Answer

3 What is the ratio for tan θ?



Answer

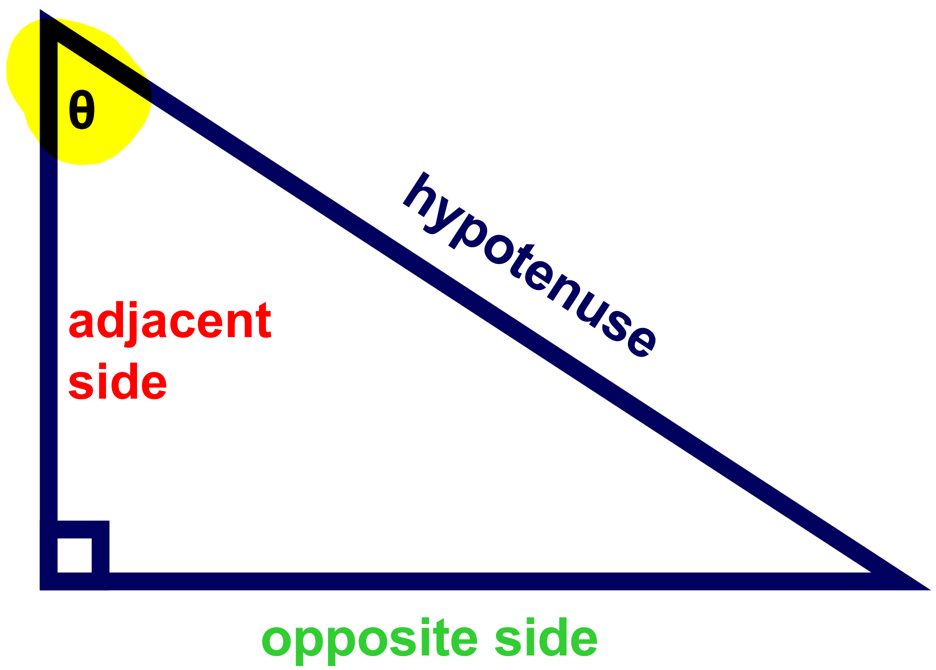
4 What is the ratio for cosθ?



Answer

Reciprocal Trig Functions

There are three more ratios that can be created comparing the sides of the triangle, **cosecant** (csc), **secant** (sec), and **cotangent** (cot):



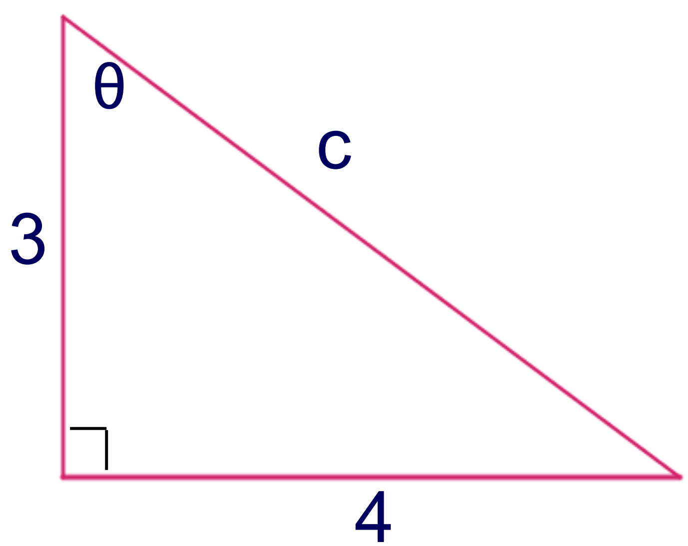
$$\csc\,\theta = \frac{1}{\sin\theta} = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{\text{hyp}}{\text{opp}}$$

$$\sec\,\theta = \frac{1}{\cos\theta} = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{\text{hyp}}{\text{adj}}$$

$$\cot\,\theta = \frac{1}{\tan\theta} = \frac{\text{adjacent}}{\text{opposite}} = \frac{\text{adj}}{\text{opp}}$$

Evaluating Trig Functions

Example: Find the values of the six trig functions of θ in the triangle below.



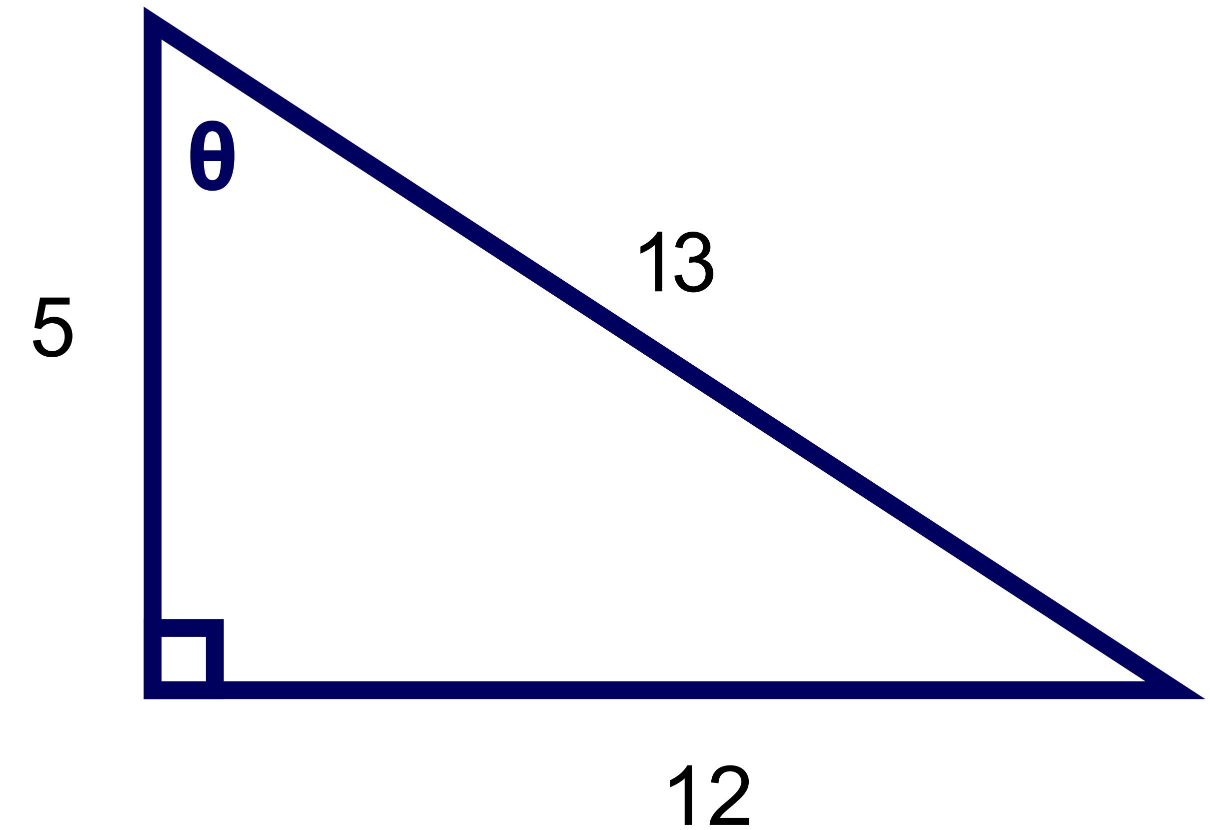
Solution: Use the Pythagorean Theorem to find the missing side: $3^2 + 4^2 = c^2$, so $c = 5$.

$\sin\theta = \dfrac{4}{5}$ $\qquad$ $\csc\theta = \dfrac{5}{4}$

$\cos\theta = \dfrac{3}{5}$ $\qquad$ $\sec\theta = \dfrac{5}{3}$

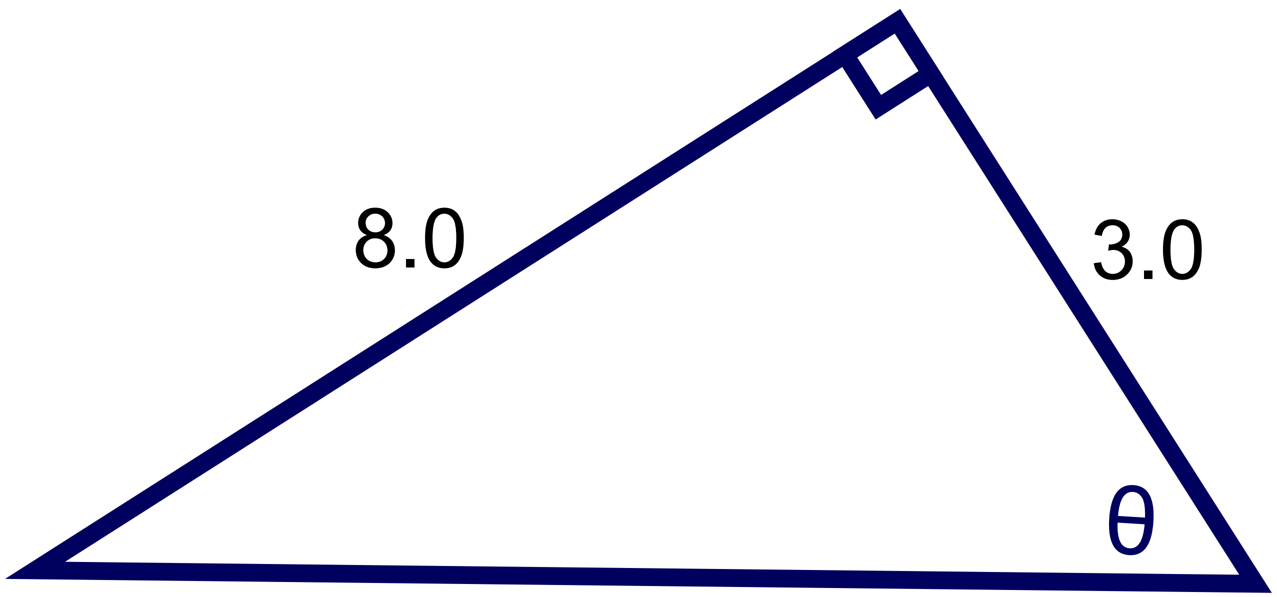
$\tan\theta = \dfrac{4}{3}$ $\qquad$ $\cot\theta = \dfrac{3}{4}$

5 What is the ratio for sec θ?



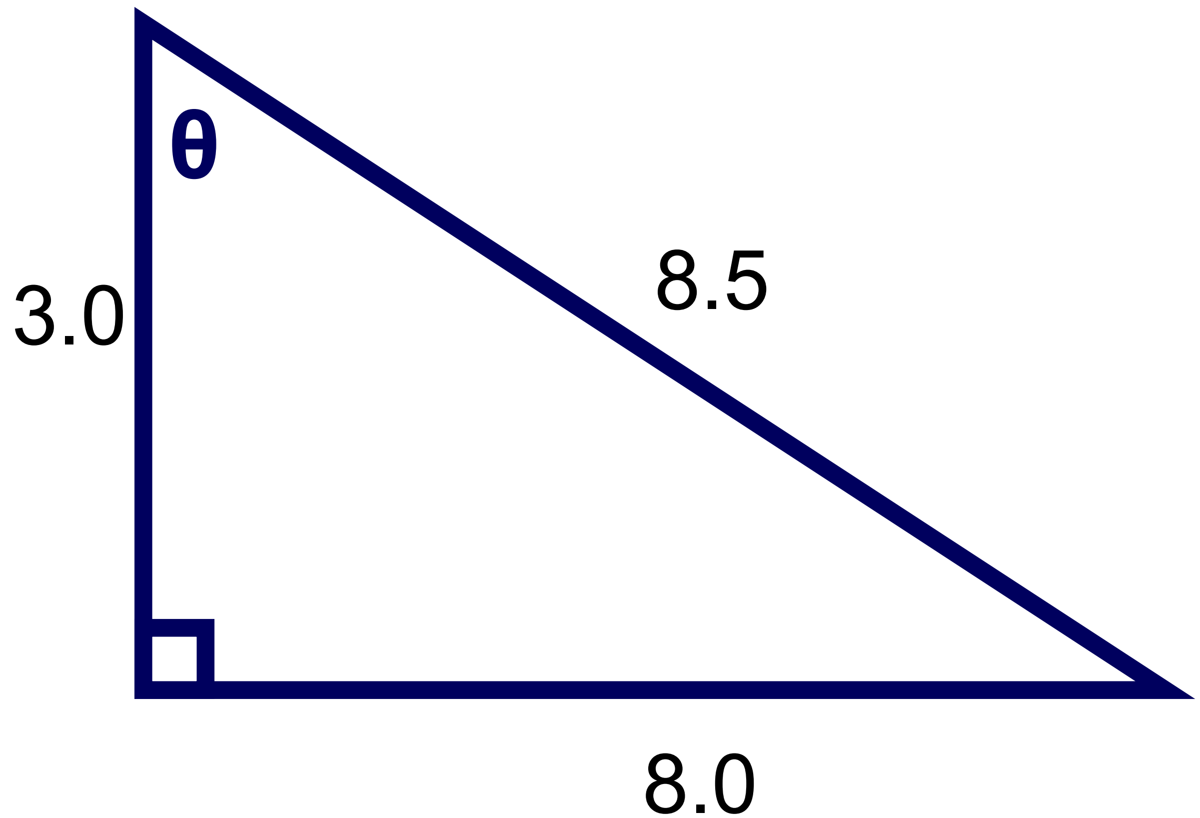
Answer

6 What is the ratio for sinθ? Enter your answer in decimal form.



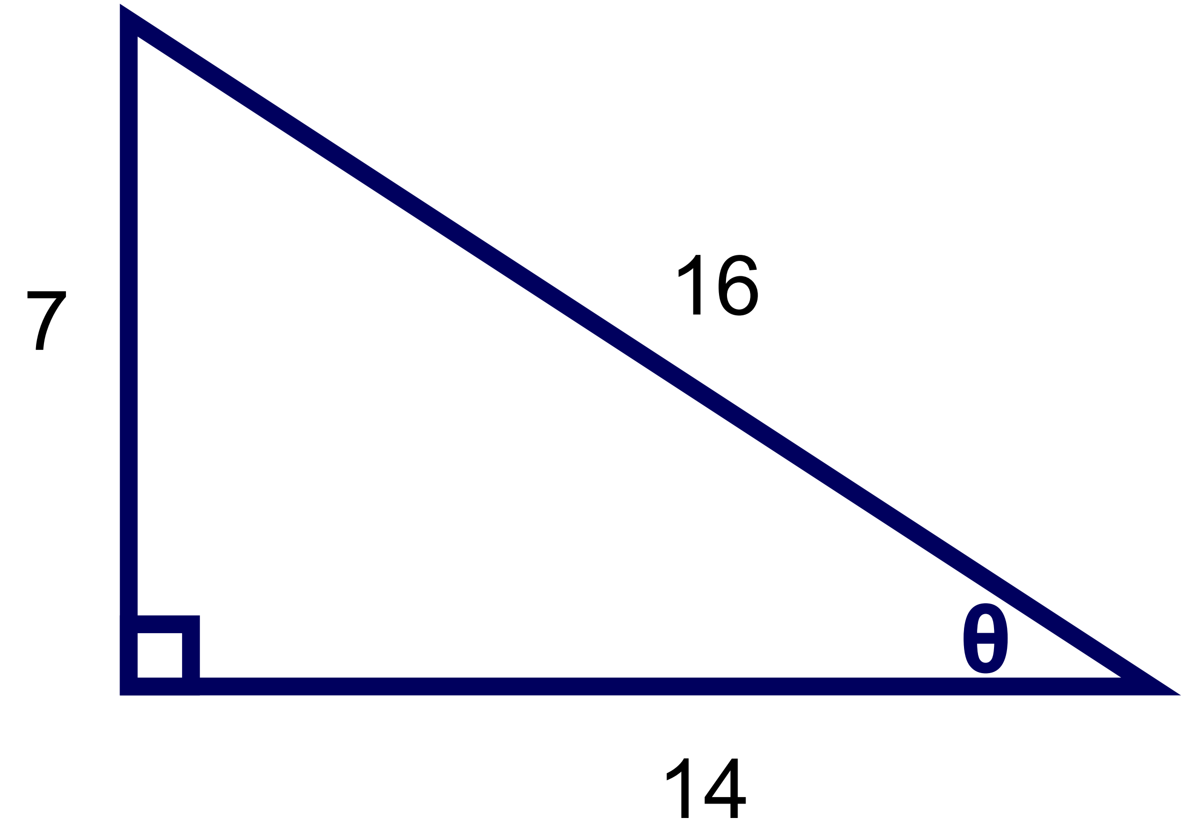
Answer

7 What is the ratio for cot θ?



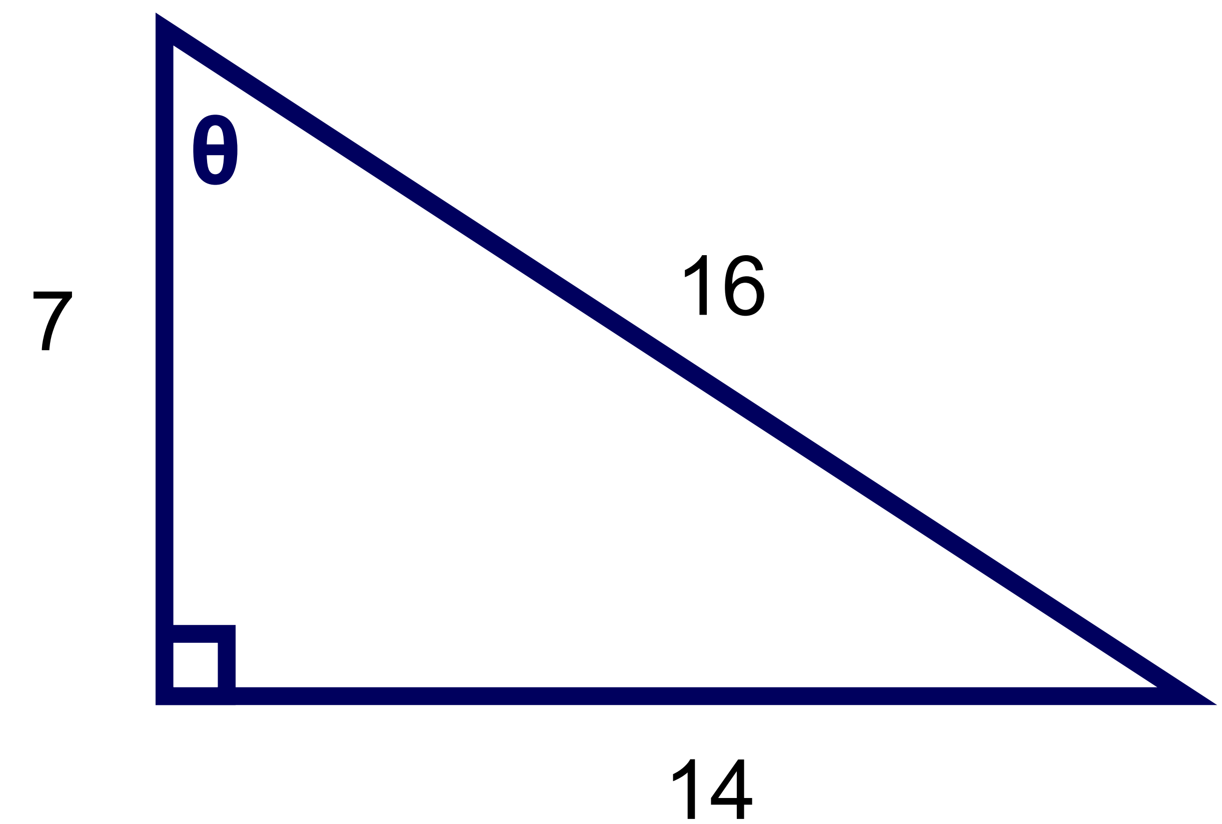
Answer

8 What is the ratio for cot θ?



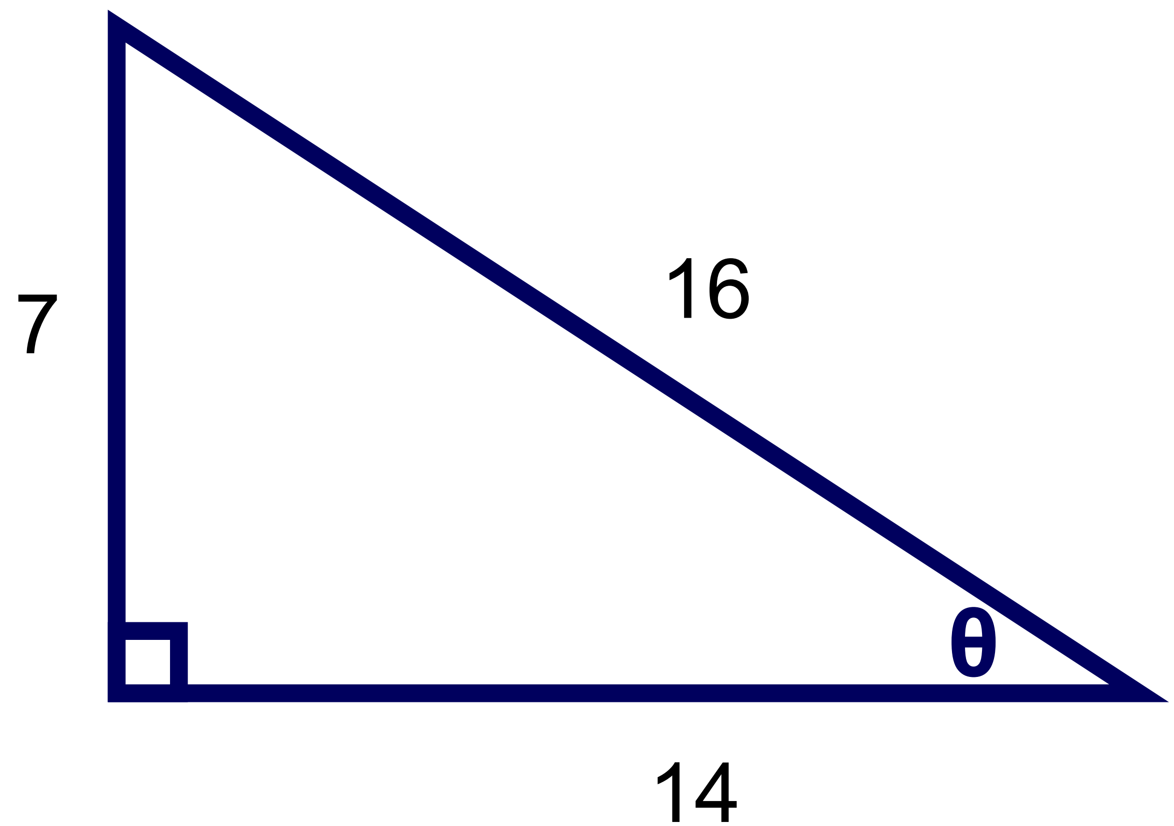
Answer

9 What is the ratio for csc θ?



Answer

10 What is the ratio for sec θ?

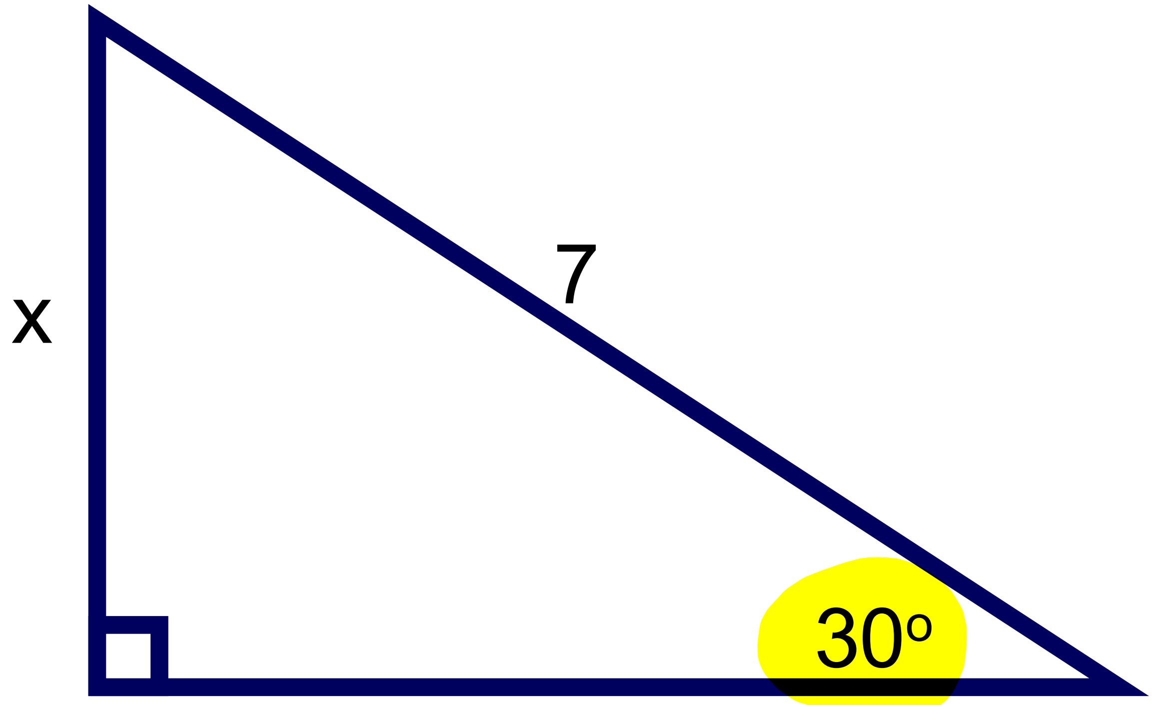


Answer

Using Trig Ratios

If you know the length of a side and the measure of one of the acute angles in a right triangle, you can use trig ratios to find the other sides.

Trigonometric Ratios



For example, let's find the length of side x.

The side we're looking for is opposite the given angle;

and the given length is the hypotenuse;

so we'll use the trig function that relates these two:

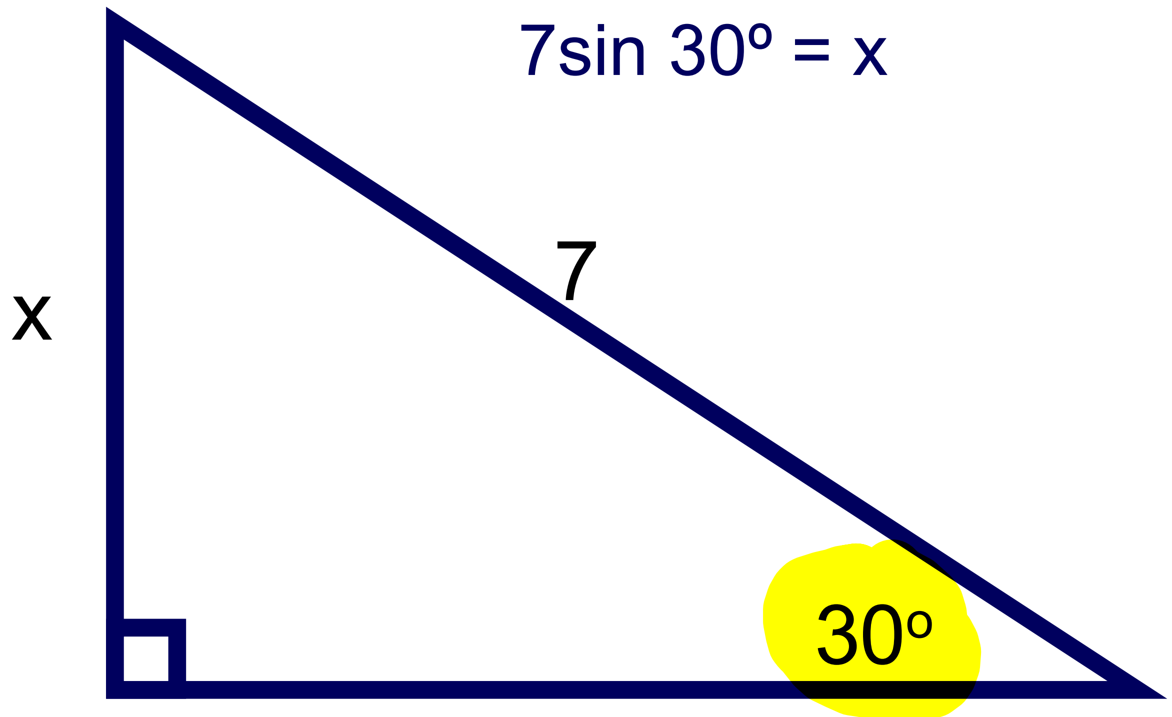
$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{\text{opp}}{\text{hyp}}$$

(continued on next slide)

Trigonometric Ratios

$\sin 30^\circ = \frac{x}{7}$

$7\sin 30^\circ = x$

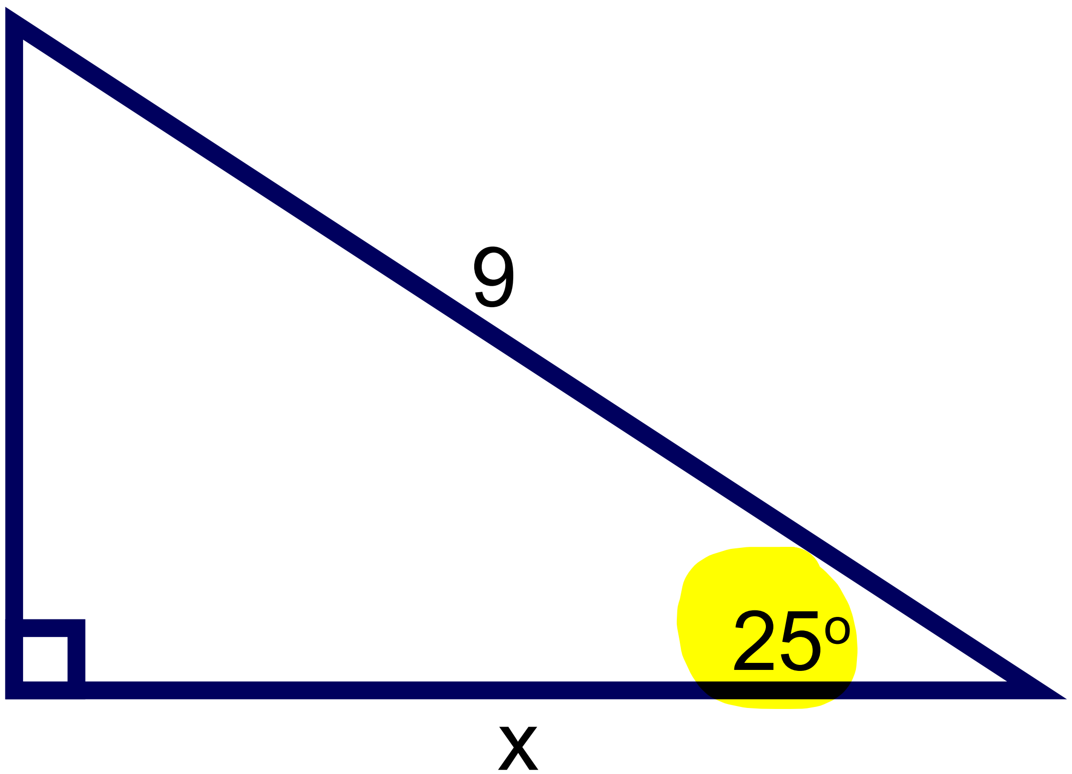


sin 30º is always equal to the same number, regardless of the size of the triangle. To find the value of sin 30º, we can use a calculator that has trig functions.

$\sin 30^\circ = 0.5$,
so $x = 7(0.5) = 3.5$

NOTE: Be sure your calculator is set to degree mode.

Trigonometric Ratios



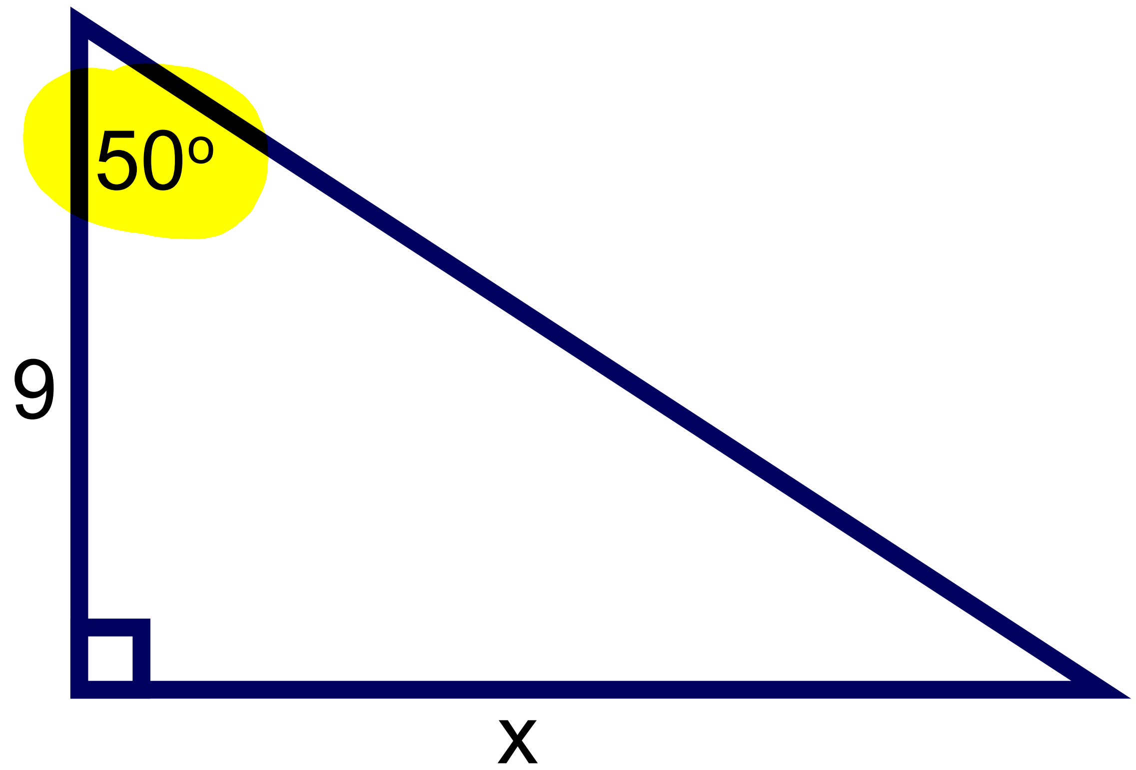
Example 2: Find the value of x.

The side we're looking for is adjacent to the given angle and the given length is the hypotenuse
so we'll use the trig function that relates these two:

$$\cos\theta = \frac{adj}{hyp}$$

click to reveal

Trigonometric Ratios



Example 3: Find the value of x.

We are looking for the opposite side, and are given the adjacent side.

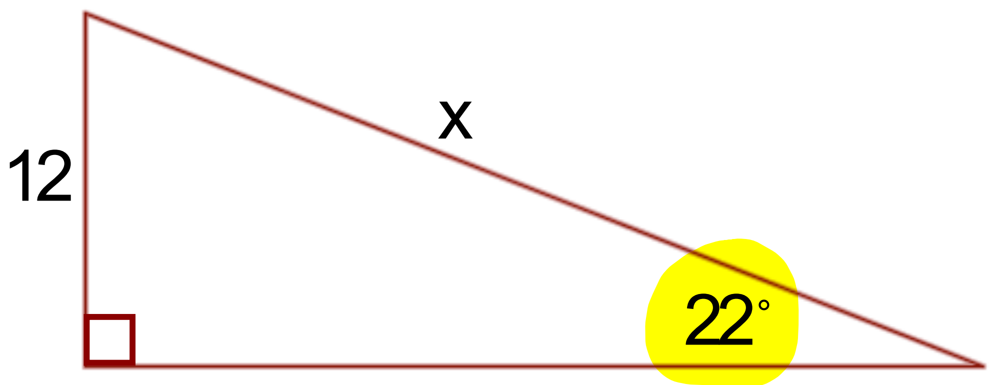
The trig function that relates these is tangent:

$$\tan\theta = \frac{opp}{adj}$$

click to reveal

Trigonometric Ratios

Example: Find the value of x.



We are looking for the opposite side, and are given the hypotenuse.

The trig function that relates these is sine:

This time the x is on the bottom. To solve we would multiply both sides by x and then divide by sin 22º. (Remember this short cut: switch the x and the sin 22º.)

$$\sin \theta = \frac{opp}{hyp}$$

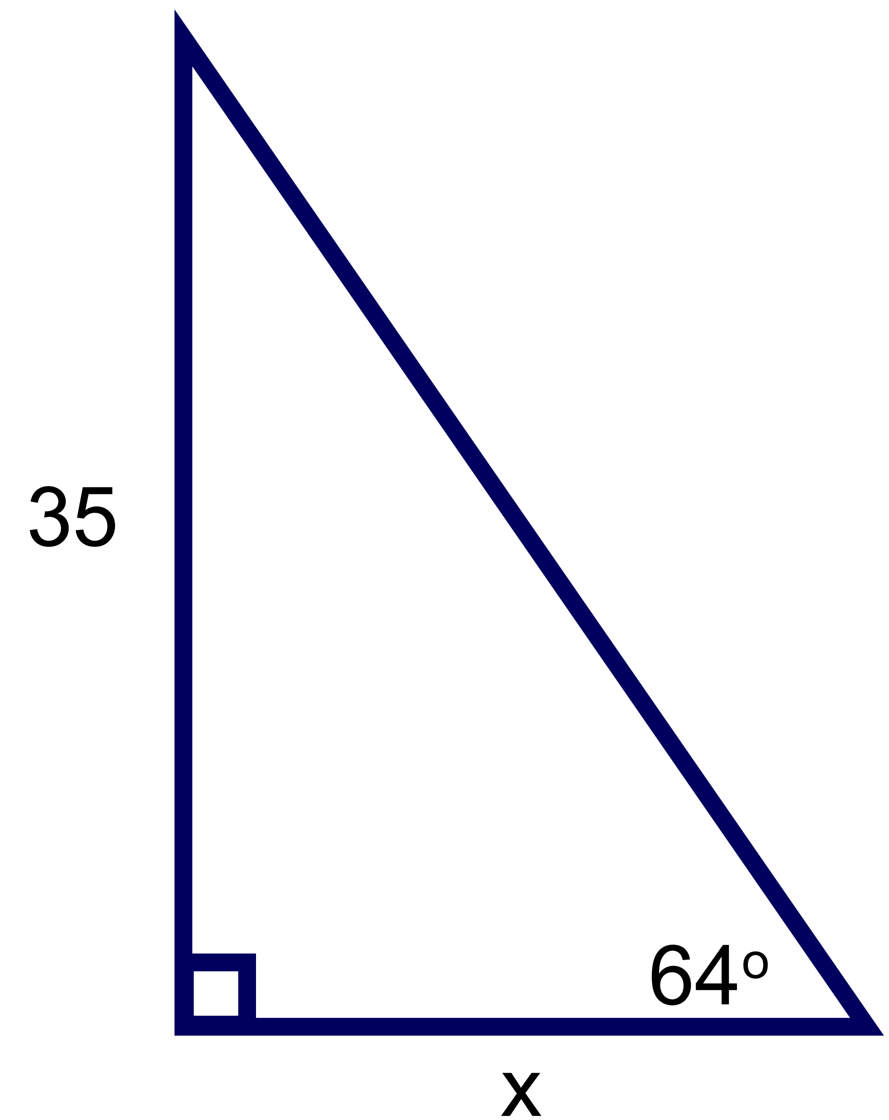
$$\sin 22^\circ = \frac{12}{x}$$

$$x = \frac{12}{\sin 22^\circ}$$

Enter this into the calculator

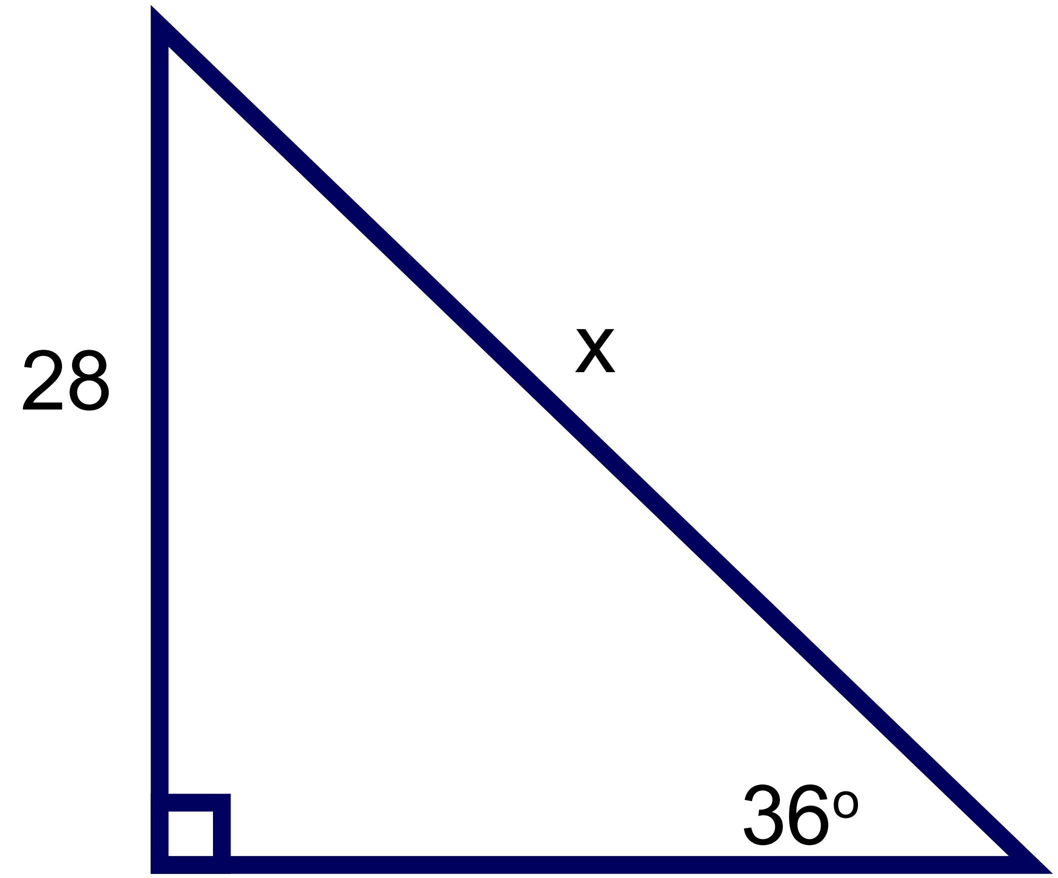
$$x \approx 32$$

11 What is the value of x?



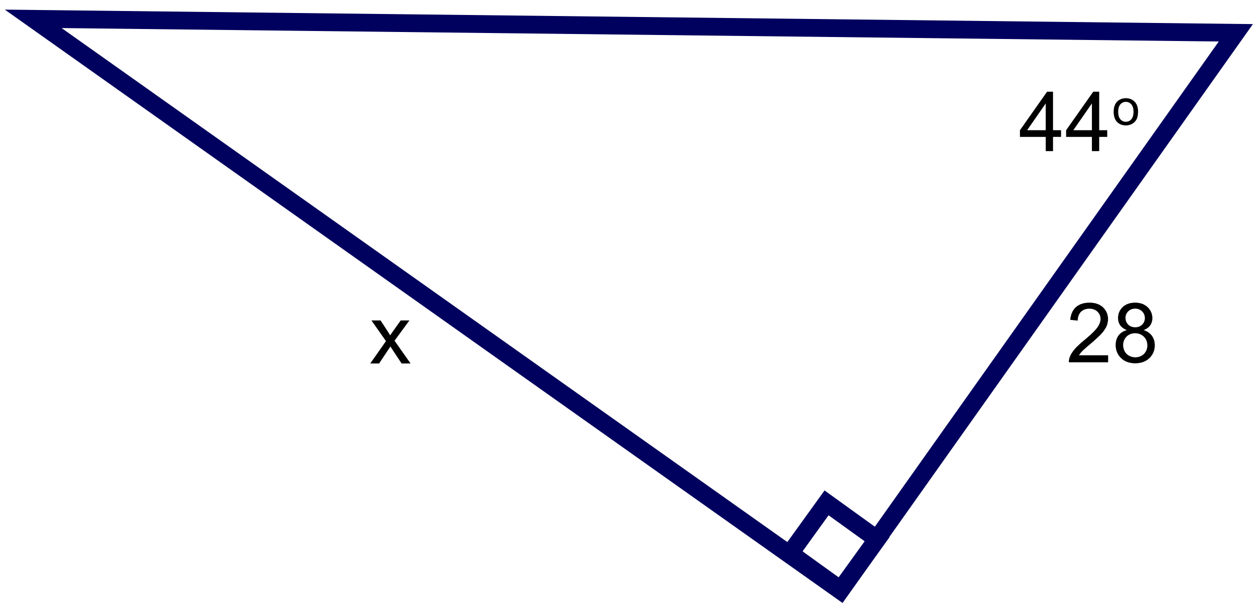
Answer

12 What is the value of x?



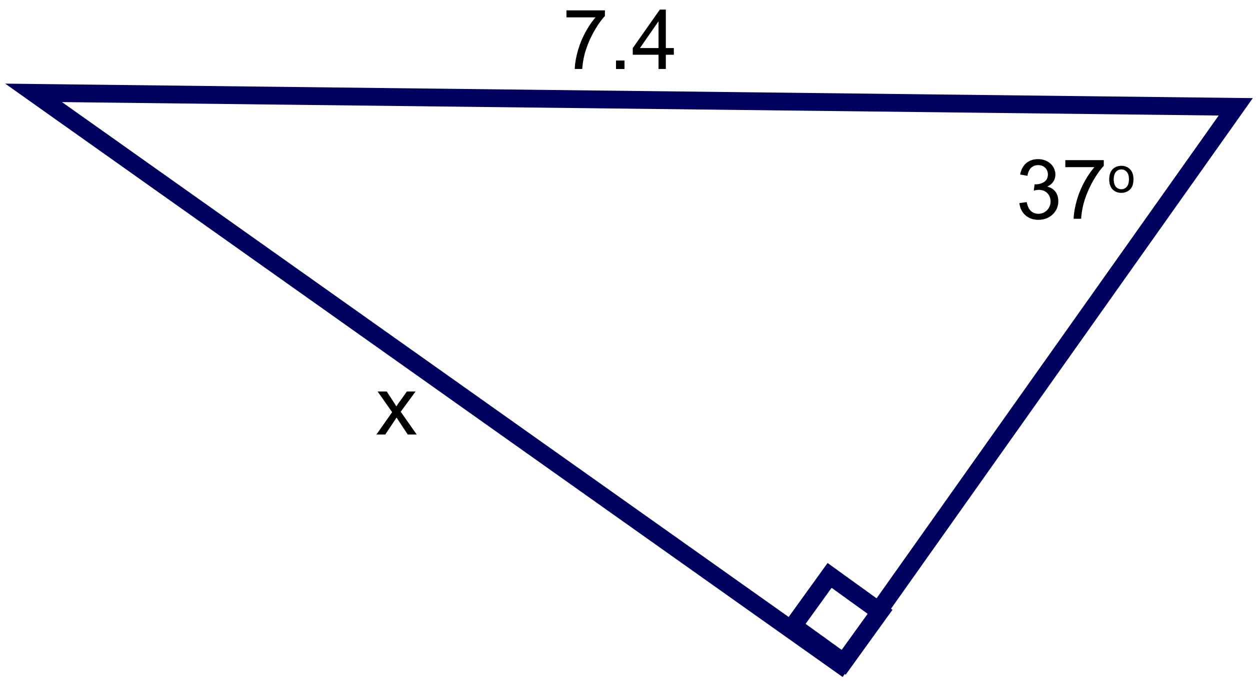
Answer

13 What is the value of x?



Answer

14 What is the value of x?



Answer