

Working with Rational Expressions

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Goals and Objectives

- Students will simplify rational expressions, as well as be able to add, subtract, multiply, and divide rational expressions.
- Students will solve rational equations and use them in applications.
- Students will graph rational functions and identify their holes, vertical asymptotes, and horizontal asymptotes.

What is a rational expression?

A **rational expression** is the ratio of two polynomials. It is written as a fraction with polynomial expressions in the numerator and denominator.

Why do we need this?

Rational expressions are often used to simplify expressions with long polynomials in both the numerator and denominator.

Since it is more efficient to work with simple problems and situations, knowing how to simplify rational expressions makes looking at graphs and other problems easier.

Rational expressions and equations are often used to model more complex equations in fields such as science and engineering. Rational expressions are applicable in working with forces and fields in physics and aerodynamics.

Vocabulary Review

Recall from Algebra I

A **monomial** is a one-term expression formed by a number, a variable, or the product of numbers and variables.

$$81y^4z \qquad \frac{rt}{6} \qquad 32,457$$

A **polynomial** is an expression that contains one or more monomials.

$$8a^3 - 2b^2 \qquad \frac{rt}{6} + \frac{a^4b}{15} \qquad 4c - mn^3$$

Vocabulary Review Continued

A **rational expression** is an expression that can be written in the form $\frac{\textit{polynomial}}{\textit{polynomial}}$, where a variable is in the denominator.

The **domain** of a rational expression is all real numbers excluding those that would make the denominator 0.

For example, in the expression $\frac{3}{x-2}$, 2 is restricted from the domain because it would create an undefined term.

$$\frac{3}{x-2} = \frac{3}{2-2} = \frac{3}{0}$$

Simplifying Rational Expressions

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Simplifying Rationals

A **rational expression** is an expression that can

be written in the form $\frac{\textit{polynomial}}{\textit{polynomial}}$, where a variable

is in the denominator.

The domain of a rational expression is all real numbers excluding those that would make the denominator 0. (This is very important when solving rational equations.)

For example, in the expression $\frac{3}{x^2 - 4}$, 2 and -2 are restricted from the domain.

Simplifying Rationals

Simplifying rational expressions is similar to reducing fractions: multiplying or dividing by 1 yields an equivalent expression.

$$\frac{28}{35} = \frac{4 \cdot 7}{5 \cdot 7} = \frac{4}{5} \cdot \frac{7}{7} = \frac{4}{5}$$

$$\frac{3m^5}{12m^2} = \frac{\cancel{3} \cdot 1 \cdot \cancel{m^2} \cdot m^3}{\cancel{3} \cdot 4 \cdot \cancel{m^2}} = \frac{m^3}{4}$$

To shorten our work, we can recall the rules of exponents: $\frac{a^m}{a^n} = a^{m-n}$

Simplifying Rationals

Monomials: reduce coefficients, simplify variables by subtracting exponents.

$$\frac{4a^2b^3}{8a^6b} =$$

click

$$\frac{(3x^2y^3)^4}{18x^5y^{10}} =$$

click

Other Polynomials: factor first, then simplify.

$$\frac{x^2 - 9}{x^2 - 6x + 9} =$$

click

Discuss: How can you check your results?

Simplifying Rationals

Remember to use properties of exponents and/or factoring to simplify the rational expressions.

$$\frac{3a^4b^7}{6a^8b^5}$$

$$\frac{(2xy^3)^3}{4x^2y^7}$$

$$\frac{x^2 - 16}{x^2 - 6x + 8}$$

5 Simplify $\frac{12x^4y^{10}}{24x^5y^9}$

A $\frac{2y}{x}$

B $\frac{1}{2}$

C $\frac{y}{2x}$

D $2xy$

Answer

6 Simplify $\frac{(4x^2y^3)^2}{24x^3y^3}$

A $\frac{xy^3}{3}$

B $\frac{2xy^3}{3}$

C $\frac{2y^3}{3x}$

D $\frac{y^3}{3}$

Answer

7 Simplify

$$\frac{x^2 + 2x - 15}{x^2 - 10x + 21}$$

A $12x - 2$

B $\frac{x + 5}{x - 7}$

C $\frac{x + 5}{x + 7}$

D $-12x + 2$

Answer

8 Simplify:
$$\frac{2x^3 + 5x^2 - 12x}{2x^2 + 3x - 20}$$

A $\frac{2x^2 - 3x}{2x - 5}$

B $-3x + 1$

C $\frac{2x^2 - 4x}{5}$

D $\frac{x - 3}{x - 5}$

Answer

9 Simplify: $\frac{3(x^3 - x^2 - 20x)}{2(x^3 - 3x^2 - 10x)}$

A $\frac{3}{2}$

B $\frac{x + 4}{x + 2}$

C 1

D $\frac{3(x + 4)}{2(x + 2)}$

Answer

Multiplying Rational Expressions

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Multiplying Rational Expressions

Recall when multiplying fractions, it's helpful to simplify before multiplying.

Example:

$$\frac{45}{8} \cdot \frac{4}{9} = \frac{5 \cdot \cancel{9}}{2 \cdot \cancel{4}} \cdot \frac{\cancel{4}}{\cancel{9}} = \frac{5}{2}$$

The same applies when working with rational expressions, though you may need to factor a polynomial and/or use rules of exponents to simplify.

Example:

$$\frac{15x^4}{5x^3 + 15x} = \frac{15x^4}{5x(x^2 + 3)} = \frac{3x^3}{x^2 + 3}$$

Discuss: Can you check the result?

Simplify First

When rational expressions are multiplied, multiply the numerators together and the denominators together.

Hint: Simplifying BEFORE you multiply will shorten your work.

Example:

Multiplying: $\frac{\text{top} \bullet \text{top}}{\text{bottom} \bullet \text{bottom}} \rightarrow \text{simplify...}$

$$\frac{8x^2}{9y^3} \cdot \frac{3y^2}{4x^3} = \frac{8x^2 \cdot 3y^2}{9y^3 \cdot 4x^3} = \frac{2}{3xy}$$

$$\frac{2}{3y} \cdot \frac{1}{x} = \frac{2}{3xy}$$

Factor First

Sometimes you must factor first in order to simplify.

$$\frac{4x^2}{4x^2 - 1} \cdot \frac{2x + 1}{2x} = \frac{2x}{(\cancel{2x + 1})(2x - 1)} \cdot \frac{\cancel{2x + 1}}{1} = \text{click}$$

$$\frac{x^2 + x - 2}{xy^2} \cdot \frac{x^3 y}{x^2 + 5x + 6} = \frac{(\cancel{x + 2})(x - 1)}{y} \cdot \frac{x^2}{(x + 3)(\cancel{x + 2})} = \text{click}$$

Multiply

$$\frac{3k^2m}{4ab^3} \cdot \frac{8ab}{9km} =$$

$$\frac{4x^2}{9x^2 - 1} \cdot \frac{3x + 1}{2x} =$$

Multiply

$$\frac{n^3 - 16n}{n^2 - 6n + 8} \cdot (n^2 - 4) =$$

10 Simplify $\frac{8x^2}{12xy} \cdot \frac{36y^2}{6x} =$

A $\frac{4}{x}$

C $4xy$

B $\frac{4x}{y}$

D $4y$

Answer

11 Simplify $\frac{18a^3b^2}{27a^2b^2} \cdot \frac{30a^2b}{6ab^2} =$

A $\frac{10a^2b}{3}$

C $\frac{5a^2b}{6}$

B $\frac{10a^2}{3b}$

D $\frac{10a}{3b}$

Answer

12 Simplify $\frac{x+5}{x^2} \cdot \frac{x}{x^2-25} =$

A $\frac{1}{x^2-5x}$

C x^2-5

B $\frac{1}{x^2-5}$

D x^2-5x

Answer

13 Simplify $\frac{m^2 + m - 12}{m^2 - 9} \cdot \frac{m + 3}{m^2 - 16} =$

A $\frac{1}{m + 4}$

C $m + 4$

B $\frac{1}{m - 4}$

D $m - 4$

Answer

Dividing Rational Expressions

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Dividing Rational Expressions

When dividing rational expressions, multiply by the reciprocal.

Examples:

$$\textit{copy} \bullet \textit{flip} \rightarrow \frac{\textit{top} \bullet \textit{top}}{\textit{bottom} \bullet \textit{bottom}} \rightarrow \textit{simplify}$$

$$\frac{x^2}{b} \div \frac{b^2}{x} = \frac{x^2}{b} \bullet \frac{x}{b^2} =$$

click

$$\frac{\frac{x}{x+4}}{\frac{x+3}{x+4}} = \frac{x}{x+4} \div \frac{x+3}{x+4} = \frac{x}{\cancel{x+4}} \bullet \frac{\cancel{x+4}}{x+3} =$$

click

14 Simplify $\frac{7}{m^2} \div \frac{14}{m} =$

A $\frac{98}{m^3}$

C $\frac{1}{2m}$

B $\frac{m}{2}$

D $\frac{m^3}{98}$

Answer

15 Simplify $\frac{y-4}{9} \div \frac{4-y}{3} =$

A $\frac{y-4}{3(y-4)}$

C $\frac{y^2-16}{27}$

B $-\frac{1}{3}$

D $\frac{1}{3}$

Answer

16 Simplify $\frac{j^2 - 9}{j + 3} \div \frac{j^2 + 2j - 15}{j - 1} =$

A $\frac{j - 1}{j + 5}$

C $\frac{j - 1}{j - 2}$

B $\frac{j - 2}{j - 1}$

D $\frac{j - 5}{j - 1}$

Answer

17 Simplify $\frac{3a^2 - 12}{2a^2 + 3a - 2} \div \frac{9a}{4a^2 - 2a} =$

A $\frac{3(a + 2)}{2(2a + 1)}$

C $\frac{3(a + 2)}{2}$

B $\frac{3(2a + 1)}{2(a - 1)}$

D $\frac{2(a - 2)}{3}$

Answer

18 Simplify $\frac{e^2 - f^2}{\frac{ef}{\frac{e-f}{e}}} =$

A $\frac{f}{e-f}$

C $\frac{e}{e+f}$

B $\frac{e+f}{e}$

D $\frac{e+f}{f}$

Answer